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SIMULATED ANNEALING FOR SELECTION OF EXPERIMENTAL REGIONS IN RESPONSE SURFACE METHODOLOGY APPLICATIONS

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ABSTRACT

In this paper we describe a methodology that includes the complementary use of simulated annealing and response surface methodology (RSM). The methodology was developed for analysis of simulations to help determine procedures for the employment of superheterodyne surveillance receivers. In this methodology, we use simulated annealing to determine near optimal solutions and to help select an initial search region from which to begin experimentation and analysis. By using this technique, we are able to take the results of an otherwise obscure function, over a limited range of the variable values, and develop a simplified, more understandable model which closely represents the actual system over the limited solution space.

1 INTRODUCTION

Advanced superheterodyne surveillance receivers may be used to determine the location of an emitter whose location was previously unknown. Effective, accurate use of such receivers in this capacity is heavily dependent on the geometry, velocity, and direction of movement among the receivers and the emitter. The problem addressed in this paper is to determine techniques which describe the combinations of factor values that will typically result in accurate identification of the location of emitters. The problem is set up so that the factors describing the employment of the receivers are constrained to a specified operability region. The emitters of concern are assumed to be located in an identified area where the receivers will focus. We used a computer simulation model to evaluate the relative accuracy of a given set of factors which describe the employment of the receivers. At one stage in the methodology, this model is used as an evaluation function for optimization techniques. At another stage, the model is used as a response for experimentation.

In this paper, we describe a methodology developed to determine techniques for employing superheterodyne surveillance receivers. The purpose of the methodology is twofold. First, we desire to gain an increased understanding of the relationships between accuracy and the employment factors involved with such receivers. Second, we wish to determine "optimal" or near "optimal" settings of factors over which we have control, (Brown and Schamburg 1997). In this methodology, we use Simulated Annealing to determine near optimal solutions and to help select an initial search region from which to begin experimentation and analysis. In order to increase understanding, we simplify a relatively complex system and its relationships through empirical model-building and the use of graduating functions. By using this technique, we are able to take the results of an otherwise obscure function, over a limited range of the variable values, and develop a simplified, more understandable model which closely represents the actual system over the limited solution space. Although the simplified model does not perfectly resemble the theoretical function, its representation is close, considering a reasonable resolution of the response values. Analysis of this simplified model provides an increased understanding of the actual system and allows us to draw conclusions that may otherwise have been unknown. Additionally, it is easier to find "optimal" or near "optimal" solutions from such models. The graduating function allows us to check the robustness of good solutions. That is, we desire solutions where slight changes in the variable values still result in relatively good accuracy. To ensure the validity of the conclusions drawn, the empirical model, the results, and conclusions are checked at appropriate steps during the methodology. Finally, we attempt to generalize our findings so that they may be used as techniques for accurate emitter location surveillance.

2 METHODOLOGY

Using the methodology, we first specify the "issues for analysis" and determine the search region. At first, we make conjectures from an understanding of likely relationships to determine the "issues of analysis" and to pick a reasonable search region. As the effort continues, we consider a larger number of variables, a larger operability region, and use an understanding gained from the previous experimentation. For the larger, more complex cases, we determine the areas of the solution space from which to begin the next set of experiments by using simulated annealing. The simulated annealing algorithm used helps determine a good solution area from which to begin the experimentation for these larger cases. The methodology is iterative and as the experimentation progresses, understanding of the system increases, resulting in an updated set of issues and an updated search region from which to consider.

The methodology developed to deal with problems of this nature, is one in which the fundamentals of response surfaces are incorporated. The iterative process of learning is roughly formalized by (Box and Draper 1987) and consists of the repeated use of the steps, "conjecture, design, experiment, and analysis." In our empirical model-building, we first "conjecture" as to the form of the model which may be used to represent the system over a given portion of the solution space. We then "design" a "suitable experiment to test, estimate, and develop a current conjectured model." We conduct the "experiment" and then the "analysis," which leads to "verification of the postulated model and the working out of its consequences, or to the forming of a new or modified conjecture (Box and Draper 1987)." We then use this empirical model to conduct further analyses and to determine good settings for the factors considered. Finally, we attempt to generalize our findings to develop techniques.

