Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 2458-9403
Vol. 3 Issue 11, November - 2016

Magnetohydrodynamic Fluid Flow Between Two Parallel Infinite Plates Subjected To An Inclined Magnetic Field Under Pressure Gradient

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***Abstract*—Magnetohydrodynamic (MHD) flow between two parallel infinite plates with inclined magnetic field under applied pressure gradient has been investigated in this paper. The differential equations governing the flow are non-dimensionalised and solved analytically with appropriate boundary conditions. The effects of Hartmann number, the angle of magnetic field inclination, pressure gradient, and Reynolds number on the flow field have been presented graphically. It was found that decrease in angle of inclination of magnetic field increases the velocity profiles while increase in the Hartmann number decreases the velocity. Increase in pressure gradient leads to increased velocity.**

Keywords—Magnetohydrodynamic fluid flow, Hartmann number, Reynolds number, Pressure Gradient.

Nomenclature

$\vec{B}$	Magnetic flux
$\vec{E}$	Electric field
$\vec{F}_b$	Body force
$\vec{H}_o$	Applied magnetic field
μ	Dynamic viscosity
υ	Kinematic viscosity
$\vec{J}$	Current density
L	Characteristic length
M	Hartmann number
P	Pressure
Re	Reynolds number
t	Time
$\vec{V}(u, v, w)$	Velocity in the x,y,z direction
ρ	Density
σ	Electrical conductivity of the fluid

I. INTRODUCTION

When a conducting fluid flows past a magnetic field, an electric field and consequently an electric current is induced. In turn the current interacts with the magnetic field to produce a body force on the fluid .Such phenomenon occurs both in nature and in man-made devices and is termed as Magnetohydrodynamics(MHD).It is an important branch of fluid dynamics. In natural phenomena MHD occurs in the sun, the Earth's interior, the ion sphere the stars and their atmosphere to mention a few examples. In the laboratory or in man-made phenomena many new devices have been made which utilizes the MHD interaction directly such as propulsion units and power generators. In the recent past, several investigations have been carried out on MHD.

Chaturani and Saxena [1] investigated a two-layered MHD model for parallel plate haemodialysis, under the influence of uniform transverse magnetic field. Agwal, Ram and Yadav [2] carried an investigation of MHD unsteady viscous flow through a porous straight channel. The study was carried out with two parallel porous flat walls in the presence of transverse magnetic field under a time varying pressure gradient. Ganesh and Krishnambal [3] studied the unsteady stoke's flow of an electrically conducting viscous incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field. Singh [4] investigated on hydromagnetic steady flow of viscous incompressible fluid between two parallel infinite plates under the influence of inclined magnetic field. Singh and Okwoyo [5] carried out a study of couette flow between two parallel infinite plates in the presence of a transverse magnetic field. Krishna [6]carried out an investigation on unsteady incompressible free convective flow of an electrically conducting fluid between two heated parallel horizontal plates under the action of magnetic field applied transversely to the flow. Sayed, et al [7] did a study of time dependent pressure gradient effect on unsteady MHD couette flow and heat transfer of a casson fluid, while Kumar et al [8] investigated on an unsteady periodic flow of a viscous incompressible fluid through a porous planer channel in the presence of transverse magnetic field.

Despite the investigations done on MHD flows, flow past parallel plates subjected to inclined magnetic field in presence of pressure gradient has received little attention. Hence the main objective of the present investigation is to obtain an exact solution of MHD fluid flow between two parallel infinite plates subjected to inclined magnetic field in presence of pressure gradient.

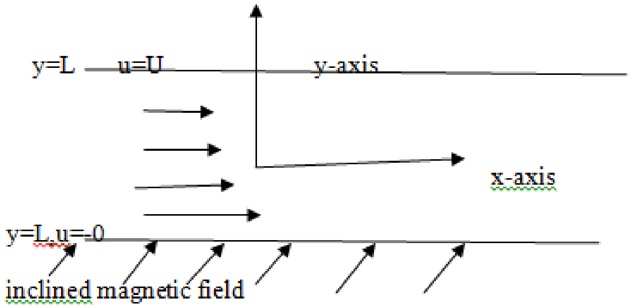

Fig.1 Geometric configuration

II.MATHEMATICAL ANALYSIS

We consider the steady viscous flow along the x-axis of an electrically conducting fluid between two horizontal parallel infinite plates situated at $y=-L$ and $y=L$. The plate at $y=-L$ is stationery while the plate at y=L is moving with uniform velocity *U.* An inclined magnetic field at an angle α is applied to the flow. There is no external applied electric field and we assume steady flow conditions have been attained. It is required to analyse the velocity profiles taking into consideration the variable pressure gradient.

