Quantum Information and Computation, Vol. 16, No. 15&16 (2016) 1365–1378

STABILITY OF TWO INTERACTING ENTANGLED SPINS INTERACTING WITH A THERMAL ENVIRONMENT

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Received Juanuary 7, 2016
Revised October 18, 2016

We study the entanglement dynamics of two entangled spins coupled with a common environment consisting of a large number of harmonic oscillators. Specifically, we study the impacts of both interaction and temperature of the environment on the dynamic quantum correlation, namely, entanglement and quantum discord of two spins via concurrence and global quantum discord criteria. In the present system, we show that the interaction between the spin sub-systems and the common environment causes environmental states to approach a composition of even and odd coherent states, which have different phases, and which are entangled with the spin states. Moreover, using the thermofield approach, we demonstrate quantum correlation stabilization as a result of increasing environmental interaction as well as increasing temperature..

Keywords: entangled spins coupled with environment, dynamic quantum correlation, concurrence, quantum discord

Communicated by: S Braunstein & G Milburn

1 Introduction

The publication of "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" [1], was undoubtedly a landmark in the field of quantum investigations, with the concept of entanglement as its spotlight. Indeed, this paper has played an important role in such varied fields as the philosophy of physics, mathematical physics, and foundation of physics [2]. An entangled state, as a state of a composite quantum system consisting of two or more subsystems, cannot be decomposed into the states corresponding to the different constituent subsystems. Moreover, the generation and control of the entangled state are important issues in the progress of quantum technologies in the field of quantum information. Indeed, the preparation and construction of entangled states have been studied extensively in quantum information theory. Recently, numerous approaches have been proposed and/or experimentally developed to generate quantum entanglement in such physical systems as beam splitters [3-7], cavity QED [8, 9], NMR systems [10, 11], and semiconductor microcavities [12]. On the other hand, all quantum systems are open in the sense that they are correlated with their surrounding environment giving rise to the expectation that their quantumness is destroyed with the evolution of the system through time [13-23]. Despite this destructive role of the environment, it has shown that interactions with the surrounding environment may lead to the production or invigoration of entanglement and quantumness in specific physical systems [24-35].

In this paper, we consider the physical dynamics that generate an entanglement between two qubits and study the dynamic entanglement between them when they interact with a common bosonic environment. The findings indicate that the physical parameter which leads to the decoherence phenomenon will increase the quantumness, i.e. the quantum entanglement, in specific situations. We will show that the interaction of the system composed of two sub-systems (two spins) with a common environment causes the system states to approach a composition of even and odd coherent states of the environment, which are entangled with systems states, and which have different phases. In this way, the common environment interaction causes the entangled spin systems to stabilize.

In the next stage of the study, the role of environment and its participation in dynamic entanglement will be investigated analytically by applying the concurrence criterion to entanglement [36, 37]. The thermofield approach [38, 39] will also be employed to investigate the role of temperature on the evolution of quantum correlation. Our findings indicate that increasing temperature leads to the early achievement of maximal entanglement; in other words, quantumness becomes stable as a result of increasing environmental temperature.

Moreover, we consider global discord as a kind of criteria to investigate quantum discord and the role of the environment in the dynamic entanglement of two qubits. Quantum discord was first introduced by Ollivier and Zurek to capture quantum correlations not only in entangled states but also in separable states [40]. Indeed, quantum discord as a kind of quantum correlation of multipartite states has been considered in several scenarios [41]. The non-unique criterion of quantum discord has been defined by generalizing the quantum mutual information to a multipartite system [42, 43]. In a different approach, quantum discord has also been defined using relative entropy [44]. Finally, a global measure of quantum discord is obtained through the systematic extension of the bipartite quantum discord, while the basic requirements of the correlation function have also been satisfied [45]. Using the global quantum discord criterion, we consider the evolution of the quantum correlation associated with a two-spin system in contact with a common environment. In this way, it will be shown that interaction with the environment and the environmental temperature will lead to the stability of the quantum correlation as the system evolves through time, a fact that is confirmed by previously reported results.

This paper is organized as follows: In Sec. 2, we introduce the dynamical entanglement model by considering the contribution by the environment. Sec. 3 exploits the concurrence and the global quantum discord to investigate the roles of interactions with environment and environment temperature in the dynamic entanglement process. Also investigated in this section is the quantum correlation using environment spectrum density. Finally, Sec. 4 is devoted to some conclusions and remarks.