In order to determine what levels of the control factors typically produce good results, which of these factors (and/or combinations of factors) have the greatest effect on accuracy, and to analyze the tradeoffs among these factors, an iterative methodology is used, the generalized steps of which are depicted in Figure 1, (Schamburg 1995), and (Brown and Schamburg 1997). The steps are briefly described as follows.

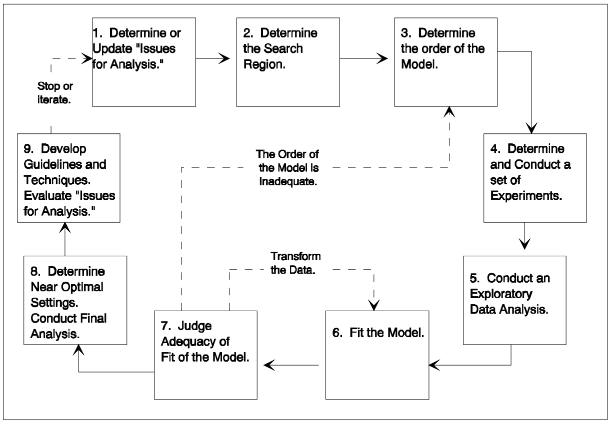

Figure 1: Diagram of the Developed Methodology, (Brown and Schamburg 1997)

Step 1. Determine or update the "issues for analysis." We first determine the issues and concepts upon which the study will focus. For example, we may be interested in finding the best deployments for airborne receivers; learning how the factors involved effect the response; how they interact; which are most important; and which settings for those factors are most favorable. The analysis at hand, in part, is defined by a situation describing the available assets, constraints, and the objective.

Step 2. Determine the search region. We determine a search region based on the focus at hand. Knowledge gained from previous iterations of the methodology is used to help determine the regions upon which the study will progress. In those larger, more complex cases, we determine the areas for experimentation through the use of simulated annealing.

Step 3. Determine the order of the model. We next conjecture as to the form of the model which may be used to represent the system over a given portion of the solution space. Appropriately determining the order of the model in this step leads to an appropriate set of experiments in the next step.

Step 4. Determine and conduct a set of experiments (computer simulations) that will yield measurements of the response of interest. The experiments are conducted through use of a computer simulation program. The program models the relative accuracy, given an given the employment factors for the receivers. This step includes determining which variables and what levels of these variables should be considered for an analysis.

Step 5. Conduct an exploratory data analysis. The exploratory data analysis is used to determine which factors and interactions are most important and how they affect the response. The exploratory data analysis, in part, addresses some of the issues and concerns brought forth at the beginning of the process. It also helps determine which terms may be most important in the model.

Step 6. Determine a mathematical model that best fits the data collected. This step requires the determination and fitting of an appropriate mathematical model from which to analyze the relationship between the input variables and the response variable. In most cases, we use least squares regression to fit the models.

Step 7. Judge the adequacy of fit of the model. The fit of the model is judged through use of statistical analysis, analysis of the mean square error, and residual analysis techniques.

1. If the model fails particular tests described in step 7, we may attempt to try a different transformation of the data or re-compute the model and return to step 7.
2. If the model does not satisfactorily predict the response, return to 3. above, make adjustments to the experiment and go through the sequence again to improve the model.
3. If the model is satisfactory, we continue the process and move to step 8.

Step 8. Determine optimal or near optimal settings and conduct final analysis. First, considering the model developed in step 6, we optimize the parameter values using linear or non-linear programming techniques. Of the factors considered, we determine which of these factors or combinations of factors have the greatest effect on the response and conduct a sensitivity analysis. The sensitivity analysis includes an analysis of the tradeoffs among these factors. We desire robust solutions. That is: solutions of concern are those in which slight deviations from the solution would still result in a relatively good response.

Step 9. Develop techniques and evaluate issues for analysis. Through the analysis and conclusions found in the steps above, we attempt to make generalizations that will be beneficial in planning and decision making for the employment of surveillance receivers. In this step we attempt to address the key issues and summarize the most important findings in our analysis.