The equations governing the flow field are:

$$\frac{\partial p}{\partial t}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0 \qquad (1)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\vec{F}_x \qquad (2a)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\vec{F}_y \qquad (2b)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\vec{F}_z \qquad (2c)$$

The flow is along the *x*-direction so the velocity profile along the *y and z*-axis *v=w=0* and the velocity *u* along the *x*-axis depends on *y* only. So “(1)” becomes;

$$\frac{\partial u}{\partial x}=0 \quad (3)$$

Since the flow is steady (does not depend on time), and *v=0*, *w=0* and using “(3)” then “(2)” above reduces to

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{F_x}{\rho} \qquad (4a)$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{F_y}{\rho} \qquad (4b)$$

We know that the body forces or the ponder motive forces $\vec{F}=\vec{F}(\vec{F}_x,\vec{F}_y,\vec{F}_z)$

$\vec{F}=\vec{J}\times\vec{B}$ therefore “(4a)” becomes

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\vec{J}\times\vec{B}}{\rho} \qquad (5a)$$

since $\vec{F}_y=0$ “(4b)” becomes

$$0=\frac{\partial P}{\partial y} \qquad (5b)$$

Since we are considering the flow in *x*-direction then the flow will be affected by the magnetic flux which is perpendicular to the flow. Since we want to study the effect of different angles of inclination of the magnetic field then the velocity and magnetic flux profiles will be;

$\vec{V}=\vec{V}(u,0,0)$ and

$\vec{B}=\vec{B}(0,Bsin\alpha,0)$ Respectively.

Now $\vec{F}_x=\vec{J}\times\vec{B}$ and since $\vec{J}=\sigma\vec{E}$, and $\vec{E}=\vec{V}\times\vec{B}$

Then we have

$$\vec{V}\times\vec{B}=\sigma\begin{vmatrix}\tilde{\imath} & \tilde{\jmath} & \tilde{k}\\ u & 0 & 0\\ 0 & Bsin\alpha & 0\end{vmatrix}$$

$$=\tilde{\imath}(0)-\tilde{\jmath}(0)+\tilde{k}(\sigma uBsin\alpha)$$

$$\vec{V}\times\vec{B}=\sigma uBsin\alpha\tilde{k}$$

Again

$$\vec{J}\times\vec{B}=\begin{vmatrix}\tilde{\imath} & \tilde{\jmath} & \tilde{k}\\ 0 & 0 & \sigma uBsin\alpha\\ 0 & Bsin\alpha & 0\end{vmatrix}$$

$$=\tilde{\imath}(0-\sigma uB^2sin^2\alpha)-\tilde{\jmath}(0)+\tilde{k}(0)$$

$$\vec{F}_x=\vec{J}\times\vec{B}=-\sigma uB^2sin^2\alpha\tilde{\imath} \qquad (6)$$

Using “ (6)”, then “(5a)” becomes

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma uB^2sin^2\alpha}{\rho} \qquad (7)$$

Using the following non-dimensional quantities

$x^+=\frac{x}{L}$, $y^+=\frac{y}{L}$, $p^+=\frac{P}{\rho U^2}$, $u^+=\frac{u}{U}$, $\frac{UL}{\upsilon}=Re$, M= $LB\sqrt{\frac{\sigma}{\mu}}$

“(7)” becomes

$$0=-Re\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-M^2sin^2\alpha u \qquad (8)$$

Suppose we let pressure gradient $\frac{\partial p}{\partial x}$ to be a variable say $(-P(x))$ so that we can write $\frac{\partial p}{\partial x}=-P$ and “ (8)” becomes

$0= PRe + \frac{\partial^2 u}{\partial y^2} - M^2 sin^2 \alpha u$ or

$$\frac{d^2u}{dy^2} - M^2 sin^2 \alpha u = -PRe \quad (9)$$

Equation (9) is the ordinary differential equation governing the flow problem which will be solved subject to the boundary conditions.

When y= −L u=0

When y= L u=U

The non-dimensional form of these boundary conditions is;

$y= -1, u= 0$

$y= 1, u= 1 \quad (10)$

III.METHOD OF SOLUTION

The ordinary differential equation "(9)" is inhomogeneous therefore we will solve the homogenous part and the inhomogeneous part separately and then add them together to get the general solution.

The auxiliary equation for the homogenous part can be written as

$$D^2u - M^2 sin^2 u = 0 \quad (11)$$

Or $(D^2 - M^2 sin^2\alpha) = 0$

$D= \pm\sqrt{M^2 sin^2 \alpha}$

$D= \pm M sin\alpha \quad (12)$

The complimentary solution is then given by

$$u_c = Ae^{Msin\alpha y} + Be^{-Msin\alpha y} \quad (13)$$

Where A and B are constants to be determined using the boundary conditions.