2 Quantum correlation Dynamics in a two-spin system in contact with A Common environment

The physical system consists of two spins which interact with a bosonic environment. The Hamiltonian of two spins, a and b, and their interactions with each other in the environment is given by:

$$\hat{H} = \hat{H}_a + \hat{H}_b + \hat{H}_\varepsilon + \hat{H}_{int}, \tag{1}$$

where,

$$\hat{H}_j = \frac{1}{2}\omega_0 \hat{\sigma}_z^j, \qquad j = a, b, \tag{2}$$

is the self-Hamiltonian of the qubits. In this case, $\hat{\sigma}_z^j$ is the usual z-component of the Pauli matrix related to the spins $j = a, b$. Eigenstates of σ_z are denoted by $|+\rangle$ and $|-\rangle$. H_ε describes the familiar self-Hamiltonian of the environment which is modeled by the harmonic oscillators,

$$\hat{H}_\varepsilon = \sum_i \omega_i \hat{a}_i^\dagger \hat{a}_i, \tag{3}$$

where, $\hat{a}_i$ and $\hat{a}_i^\dagger$ are annihilation and creation operators, respectively.

The interactions between the spins and the environment are described by the following

Hamiltonian

$$\hat{H}_{int} = \frac{1}{4}\left(\hat{\sigma}_-^a\hat{\sigma}_+^b + \hat{\sigma}_+^a\hat{\sigma}_-^b\right) \otimes \left[\sum_i \left(g_i\hat{a}_i^\dagger + g_i^*\hat{a}_i\right)\right], \tag{4}$$

where g_i's are coupling coefficients. A better understanding of this dynamics may be obtained by switching to the interaction picture. In the interaction picture, $\hat{H}_{int}(t)$, is given by

$$\hat{H}_{int}(t) = e^{i\hat{H}_0 t}\hat{H}_{int}e^{-i\hat{H}_0 t} = \frac{1}{4}\left(\hat{\sigma}_-^a\hat{\sigma}_+^b + \hat{\sigma}_+^a\hat{\sigma}_-^b\right) \otimes \Big[\sum_i \left(g_i\hat{a}_i^\dagger e^{i\omega_i t} + g_i^*\hat{a}_i e^{-i\omega_i t}\right)\Big], \tag{5}$$

where $\hat{H}_0 = \hat{H}_a + \hat{H}_b + \hat{H}_\varepsilon$. By considering the time evolution operator of the whole system, in the weak coupling limit,

$$\mathcal{U}(t) = \mathcal{T}_\leftarrow \exp\left[-i\int_0^t dt'\hat{H}_{int}(t')\right] \tag{6}$$

in which $\mathcal{T}_\leftarrow$ denotes the time-ordered product of operators and this fact that the commutator of the operators $H_{int}(t)$ and $H_{int}(t')$ is c-number, $[H_{int}(t), H_{int}(t')] = -2i\sum_i |g_i|^2 \sin\left(\omega_i(t-t')\right)$, we can rewrite the time evolution operator as a phase factor and the ordinary equivalent of the relation (6), i.e.,

$$\mathcal{U}(t) = e^{i\phi(t)}U(t) = e^{i\phi(t)}\exp\left[-i\int_0^t dt'\hat{H}_{int}(t')\right]. \tag{7}$$

As a result of this fact that the phase factor has no role in our study (for further details, see [17, 18, 46]), we can define the effective time evolution operator $U(t)$ as the following:

$$U(t) = \exp\Big[\frac{1}{4}\left(\sigma_-^a\sigma_+^b + \sigma_+^a\sigma_-^b\right) \otimes \Big(\sum_i \left(\lambda_i(t)\hat{a}_i^\dagger - \lambda_i^*(t)\hat{a}_i\right)\Big)\Big]. \tag{8}$$

in which,

$$\lambda_i(t) = \frac{g_i}{\omega_i}\left(1 - e^{i\omega_i t}\right). \tag{9}$$

Note that the above-mentioned relation states that the coupling constant changes as a consequence of interaction with environment.

Moreover, we assume that there is no correlation between the system and the associated environment at $t = 0$. Also, at the initial time, $t = 0$, we suppose that the spin sub-systems are entangled. Thus, the state of the whole system at time zero may be written as

$$|\Psi(0)\rangle = \left(\cos\frac{\theta}{2}|-,+\rangle + e^{-i\phi}\sin\frac{\theta}{2}|+,-\rangle\right) \otimes |\Phi_\varepsilon\rangle, \tag{10}$$

where, $|\Phi_\varepsilon\rangle$ refers to the state of environments at $t = 0$; θ and ϕ, as the control parameters of entanglement, are two angles which take values in the intervals $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$, respectively. By using Relation (8), the time evolution of the total system is obtained as