The simulated annealing algorithm developed for step 2 determines near optimal deployments of the receivers and is used to help determine an appropriate initial search region for further steps in the methodology. Simulated annealing is an optimization technique based on concepts adapted from statistical mechanics (Brown, Pittard, and Sappington 1993). Annealing is a physical process in which the purpose is to minimize the free energy of a solid and thus reach a crystallized state with a perfect lattice. The process involves two steps. First, the temperature is increased to a maximum value at which the solid melts. Second, the temperature is decreased carefully until the particles arrange themselves in the ground state of the solid. The cooling must be done carefully so that the solid does not get trapped into locally optimal lattice structures with crystal imperfections. The converse of this process is known as quenching. The quenching process is one in which the temperature is instantaneously lowered and thus results in an unstable state (Aarts and Korst 1989). Inspired by process annealing, an important characteristic in simulated annealing is the selection of an appropriate cooling schedule so that the properties achieved are better than those obtained from quenching. (Brown 1994) gives the following linkages in an analogy between optimization problems and the simulation of annealing in solids, (Ignizio 1994).

1. Simulated Annealing → Discrete Optimization Procedure
2. Ground State → Global Optimum
3. Metastable States → Local Optima
4. Energy → Cost

The typical simulated annealing procedure requires an iterative sequence of the following steps until some stopping criterion is met.

Step 1. Select the initial parameters and the current solution.

Step 2. Obtain a neighbor solution to the current solution.

Step 3. Evaluate the neighboring solution against the current solution. If it is better, then make it the current solution. If it is not better, make it the current solution according to some probability (otherwise keep the current solution).

Step 4. Revise the parameters and return to step 2 (Brown 1994) and (Ignizio 1994).

Three of the important parameters for the simulating annealing procedure include:

1. Initial temperature - controls the initial probability of accepting a non improving solution in step 3.
2. Chain length - specifies the number of iterations that will be run at a specified temperature.
3. Cooling schedule - used to decrease the temperature. The cooling schedule specifies the fraction of the current temperature that will be used as the next temperature once we have completed a number of iterations equal to the chain length, (Brown 1994) and (Ignizio, 1994).