The particular integral of equation "(9)" is given by

$u_p = \left(\frac{1}{D^2 - Q}\right) PRe$ where $Q= M^2 sin^2 \alpha$

$= \frac{PRe}{Q}$

$$u_p = \frac{PRe}{M^2 sin^2 \alpha} \quad (14)$$

The general solution is got by adding "(13)" and "(14)" as follows

$$u= Ae^{Msin\alpha y} + Be^{-Msin\alpha y} + \frac{PRe}{M^2 sin^2\alpha} \quad (15)$$

The above equation will be solved subject to the boundary conditions "(10)"

Let $M\ sin\alpha$ be m and $\frac{PRe}{M^2 sin^2\alpha}$ be R so that "(15)" becomes

$$u= Ae^{my} + Be^{-my} + R \quad (16)$$

The boundary conditions are

Solving "(16)" subject to the boundary conditions we have,

$$0= Ae^{-m} + Be^{m} + R \quad (17a)$$

$$1= Ae^{m} + Be^{-m} + R \quad (17b)$$

Multiplying " (17a)" by e^{-m} and "(17b)" by e^{m} and subtracting we get

$$e^m = Ae^{2m} - Ae^{-2m} + Re^m - Re^{-m}$$

$$e^m = A(e^{2m} - e^{-2m}) + R(e^m - e^{-m})$$

$$A= \frac{e^m - R(e^m - e^{-m})}{(e^{2m} - e^{-2m})}$$

Similarly we multiply "(17a)" by e^m and "(17b)" by e^{-m} and subtract to get

$$e^{-m} = Be^{-2m} - Be^{2m} + Re^{-m} - Re^{m}$$

$$-e^{-m} = B(e^{2m} - e^{-2m}) + R(e^m - e^{-m})$$

$$B= \frac{e^{-m} - R(e^{-m} - e^{m})}{(e^{-2m} - e^{2m})}$$

Substituting the values of A and B we get

$$u= \frac{e^m - R(e^m - e^{-m})}{(e^{2m} - e^{-2m})} e^{my} + \frac{-e^{-m} - R(e^{-m} - e^{m})}{(e^{-2m} - e^{2m})} e^{my} + R \quad (18)$$

$$u= \frac{(e^{m+my}) - R\sinh m e^{my} - R\sinh m e^{-my} - U(e^{-m-my})}{(e^{2m} - e^{-2m})} + R$$

$$u= \frac{(e^{m+my} - e^{-(m+my)}) - R\sinh m(e^{my} + e^{-my})}{\sinh 2m} + R$$

$$u= \frac{(e^{m(1+y)} - e^{-m(1+y)}) - R\sinh m(e^{my} + e^{-my})}{\sinh 2m} + R$$

$$u= \frac{\sinh m(1+y) - R\sinh m\,(\cosh my)}{\sinh 2m} + \frac{PRe}{M^2 sin^2\alpha} \quad (19)$$

Putting back the values of m and R we have

$$u = \frac{\sinh(M\sin\alpha(1+y)) - \frac{PRe}{M^2 sin^2\alpha}\sinh(M\sin\alpha)\cosh(M\sin\alpha y)}{\sinh(2M sin\alpha)} + \frac{PRe}{M^2 sin^2\alpha} \quad (20)$$

IV.RESULTS

Using " (20)" we analyse the effects of different parameters on velocity. The analysis is done using graphs as shown below: Throughout the four figures we have taken the standard values for the Hartmann number to be 2, the angle of inclination to be 60^0, the Pressure gradient to be 2 and the Reynolds number to be 2.

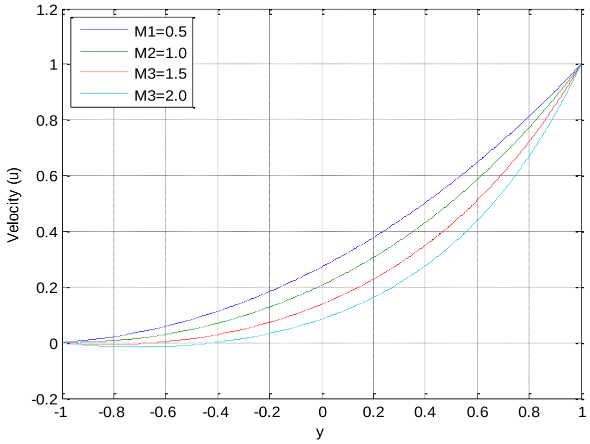

Fig. 2 Velocity distributions for different values of Hartmann Number (M)