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle = |-,+\rangle|\varepsilon_1(t)\rangle + |+,-\rangle|\varepsilon_2(t)\rangle, \tag{11}$$

where,

$$
\begin{aligned}
|\varepsilon_1(t)\rangle &= \frac{1}{2}\Big(\cos\frac{\theta}{2}\left[\prod_i D_{even}(+\lambda_i)|\Phi_\varepsilon\rangle\right] + e^{-i\phi}\sin\frac{\theta}{2}\left[\prod_i D_{odd}(+\lambda_i)|\Phi_\varepsilon\rangle\right]\Big), \\
|\varepsilon_2(t)\rangle &= \frac{1}{2}\Big(\cos\frac{\theta}{2}\left[\prod_i D_{odd}(+\lambda_i)|\Phi_\varepsilon\rangle\right] + e^{-i\phi}\sin\frac{\theta}{2}\left[\prod_i D_{even}(+\lambda_i)|\Phi_\varepsilon\rangle\right]\Big). \quad (12)
\end{aligned}
$$

In these equations, the even and odd displacement operators, i.e., $D_{even}(\lambda_i)$ and $D_{odd},(\lambda_i)$ respectively, are defined as

$$
\begin{aligned}
D_{even}(\lambda_i) &= D(+\lambda_i) + D(-\lambda_i), \\
D_{odd}(\lambda_i) &= D(+\lambda_i) - D(-\lambda_i). \quad (13)
\end{aligned}
$$

Here, the displacement operator $D(\lambda_i)$ is given by

$$
D(\lambda_i) = \exp\left[\left(\lambda(t)\hat{a}^\dagger + \lambda^*\hat{a}\right)\right]. \quad (14)
$$

One can obtain the reduced density matrix of the spin sub-systems as follows:

$$
\rho_{a,b} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (15)
$$

where,

$$
\begin{aligned}
\rho_{22}(t) = \langle\varepsilon_1(t)|\varepsilon_1(t)\rangle = \frac{1}{2} + \frac{1}{2}r(t)\cos\theta, \\
\rho_{33}(t) = \langle\varepsilon_2(t)|\varepsilon_2(t)\rangle = \frac{1}{2} - \frac{1}{2}r(t)\cos\theta, \quad (16)
\end{aligned}
$$

and

$$
\rho_{23}(t) = \rho_{32}^*(t) = \langle\varepsilon_1(t)|\varepsilon_2(t)\rangle = \frac{1}{2}\sin\theta\left(\cos\phi - ir(t)\sin\phi\right). \quad (17)
$$

In these equations,

$$
r(t) = \prod_i \langle\Phi_\varepsilon|D^\dagger(-\lambda_i)D(+\lambda_i)|\Phi_\varepsilon\rangle, \quad (18)
$$

is the decoherence factor of a superposition of two states of a qubit [14, 15, 20].

3 Dynamic Entanglement and Quantum Correlation of Two Qubits in Bosonic Environment

The main objective of the present contribution is to study the quantum correlation evolution in this system. To this end, we investigate the entanglement dynamic as a function of time through the evolution of concurrence. In the case of two-qubit systems, this measure quantifies the entanglement and has a range of values from 0, for separable states, to 1, for maximal entanglement states. In this case, the concurrence of the density matrix (15), is obtained as

$$
\mathcal{C}\left(\rho_{a,b}\right) = max\left\{0, \sqrt{\lambda_+} - \sqrt{\lambda_-}\right\}, \quad (19)
$$

where, $\lambda_{\pm}$ represents the roots of the eigenvalues of matrix $R^{a,b}$ defined as follows

$$R^{a,b} = \rho_{a,b}\bar{\rho}_{a,b}. \tag{20}$$

where, $\bar{\rho}_{a,b}$ is defined by

$$\bar{\rho}_{a,b} = \left(\sigma_y^a \otimes \sigma_y^b\right) \rho_{a,b}^* \left(\sigma_y^a \otimes \sigma_y^b\right), \tag{21}$$

where, $\rho_{a,b}^*$ represents the complex conjugate matrix $\rho_{a,b}$. In this case, using a straightforward calculation, the concurrence $\mathcal{C}(\rho_{a,b})$ is given by

$$\mathcal{C}(\rho_{a,b}) = \sin\theta\sqrt{(1 - r^2(t)\cos^2\theta)\left(\cos^2\phi - r^2(t)\sin^2\phi\right)}. \tag{22}$$

It is worth mentioning that, if $\theta = 0$, the concurrence $\mathcal{C}(\rho_{a,b})$ will be independent of the environment and its value will be equal to $\mathcal{C}(\rho_{a,b}) = 0$; in other words, the non-entangled state is achieved. In addition, in the case of initial Bell states, we can obtain a stable maximal entanglement independent of environment effects as the system evolves through the time.