(Aarts and Korst 1989) shows that the simulated annealing algorithm will asymptotically converge to the global optimal solution. However this behavior can only be approximated in polynomial time at the expense of optimality (Brown, Pittard, and Sappington 1993). Figure 2 shows the general structure of the simulated annealing used in this problem. The following paragraphs further describe the evaluation function, the perturbation operation, constraints, and algorithm parameters, (Brown and Schamburg 1997).

```
Let     s, s' be the current and perturbed solutions, respectively
        f(s), f(s') be the evaluation function values of the current and perturbed solutions, respectively
        C be a temperature parameter
        $C_0$ be an initial temperature
        M be a number of iterations at a temperature
        d be a temperature decrement parameter (between .8 and .99)
        mutate be a perturbation selection operation
$k \leftarrow 0$
Repeat
    FOR $i \leftarrow 1$ to M DO
        Generate s' (by perturbing s in the dimensions chosen according to the mutate operation)
        if $f(s) \leq f(s')$, then $s \leftarrow s'$
        else
            if $\exp((f(s) - f(s')/C_k) >$ random [0, 1) then $s \leftarrow s'$
    END
    Calculate $C_{k+1}$ ($C_{k+1} \leftarrow C_k * d$)
Until Stop Criterion (When either little or no change has occurred for a given number of iterations)
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Figure 2: General Structure of the Simulated Annealing Procedure, (Aarts and Korst 1989) and (Brown, Pittard, and Sappington 1993)

1. Evaluation Function. A computer simulation program was used to model the relative accuracy for the simulated annealing algorithm. The program models the relative accuracy at a specified point, given parameter inputs such as receiver locations, receiver velocities, and measurements used. The simulated annealing algorithm was made such that it could interface with the simulation model and change the necessary parameters to get the respective evaluation function.

Although the general structure of the simulated annealing used in this problem remained relatively constant, three different evaluation functions were used depending on the situation under consideration. In the first set of problems, the evaluation function was the accuracy at a specified point. In the second set of problems, the evaluation function was the average relative accuracy over a specified area. The average relative accuracy was calculated slightly different depending on whether or not moving receivers were used in determining the emitter locations. In those problems where moving receivers were used, the average accuracy across the area was calculated at 1 km increments along the routes in the current solution. The average accuracy values at each increment were then averaged to give the response value. The following expressions show the calculations described above where the evaluation function is given by $E[E[A]]$.

$$E[A] = \frac{1}{n}\sum_{i=1}^{n} A_i$$

$$E[E[A]] = \frac{1}{l}\sum_{k=1}^{l} E[A]$$

where A is the relative accuracy measure, $E[A]$ is the average accuracy across the area, n is the number of points in the area from which the accuracy measure was calculated, and l is the number of 1 km increments for the routes.

2. Perturbation Operation. In each iteration of the procedure described in Figure 2, each variable was selected for perturbation according to some probability. We typically gave each variable a probability of 0.5 for being selected for perturbation such that if selected, the variable would have an equal chance of being decremented as it would have for being incremented. The procedure then uses direction cosines for obtaining a random direction. The use of direction cosines for determining a random direction is described in (Bohachevsky, Johnson, and Steom 1986). In this procedure, a random number, θ_i, is chosen from a uniform distribution on [0, 1], for each of n variables. The direction cosine is then calculated by $U_i = +/-(\theta_i * (\sum_{i=1}^{n} \theta_i^2)^{-1/2})$. The direction cosine, U_i, is then multiplied by some fraction, Δr, of the range for the given variable. We used values between 0.04 and 0.10 for Δr, with Δr = 0.07 being the predominately used value, depending on the chain length and the cooling schedule. Each variable is then perturbed according to U_i*Δr. This results in a new perturbed solution, s'.

3. Constraints; Invalid s'. To prevent s' from violating some constraint, s' was adjusted by having reflection off the boundaries. (Brown, Pittard, and Sappington 1993) used this technique in development of SPA, Sensor Placement Analyzer. If, for example, receiver i exceeds the maximum value of one of its location parameters, that location parameter is decremented by the value to which it exceeded the maximum constraint. Keeping invalid solutions and making these adjustments proved to result in better solutions than those cases where invalid solutions were simply thrown away.