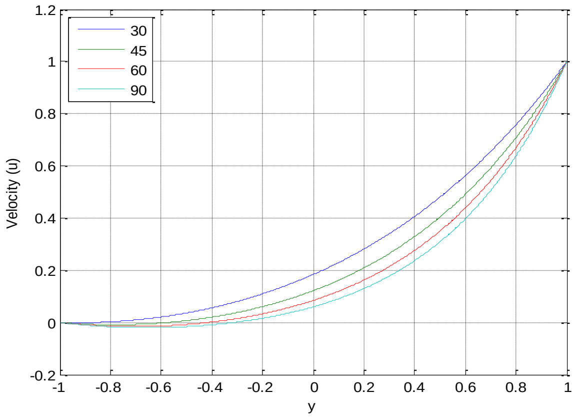

Fig.3 velocity profiles for different inclination of magnetic field

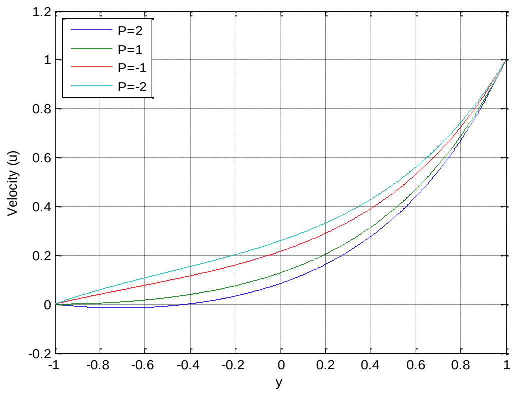

Fig.4 Velocity profiles for different values of pressure gradient (P)

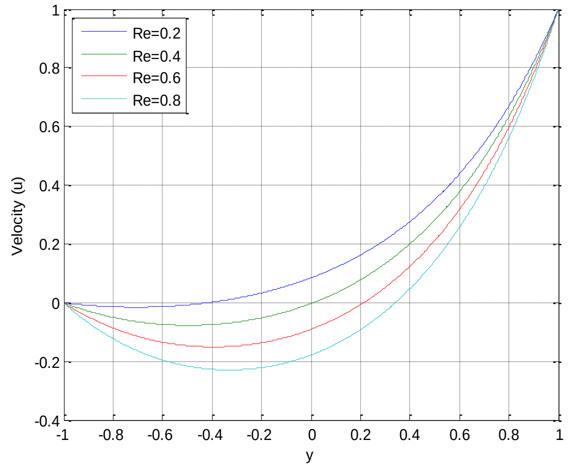

Figure 5.velocity distribution for different values of Reynolds number (Re)

V. DISCUSSIONS

From Figure 2 it is observed that the increase in Hartmann number leads to a decrease in velocity. This is because the Hartmann number is the ratio of magnetic force to viscous force and so the larger the Hartmann number the stronger the magnetic force and the smaller the viscous force. So it is clear from the figure that increasing the Hartmann number reduces the velocity due to the action of Lorentz force. Figure 3 shows the effect of the angle of inclination of magnetic field on the flow velocity. The smallest angle of inclination is 30^0 and from the figure we observe that when the angle of inclination is small then the velocity is high and as the angle of inclination increases the velocity decreases. At 90^0 it indicates a special case where the effect of fluid flow is observed at transverse magnetic field which is the normal MHD flow.

Figure 4 illustrates the effect of pressure gradient on the flow velocity. The figure shows that due to the force the pressure will have on the fluid an increase from negative to positive pressure gradient leads to an increase in flow velocity.

Figure 5 shows the effects of the Reynolds number on the velocity of the flow. We observe that an increase in Reynolds number leads to a decrease in the velocity of the flow while a decrease of the Reynolds number leads to an increase in the flow velocity. This is due to the viscous force of the fluid since the Reynolds number is the ratio of viscous force to inertia force.

VI.VALIDATION OF RESULTS

Our results were compared with those of Singh [4] who considered hydromagnetic flow of viscous incompressible fluid flow between two parallel infinite plates under the influence of inclined magnetic field. He used analytical method to obtain the velocity profiles. The results coincides with our findings that the velocity profiles decreased as the strength of magnetic field was increased and also with the increase

However he did not consider the effect of pressure gradient and the Reynolds number on the flow. We found that increase in pressure gradient leads to increase in velocity while increase in Reynolds number leads to a decrease in velocity.

VII.CONCLUSION

From this investigation we conclude that increasing the magnetic force (M) reduces the velocity of the flow. This is due to the action of the Lorentz force on the fluid. Reducing the angle of inclination of the magnetic field and the strength of the magnetic field leads to an increase in velocity this is mainly because the magnetic field slows down the motion of the fluid so the stronger the magnetic field the slower the motion of the fluid. Increasing the pressure gradient on the flow leads to an increase of the flow velocity. This is because the pressure gradient will tend to push the fluid thus increasing its velocity.

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