On the other hand, we examine the global quantum discord criterion as a measurement of quantum correlation [45]. Indeed, quantum entanglement is an evidence of quantum correlations, but it does not guarantee to include all quantum correlations. Although separable quantum states are not entangled and are neither included in quantum entanglement measures, they possess a kind of quantum correlation. Indeed, this quantity defines the degree of quantum correlations and is defined as the difference between two expressions of mutual information in quantum while they are considered to be identical in the classical terms [47]. In classical information theory, mutual information is the correlation between random variables and takes the following form for a bipartite system [40]:

$$\mathcal{J}(\mathcal{X},\mathcal{Y}) = \mathcal{H}(\mathcal{X}) - \mathcal{H}(\mathcal{X}|\mathcal{Y}), \tag{23}$$

where, $\mathcal{H}$ is the Shanon entropy and is given by $\mathcal{H} = -\sum \mathcal{P}(\mathcal{X} = x)\log\mathcal{P}(\mathcal{X} = x)$. Here, $\mathcal{P}(\mathcal{X})$ is the probability distribution for the random variable $\mathcal{X}$ to have the x-value. $\mathcal{H}(\mathcal{X}|\mathcal{Y})$ is the conditional entropy and may be written as:

$$\mathcal{H}(\mathcal{X}|\mathcal{Y}) = \mathcal{H}(\mathcal{X},\mathcal{Y}) - \mathcal{H}(\mathcal{Y}), \tag{24}$$

where, $\mathcal{H}(\mathcal{X},\mathcal{Y})$ is the joint entropy; i.e., both $\mathcal{X}$ and $\mathcal{Y}$ occur. Mutual information could also be written differently as follows [48]

$$\mathcal{I}(\mathcal{X};\mathcal{Y}) = \mathcal{H}(\mathcal{X}) + \mathcal{H}(\mathcal{Y}) - \mathcal{H}(\mathcal{X},\mathcal{Y}). \tag{25}$$

It is evident that two Eqs. (23) and (25) are equivalent in the classical theory but that they behave differently when they are extended to quantum systems. The difference would lie in the term for quantum discord. In the context of quantum mechanics, $\mathcal{H}$ would explain the Von-Neumann entropy $\mathcal{S}$ which is defined in terms of density matrix as

$$\mathcal{S} = -Tr_{\mathcal{X}}\rho_{\mathcal{X}}\log_2\rho_{\mathcal{X}}. \tag{26}$$

Thus, for a bipartite system, Eq (25) would take the following form:

$$\begin{aligned} \mathcal{I}(\mathcal{X};\mathcal{Y}) &= \mathcal{S}(\mathcal{X}) + \mathcal{S}(\mathcal{Y}) - \mathcal{S}(\mathcal{X},\mathcal{Y}) \\ &= -Tr(\rho_{\mathcal{X}}\log_2\rho_{\mathcal{X}}) - Tr(\rho_{\mathcal{Y}}\log_2\rho_{\mathcal{Y}}) \\ &+ Tr(\rho_{\mathcal{XY}}\log_2\rho_{\mathcal{XY}}). \end{aligned} \tag{27}$$

In addition, Eq. (23) would change in the quantum system. Since the conditional entropy requires the state $\mathcal{X}$ to be in a given state A, an optimized measurement approach will need to be adopted [48]. This will be achieved by introducing some projection operators. Applying the optimized measurement approach would change Eq (23) into its quantum counterpart below [49, 50]:

$$\mathcal{J}(\mathcal{X}; A) = \mathcal{S}(\mathcal{X}) - min_{\pi_i}[\mathcal{S}(\rho_{\mathcal{X}|\pi_i^A})]. \tag{28}$$

Evidently, two expressions for $\mathcal{J}$ differ in their second term which is now the optimized measurement of state $\mathcal{X}$ corresponding to projections π_i^A. The state is given by [51]

$$\rho_{\mathcal{X}|\pi_i^A} = \frac{1}{P_i}\pi_i^A \rho_{\mathcal{X},A}\pi_i^A, \tag{29}$$

where, P_i is equal to $Tr_{\mathcal{X},A}(\pi_i^A \rho_{\mathcal{X},A})$. This is the probability for each measurement to have a given value. The difference between two mutual information pieces, i.e., mutual classical and quantum information, is defined as the quantum discord [40]:

$$\mathcal{D}(A:B) = \mathcal{I}(A:B) - \mathcal{J}(A:B) = \mathcal{S}(A) + \mathcal{S}(B) - \mathcal{S}(A,B) - \mathcal{S}(A) + min_{\{\pi_i^A\}}\mathcal{S}(A|B). \tag{30}$$