4. Parameters. The initial temperature for the simulated annealing was chosen so that nearly all solutions would be accepted at the beginning of the procedure. This was accomplished by taking a random sample of the response over the given solution space and selecting an initial temperature that would give a high probability, 0.95 or greater, for accepting some of the worse changes encountered. The cooling schedule was based upon the value of d (see Figure 2) used for a given problem. d ranged in value between 0.90 and 0.99 and was problem dependent. In smaller problems where the evaluation function calculation and the perturbation operation is rather quick, larger values of d could be used to provide a slower cooling schedule. In larger problems, in order to get solutions within a reasonable time, a quicker cooling schedule had to be used where d ranged between 0.90 and 0.95. The chain length value ranged between 5 and 40 depending on the problem as well. In those problems where solution time was long, we found that use of a shorter chain length in order to have a larger d typically gave better results than the converse. The search is stopped when little improvement is found after a given number of iterations or when the search exceeds a given number of iterations.

3 APPLICATION OF THE SIMULATED ANNEALING TO DETERMINE THE EXPERIMENTAL REGION

To show how the developed methodology is employed, we demonstrate the steps taken in the analysis of a scenario in which 3 airborne receivers are employed to determine the location of emitters. The analysis is focused on finding parameter values which describe the flight routes for accurate surveillance over an area 70km in depth (y axis) by 74km in width (x axis) when the velocities of the platforms are held constant. Therefore, the area where the receivers will focus extends from 0km to -70km in depth (y axis) and -37km to 37km in width (x axis). In this analysis, we consider some of the tradeoffs encountered by changes in the parameter values to show how robust the solution might be in application. Additionally, we compare several sets of parameter values describing flight routes for 3 aircraft, so that differences may be identified. We also compare the best sets of parameters for this situation with the best found for a 2 aircraft problem to show the potential improvement from adding a third aircraft. As was mentioned in the description of the methodology, we use insight gained from the analysis of previous cases to our advantage while going through the analysis of this situation. The analysis of previous three receiver situations and the 2 airborne receiver situation are especially of interest in this analysis.

The following is the initial list of the issues for analysis involved in the investigation of this situation. The list contains issues that we wish to resolve or specific questions that we wish to answer as the study progresses.

1. What are the best sets of parameters used to describe the flight routes for 3 airborne receivers? That is, what combinations of flight directions and route locations provide the most accurate surveillance over the entire area?
2. How much do the parameter values of the third receiver affect the accuracy in the surveillance? How do the relative accuracy values for three airborne receivers compare to those found when only two airborne receivers were used?
3. Of the parameters investigated, which have the greatest impact on the accuracy? Which parameters are less important?
4. Are the best flight routes for 2 of the 3 airborne receivers similar to the flight routes found when only 2 receivers were used?

The initial region of operability for this problem is defined by the variables describing the receivers' routes (location, direction, and length of the routes) for the 3 receivers and the constraints for an area that extends from -37km to 37km in width (x axis) and from -140km to -90km in depth (y axis). The issues for analysis help us to focus on the important parameters for this situation. The understanding gained as a result of analysis of the previous situations give some idea where good solutions may exist within the constraints of the entire solution space (region of operability) for this situation. At this point in the analysis, we have already looked at many different scenarios. The previous analysis, was one with only 2 airborne receivers where the response was the average accuracy over the entire area described above. The starting point of the routes for the best solution found in this scenario 26 is presented in Figure 3 below. The best solutions to the 2 airborne receiver situation were all ones where the flight route of the left receiver started at the left boundary and the right receiver ended at the right boundary. Figure 3 shows the starting point where the right receiver is at the right boundary. Additionally, the flight routes for these solutions were ones where the left most receiver was traveling in a slightly negative direction (about -0.3 radians) to that which is parallel to the x axis and the right most receiver was traveling in a slightly positive direction (about 0.3 radians).

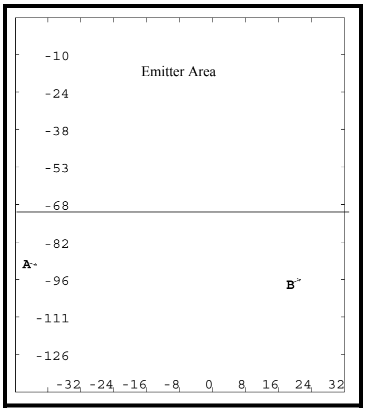

Figure 3: Solution to the 2 Airborne Receiver Scenario Showing the Initial Position of 2 Receivers for the Best Solution Found.

Analysis of the lengths of the flight routes for the 2 airborne receiver problem indicated that shorter flight routes are preferred to longer ones. It was also found that the minimum elevation of 1 km is better in terms of accuracy than the higher elevations considered.

In order to determine the initial search region for the experimentation in this situation, we use the understanding gained from the previous situations and the results of the simulated annealing algorithm described in above. We ran the simulated annealing program 6 times for this problem with the velocity held constant at 50 m/s. The best solution found by the simulated annealing is given in Table 1 below. In this table, the x and y coordinate positions given are the starting positions of the flight routes. The direction and length of the routes are also given.

Table 1: Best Simulated Annealing Solution to the 3 Airborne Receiver Situation

Receiver	Starting x coordinate (km)	Starting y coordinate (km)	Direction (radians)	Route Length (km)	Average Accuracy Measure
1	-36.199	-109.459	-0.292	10.100	
2	-8.000	-117.000	2.859	10.100	
3	27.441	-93.633	0.264	10.100	116.703

Some trends were noted in the solutions found by the simulated annealing. First, in each of the solutions, receivers 1 and 3 have positions and directions similar to those found for 2 aircraft in the previous problem. The starting x coordinate position and the direction of receiver 2 is similar in each of the solutions. The starting x coordinate position for receiver 2 is between receiver 1 and 3 but to the left of the center of the area of operation. The direction for receiver 2 is almost the opposite of that for receiver 1, in each case.

As the results of the previous situations show, it seems that one of the over-riding factors involved with the deployment is the establishment of a long baseline between a pair of receivers employed. We therefore hold the x coordinate starting position of receiver 1 (the left most receiver) and the x coordinate ending position for receiver 3 (the right most receiver) constant at the boundaries of the operability region (x coordinate of -37km and 37km respectively). Holding these values constant seems reasonable because of the potential angles subtended at the targets by a the pair of receivers. As the previous solutions indicate, the outer most receivers are typically constrained by the unit boundaries. These longer baselines seem to give better results. We also hold the y coordinate ending position constant for receiver 3 at the y constraint (-90 km). This seems to be advantageous because of the effects of range on the signal to noise ratio (SNR). In each of the solutions found, the ending position for the right most receiver seems to be on or near this constraint. We investigate the y coordinate value for the other two receivers to allow for possible differences in geometry, that they cause among receivers and the emitter area, that may or may not be advantageous in surveillance over the entire emitter area.

In the previous situation, we investigated the effects of changes in elevation and found them to be less important than other effects. Although it was found that lower elevation is typically preferred in accurate identification of emitter locations, a reasonable range in values for elevation (between 0 and 5 km) is much less than the range in values for x and y coordinate positions. For that reason, we hold the elevation for each of the receivers constant at 2 km.

In the analysis of the previous situation, we also investigated the effects of changes in route length. It seems evident by looking at that analysis and the results of the simulated annealing that shorter flight routes are preferred for accurate surveillance. If there exists a single best position for moving receivers, longer flight routes could only detract from the solution causing greater deviation in positions than shorter flight routes. Additionally, longer flight routes result in shorter baselines between moving receivers because of the boundary constraints. We therefore hold the length of the flight route constant at a value of 10 km. This allows for about 200 seconds or 3 1/3 minutes of surveillance before turns.

The focus of this analysis is on the direction and location of the flight routes, their relationships with the other factors considered, and their effects on accuracy. The direction of movement of the platforms is with respect to the x axis where 0 radians indicates movement parallel to the x axis, from left to right. The initial set of variables under consideration in this problem includes: y_0, d_0, x_1, y_1, d_1, d_2 (the y position for receiver 0, the direction of movement for receiver 0, the x position for receiver 1, the y position for receiver 1, the direction for receiver 1, and the direction for receiver 2). The initial search region for these variables is the neighborhood of the best simulated annealing solutions. Initially, we consider a range in the location parameters of about 4 km and a range in the direction parameters of about 0.4 radians.