In order to calculate the quantum discord, we use the global discord to quantify the quantum discord. Applying the approach introduced in Ref. [45], the global discord is calculated in the present contribution from the following density matrix:

$$\rho = \frac{1-p}{2}\hat{I} + p\hat{\rho}_{a,b}, \qquad 0 \le p \le 1, \tag{31}$$

where, $\hat{\rho}_{a,b}$ is given by Eq. (15).

3.1 Environment in the ground state

We assume that each oscillator in the environment is initially in the ground state $|0\rangle$,

$$|\Phi_\varepsilon(t=0)\rangle = \left(\prod_i |0_i\rangle_a\right) \otimes \left(\prod_i |0_i\rangle_b\right), \tag{32}$$

where, index i runs over all the environment oscillators. It is worth mentioning that by considering the environment in the ground state, the environment states correspond to the composition of even and odd coherent states are entangled with the spin systems of different phases. A similar phenomenon happens in the case of the decoherence process of the spin-boson model [15]. Therefore, Zurek's claim may be generalized by saying that the interaction of the spin systems with the boson environment causes the environment states to approach coherent states [16, 15]. Moreover, using Relation (14), we will have:

$$r(t) = \exp\left[\sum_i -4\frac{|g_i|^2}{\omega_i^2}(1-\cos\omega_i t)\right]. \tag{33}$$

In addition, by using a spectral density $\sum_i |g_i|^2 \to \int_0^\infty d\omega J(\omega)$ [18],

$$J(\omega) = 4J_0\omega e^{-\omega/\Lambda}, \tag{34}$$

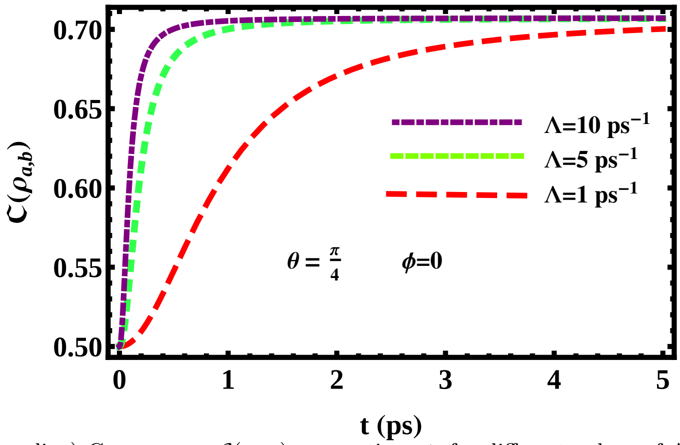

Fig. 1. (Color online) Concurrence $\mathcal{C}(\rho_{\mathsf{a,b}})$ versus time, t, for different values of Λ, with special cases of θ, and ϕ.

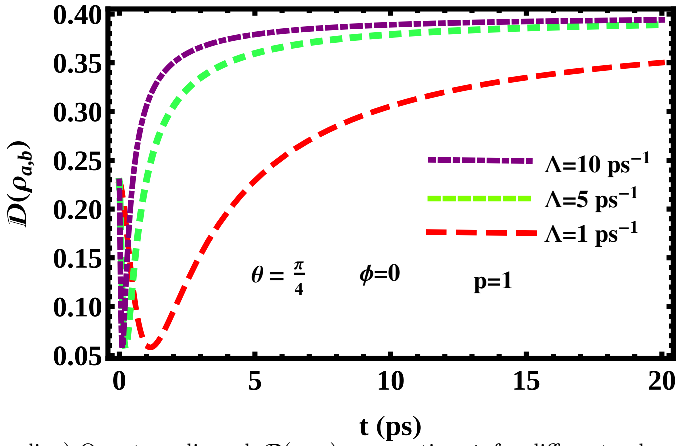

Fig. 2. (Color online) Quantum discord, $\mathcal{D}(\rho_{\mathsf{a,b}})$, versus time t, for different values of Λ, with the special cases of $\theta = \pi/4$, $\phi = 0$ and $p = 1$.

in which Λ, as a criterion of interaction with environment, is called cut-off frequency, and J_0 is a dimensionless constant, we can rewrite the Relation (33) as follows:

$$r(t) = \frac{1}{\sqrt{1 + \Lambda^2 t^2}}. \tag{35}$$

in which is supposed $J_0 = 1$.

Figure 1 illustrates the behaviour of Concurrence $\mathcal{C}(\rho_{a,b})$ as a function of time t. According to this Figure, the environment causes the concurrence and, thereby, the entanglement of the quantum state to increase. It is also seen in this Figure that an increase in Λ, i.e., the contribution by the environment, causes the maximal entanglement in this case to obtain earlier.

Figure 2 illustrates the behaviour of the global discord as a function of time, t. Clearly the correlation between the states of two-state system increases with both time elapsed and increasing interaction of environment states with the qubit system. The quantum discord

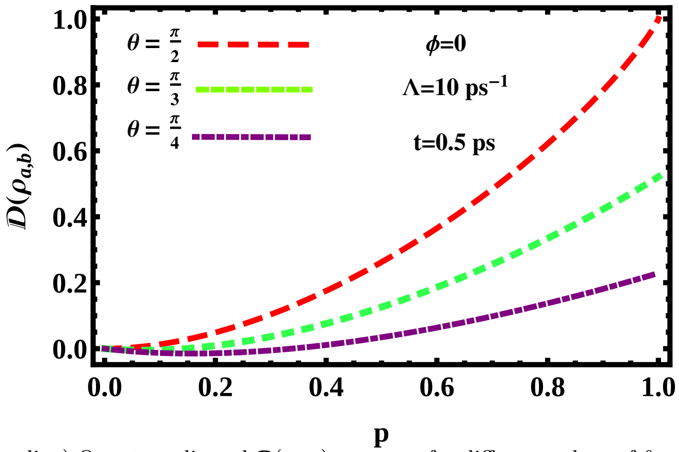

Fig. 3. (Color online) Quantum discord $\mathcal{D}(\rho_{\mathsf{a,b}})$ versus p, for different values of θ, with the special cases of Λ and ϕ, at $t = 0.5$ (ps).

follows the scenario just described, except in the case of the onset of the interaction between the spin system and the environment. According to Figure 3 the correlation increases, as already expected, with increments in the p-parameter, as a function of the purification control parameter.

3.2 Environment in the thermal state

In this subsection, we study a more general case in which the environment has a nonzero temperature, i.e. the environment oscillators are in the thermal states at the initial time, i.e.,

$$\hat{\rho}_\varepsilon = \bigotimes_i \frac{1}{Z_i(\beta)} e^{-\beta \hat{H}_{\varepsilon_i}} = \bigotimes_i \frac{1}{Z_i(\beta)} e^{-\beta\hbar\omega_i \hat{n}_i}, \tag{36}$$

where, $\beta = 1/k_B T$ in which k_B is the Boltzmann's constant. The partition function for the ith mode is defined by

$$Z_i(\beta) = Tr(e^{-\beta \hat{H}_{\varepsilon_i}}) = \frac{1}{1 - e^{-\beta\hbar\omega_i}}. \tag{37}$$

In thermofield dynamics [38, 39], a thermal vacuum state is constructed by doubling the degrees of freedom in the Hilbert space so that the ensemble average values of physical quantities, $\hat{A}$, become the thermal vacuum expectation values of the operators corresponding to those physical quantities,

$$\langle \hat{A} \rangle = \langle 0(\beta)|\hat{A}|0(\beta)\rangle = \frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} \langle n|\hat{A}|n\rangle. \tag{38}$$

Using the thermofield dynamics approach, we attribute the thermal vacuum states $|0(\beta)\rangle_i$ to the oscillators in the bath. The thermal vacuum state can be written as

$$|0(\beta)\rangle_i = \frac{1}{Z_i^{1/2}(\beta)} \sum_{n_i} e^{-\beta\hbar\omega_i n_i/2} |n_i, \tilde{n}_i\rangle = \hat{U}_i(\beta)|0_i, \tilde{0}_i\rangle, \tag{39}$$

where, the Bogoliubov transformation $\hat{U}_i(\beta)$ may be defined as follows [38]:

$$U_i(\beta) = e^{-i\hat{G}_i(\beta)} = \exp\left[-i\vartheta(\beta)\left(\hat{\tilde{a}}_i \hat{a}_i - \hat{\tilde{a}}_i^\dagger \hat{a}_i^\dagger\right)\right], \tag{40}$$