In the analysis of this situation, we do not only desire to improve the current solution, but we want to consider the relationships between the locations and the directions of the platforms. We also conjecture that there are relationships between these factors. Additionally, it is doubtful that there is a linear relationship between the directions and the response variable. We suspect curvature in the response from changes in the direction variables as was found in previous problems. Therefore, we start this analysis desiring to fit a full quadratic model. It is believed at this point that, at least through the use of appropriate transformations, a second order model will adequately represent the relationships because of the limited size of the solution space considered. We hope that the ranges selected for the control variables causes enough variability in the response variable, however we desire to fit a relatively simple and understandable graduating function to the data.

In order to detect the potential curvature and obtain an adequate representation of the response, the design chosen in this situation is a 3^6 factorial design. This seems reasonable in order to gain an understanding of the solution space considered. The factors of interest and their respective input values for this situation include:

1. y_0 - the location of receiver 0 in km on the y axis. (-111, -109, -107)
2. d_0 - the direction of receiver 0 in radians. (-0.5, -0.3, -0.1)
3. x_1 - the location of receiver 1 in km on the x axis. (-10, -8, -6)
4. y_1 - the location of receiver 1 in km on the y axis. (-119, -117, -115)
5. d_1 - the direction of receiver 1 in radians. (2.7, 2.9, 3.1)
6. d_2 - the direction of receiver 2 in radians. (0.1, 0.3, 0.5)

4 A COMPARISON OF SOLUTIONS RESULTING THROUGH USE OF THE METHODOLOGY

We now look at the distribution of the accuracy values over the entire area for the length of the flight tracks. We do this for several solutions to show the differences in the distributions. In each case the length of the flight routes are 10 km and the velocities are 50 m/s. In this analysis, we take 25 relative accuracy values across the area at three evenly spaced points along the flight route. So, there are 75 accuracy values from which to compare the cases under study. The following are the cases used in this comparison, (Brown and Schamburg 1997). Table 2 gives the starting positions and the directions.

Table 2: Starting Positions for Each Case for the Comparison of Accuracy Distributions

	x_0	y_0	d_0	x_1	y_1	d_1	x_2	y_2	d_2
1	-37	-90	0.0	-5	-90	0.0	27	-90	0.0
2	-37	-90	0.0	5	-90	π	27	-90	0.0
3	-37	-90	-π/4	-5	-105	0.0	29.9	-97.0	π/4
4	-37	-109	-0.29	-8	-117	2.86	27.3	-92.6	0.26
5	-37	-90	-0.32	4	-90	3.14	27.4	-92.8	0.29

Case 1: In case 1, 3 airborne receivers are flying in the same direction (0.0 radians) along the -90 km y constraint. The receivers are spaced so that the left most receiver starts at -37 km on the x axis, the middle receiver starts at -5 km on the x axis, and the right most receiver starts at 27 km on the x axis.

Case 2: Case 2 is the same as case 1 except the middle receiver is flying the opposite direction. That is, the middle receiver starts at 5 km on the x axis and has the direction of π radians.

Case 3: In case 3, 3 airborne receivers are flying in a semicircle type of pattern where they have directions of -π /4 radians, 0.0 radians, and π/4 radians. The second aircraft starts at -5 km and -105 km on the x and y axis respectively.

Case 4: Case 4 is the best solution found from use of the simulated annealing. This solution was presented previously in Table 1.

Case 5: Case 5 is the solution to the nonlinear program used to optimize the resulting response surface function.

Figure 4 shows the individual box plots for each of the five cases. The 95% confidence regions for the medians of each show that there is a marked difference between those of cases 1 and 3 and those of the other cases. Additionally, the distributions of cases 1 and 3 are much wider than those of the other cases. In comparing cases 2, 4, and 5 only, it is difficult to detect much of a difference among the plots although the distribution of case 5 seems to be slightly lower than the other two.

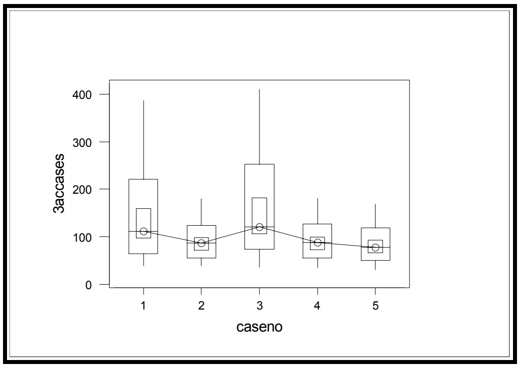

Figure 4: Individual Box Plots for Each of the Five Cases, (Brown and Schamburg 1997)

Table 3 gives the results of the Kruskal-Wallis test for cases 2, 4, and 5 only. The results indicate that although the medians and average ranks are slightly different, the null hypothesis would only be rejected at the 0.44 significance level. Therefore, we accept the null hypothesis. There is not much difference in the distributions of cases 2, 4, and 5.

Table 3: Results of the Kruskal-Wallis Test for Cases 2, 4, and 5