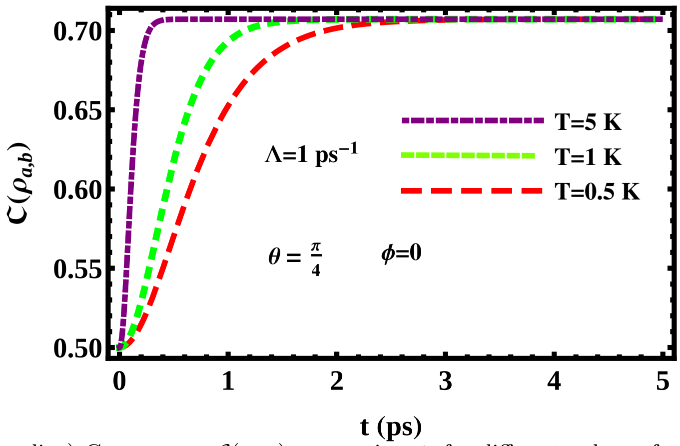

Fig. 4. (color online) Concurrence $\mathcal{C}(\rho_{\mathrm{a,b}})$ versus time t, for different values of temperature T, with special cases of Λ, θ and ϕ.

where, $\tanh\vartheta(\beta) = \exp\left[-\hbar\beta\omega/2\right]$.

The whole state of the environment in the thermal equilibrium, that is, thermal vacuum states, is given by,

$$|\Phi_\varepsilon(t=0)\rangle = \prod_i |0(\beta)\rangle_i, \tag{41}$$

where, the index i runs over all the environmental oscillators. In this case, the thermal annihilation and creation operators, respectively, can be naturally introduced by the following relations:

$$\begin{aligned} \hat{a}(\beta) &= \hat{U}(\beta)\hat{a}\hat{U}^\dagger(\beta) = \cosh\vartheta(\beta)\hat{a} - \sinh\vartheta(\beta)\hat{\tilde{a}}^\dagger, \\ \hat{\tilde{a}}(\beta) &= \hat{U}(\beta)\hat{\tilde{a}}\hat{U}^\dagger(\beta) = \cosh\vartheta(\beta)\hat{\tilde{a}} - \sinh\vartheta(\beta)\hat{a}^\dagger. \end{aligned} \tag{42}$$

Using the relations in (42), we may obtain (43) bellow:

$$D\left(\lambda_i(t)\right)|0(\beta)\rangle_i = \prod_i D\left(\lambda_i(t)\cosh\vartheta(\beta)\right)|0_i\rangle \otimes \tilde{D}\left(-\lambda_i^*(t)\sinh\vartheta(\beta)\right)|\tilde{0}_i\rangle, \tag{43}$$

which is proportional to the thermal coherent state. Consequently, with some calculations, $r(t)$ is given by

$$r(t) = \exp\left[-\sum_i \frac{4|g_i|^2}{\omega_i^2}\coth\frac{\omega_i\beta}{2}\left(1-\cos\omega_i t\right)\right]. \tag{44}$$

Moreover, using the spectral density (34), $r(t)$ is approximately given by [18]:

$$r(t) \approx \frac{1}{\sqrt{1+\Lambda^2 t^2}}\frac{\pi t}{\beta\sinh\left(\frac{\pi t}{\beta}\right)}. \tag{45}$$

Figure 4 gives the time evolution of concurrence $\mathcal{C}(\rho_{a,b})$, when the temperature of environment is changed. According to this Figure, both the increased temperature and the enhanced

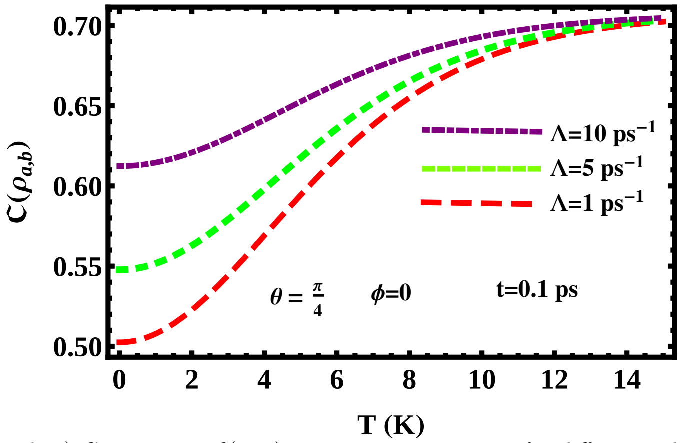

Fig. 5. (color online) Concurrence $\mathcal{C}(\rho_{\mathsf{a,b}})$ versus temperature T, for different values of Λ, with special cases of θ and ϕ at $t = 0.1$(ps).