```
LEVEL     NOBS    MEDIAN  AVE. RANK    Z VALUE
  2       75      86.46       116.8       0.61
  4       75      87.79       117.1       0.66
  5       75      77.75       105.2      -1.28
OVERALL  225                  113.0
H = 1.63  d.f. = 2  p = 0.444
```

Figures 5 through 9 show the contour plots over the emitter area for the center points of the flight routes for each of the five cases under study, (Brown and Schamburg 1997). The contour plots presented represent the lines of constant accuracy values over the emitter area. The contour plots show that cases 1 and 3 have an extra contour line of 250 meters. This line is not present in the other cases, indicating that they would not produce any accuracy values so large. There are only slight differences among the contour plots for cases 2, 4, and 5. The contour lines for case 5 seem to be slightly farther back than those of cases 2 and 4. In this sense, the performance of case 5 is preferred.

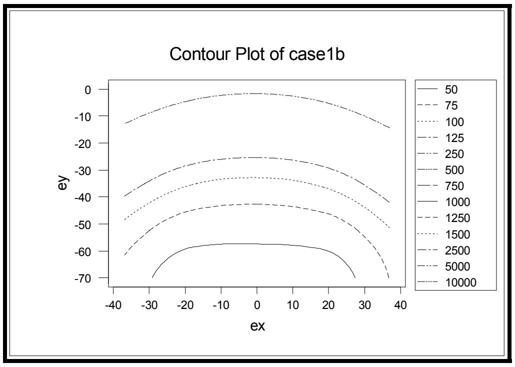

Figure 5: Contour Plot of Constant Accuracy for the Center Point of Case 1

5 SUMMARY AND CONCLUSIONS

The methodology presented here is intended to be iterative and flexible. This iterative nature helps verify the conclusions drawn from previous phases of the process. It addi-

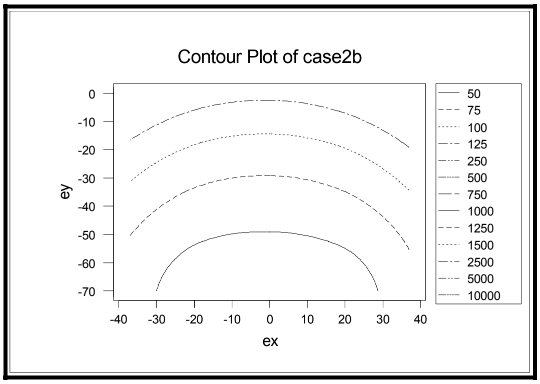

Figure 6: Contour Plot of Constant Accuracy for the Center Point of Case 2, (Brown and Schamburg 1997)

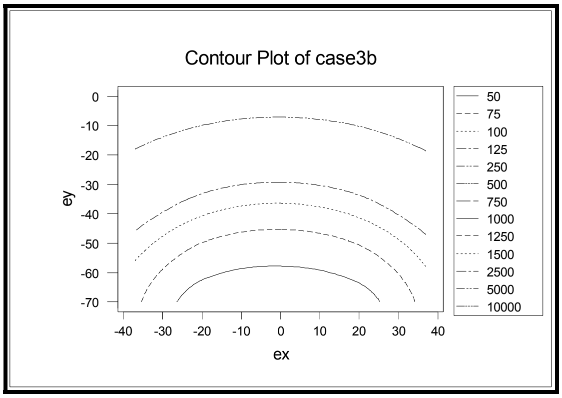

Figure 7: Contour Plot of Constant Accuracy for the Center Point of Case 3, (Brown and Schamburg 1997)

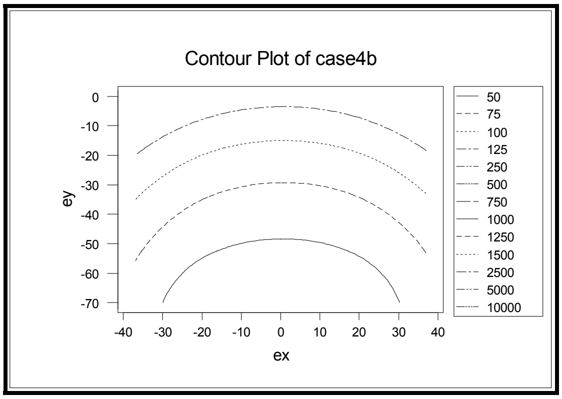

Figure 8: Contour Plot of Constant Accuracy for the Center Point of Case 4

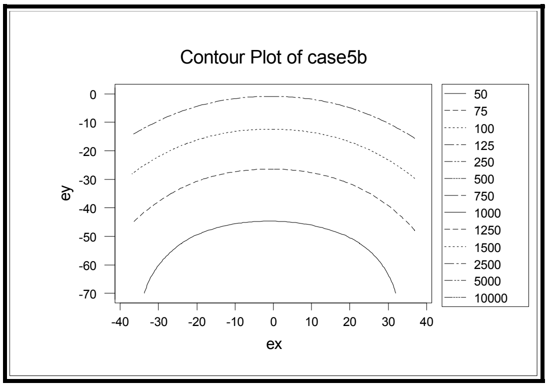

Figure 9: Contour Plot of Constant Accuracy for the Center Point of Case 5

tionally leads to increased understanding of the relationships involved in accurate surveillance. The study should be set up so that one may gain information from the analysis of a given scenario that may be beneficial in the analysis of upcoming scenarios. The methodology is also intended to be flexible. The steps and tools described above should be adapted to the problem and the issues at hand. The techniques included in this methodology are intended to:

1. result in good, robust solutions for the accurate deployment of emitter surveillance systems, and
2. improve understanding of the relationships involved in accurate emitter surveillance.

In this paper we have demonstrated the use of the developed methodology for analysis of a situation where three airborne receivers are used to cover the entire emitter area. Initially, we develop the focus of the analysis so that the investigation is pointed at answering the important issues. Through use of the methodology, we have found a good solution for accurate emitter surveillance using three receivers. The solution seems relatively robust and is comparable to our findings in analysis of other situations. The distributions of the accuracy values for several sets of parameters have been compared to determine which sets are significantly better. The results have been, not only a series of good solutions, but some useful conclusions and generalizations for accurate emitter surveillance.

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