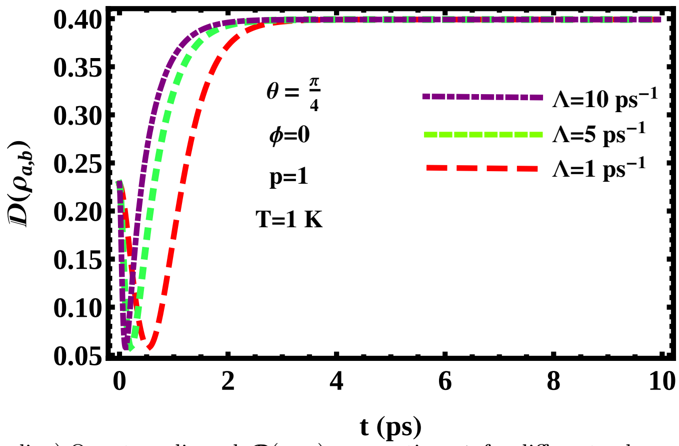

Fig. 6. (color online) Quantum discord, $\mathcal{D}(\rho_{\mathsf{a,b}})$, versus time, t, for different values of temperature, T, with the special cases of Λ, θ and ϕ.

contribution by the environment cause the maximal entanglement to be obtained earlier. Figure 5 depicts the temperature dependence of concurrence at the special time $t = 0.1$(ps) for different values of Λ. This Figure also confirms the results presented in Figure 4.

The time dependence of quantum discord is shown in Figure 6, indicating that the correlation between the spin sub-systems in the present system increases with both increments in temperature and the interactions between the environment and the systems. Figure 7 illustrates the temperature dependence of the quantum correlation for the quantum system at a specific time. At earlier times, the quantum discord decreases with increasing temperature at a fixed time. Subsequently, the quantum discord increases when the interaction of the environment is considered be weak. The same depicts exhibits the role of cut-off frequency associated with the environment.

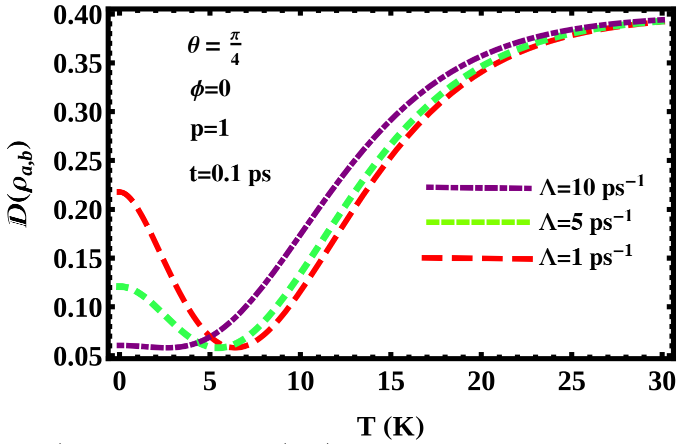

Fig. 7. (color online) Quantum discord, $\mathcal{D}(\rho_{\mathsf{a},\mathsf{b}})$, versus temperature, T, for different values of Λ, with special cases of θ and ϕ at $t = 0.1$(ps).

4 Conclusion and Remarks

We have studied the entanglement dynamics of two entangled spin systems coupled with a common environment consisting of a large number of harmonic oscillators. In the studied model, we have investigated the impacts of both the interaction and the temperature of the environment on the dynamics of quantum correlation of two initially entangled spins. Entanglement and quantum discord were studied using the concurrence and global quantum discord criteria and the entanglement dynamics of this spin-boson model was analytically investigated. Our study revealed that the interaction of the spin sub-systems with the common environment led to entangled environmental states of different phases, each composed of even and odd coherent states. Moreover, applying the thermofield approach, we have demonstrated that the common environment temperature as well as its interaction with the system causes the entanglement phenomenon to become robust as the system evolves in time.

Acknowledgement

Authors gratefully acknowledge Mr. Joseph Bass, University of North Florida (UNF), for some useful discussions and editing of this paper. This work was partly sponsored by the National Natural Science Foundation of China under Grants No 61550110245, 61322501, 61275183, and 61574127 within the National Program for Special Support of Top-Notch Young Professionals, the Program for New Century Excellent Talents (NCET-12-0489) in University, the Fundamental Research Funds for the Central Universities, and the Innovation Joint Research Center for Cyber-Physical-Society System. Also, M. B. H. wishes to thank The Office of Graduate Studies of the University of Isfahan, and Z. H. and B. M. wish to thank Isfahan University of Technology for their support.

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