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Implementations of TSP-VRP Variants for Distribution Problem

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Abstract

Problems of distributing goods from one or more depots to customers generally become substantial problems for many companies or factories. Those distribution problems have been modeled using TSP (Transportation Salesman Problem) or VRP (Vehicle Routing Problem) and its variants. The refinement of the variants is intended to have the fittest models for real life problems concerning distribution problems. In this paper, variants of TSP and VRP are implemented using Delphi programming language. The methods of TSP used in this paper are sequential insertion heuristic, nearest neighbor heuristic, and cheapest link. The TSP variants which are implemented are multiple TSP (mTSP), and TSP with Time Window (TSPTW). On the other hand, VRP variants used are CVRP, MDVRP, MTVRP, VRPTW, MFVRP, VRPSDP and the method for each variant is the sequential insertion. The application program obtained is tested with the data from TSPLIB, a library of sample instances for the TSP and related problems from various sources and of various types. We also provide some examples on distribution problems.

Keywords: TSP variants, VRP variants, distribution problems.

Introduction

Distribution problem is one of the important issues in a number of companies. Distribution relates to the problem of providing goods or services from a depot (company) to customers. The condition of customer location that is spread over an area often causes the vehicle to be driven far and inefficient. The expected efficiency is time, the length of the route, and the number of goods that can be brought [13]. The problem can be modeled using graph theory through the Traveling Salesman Problem (TSP) and Vehicle Routing Problem (VRP).

Some constraints which appear in the distribution problems are condition of customer locations which are spread, the number various demands, the limited capacity of the vehicles, which eventually lead to the inefficient route.

The research on TSP problems and a method for finding the solution can be seen in [2], [3]. To adjust with constrains, variants of the TSP can be built. For example, TSP variant with additional constraints of each point (customer) visited having a time window is TSP with Time Windows (TSPTW). The research on methods to find solutions of TSPTW can be seen in [10], [11]. TSP variant with an additional starting point and the destination point that can be more than one depot is Multiple TSP (MTSP). Research on methods to find a solution of the MTSP can be seen in [1], [12]. Another variant is Dynamic TSP (DTSP). It is TSP variant with non-fixed destination point addition so that there is addition or subtraction point of destination at the time of constructing the route [6]. Another one is TSP with Precedence Constraints (TSPPC), that is TSP variant with the additional constraint that is the sequence of points that should be visited first. These variants are developed in order to model applications in the better real world that fits with the necessary requirements.

TSP can be expanded into Vehicle Routing Problem (VRP) problems. VRP can be defined as problems of searching for the optimal route to deliver goods from one or more depot to the customers, or take goods from acustomerto one or more depots, which meet a number of constraints. As TSP, a number of new variants of VRP appear to adjust to the constraints such as travel costs, a number of depots, delivery time, and any goods pick-up instead of goods. These variants are developed to model VRP applications to be better as the necessary requirements and constraints. A number of VRP variants identified include:

- Capacitated Vehicle Routing Problem (CVRP) where each vehicle has uniform capacity with only one commodity [17],
- MultiDepot Vehicle Routing Problem (MDVRP) which is VRP with more than one depot which serves customers [8]. MDVRP solution characteristics can be seen in [19].
- Vehicle Routing Problem with Backhauls (VRPB) is VRP in which customers can ask one of the delivery or pickups with the requirement that in each vehicle route, pickup is done after all the deliveries to customers are completely done [17],
- Vehicle Routing Problem with Time Windows (VRPTW) which adds time window on each customer to receive the goods [9],
- Multiple-Trip Vehicle Routing Problem (MTVRP) is VRP which enables each vehicle to have multiple trips when distributing goods and time horizon for overall customer service, so each vehicle can have more than one route in the planning period. MTVRP solution characteristics can be seen in [20].
- Mixed Fleet Vehicle Routing (MFVRP) which is the development of VRP with the addition of constraints for a number of vehicles which has a different capacity [17].

To facilitate the use of various VRP variants that is suitable with the problems, the modifications and the combination of various methods of TSP and TSP variants, as well as variants of VRP into a computer program are needed to make, and conducting test of the application results which has been made is required.

Mathematical Model and Solution

In mathematics, TSP formulations can be expressed as a graph $G = (V,A)$ with $V = \{1, 2, 3, ..., n\}$ denote the set of graph's vertices that shows the location of the city and $A = \{(i,j) \mid i,j \in V, i \neq j\}$ as the set of edges denote connecting roads between cities. Suppose x_{ij} is the distance from city i to city j, then the TSP goal is to minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i,j}$. Research for method of finding TSP solutions, for example is performed by [3], [5] and [6].

The formulation of TSPTW can be defined as a graph $G = (V, A)$ with V is the set of graph's vertices and A as the set of edges. The set V has N vertices, including a depot which is denoted by 0 and a set of a number of customers expressed by $V \setminus \{0\}$. The edge set consists of a pair of vertices namely $A = \{(i, j) \mid i, j \in V, i \neq j\}$. Each edge (i, j) has travel cost c_{ij} between vertex i and j. The research for methods for finding TSPTW solutions, for example, has made by [10] and [11].

MTSP formulation can be defined as a graph $G = (V, A)$ with V is the set of vertices and A is set of edges. The set V has N vertices with the starting point (depot) can be more than one. The edge set consists of a pair of vertices namely $A = \{(i, j) \mid i, j \in V, i \neq j\}$. Each side (i,j) has a *travel cost* c_{ij} between vertex i and j. The objective function of MTSP is to find Hamiltonian cycle with minimum cost if there is an additional more than one starting point and the destination point [1]. The research on methods for finding MTSP solution can be seen in [12], [14] and [18].

Before implementing a VRP variant, formulations of VRP and its variants are given. VRP problem can be illustrated by the graph, for example$G = (V, E)$ is a simple connected graph with:

- The set $V = \{0, N\}$ is the vertice set on the graph, with 0 is the depot and $N = \{1, 2, 3, ..., n\}$ is a set of customers to be served.
- The set $E = \{(i, j) \mid i, j \in V, i \neq j\}$ is the edge set that connects the depot with customers and between customers.
- Distance from the customer i to customer j is denoted by $c_{ij.}$
- Distance from the depot to the customer i is denoted by c_{0i} and from customers i to depot is denoted by c_{i0}.
- $K = \{k_1, k_2, ..., k_m\}$ is the set of identical vehicles used to serve customers.
- The vehicle capacity is denoted by Q.
- The demand of each customer i is denoted by $q_i = \forall i \in \{1, 2, ..., n\}$.

Mathematics formulation of the objective function:

$$\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}^{k}$$

where

$$x_{ij}^{k} = \begin{cases} 1, \text{if } k \text{ } is \text{ run from } i \text{ to } j, i \neq j \\ 0, \text{for the others} \end{cases}$$

Constraints that must be met in resolving VRP problems are as follows:

1. Every customer is visited only once and only by one vehicle.

$$\sum_{k\in K} \sum\nolimits_{i\in N}^{k} x_{ij} = 1, \forall i \in V - \{0\}$$

2. Each vehicle route begins and ends at the depot.

$$\sum_{j\in N-\{0\}} x_{oj}^{k} = 1, \forall k \in K$$

$$\sum_{j\in N-\{0\}} x_{jo}^{k} = 1, \forall k \in K$$

3. The total number of requests that can be served in one route by a vehicle does not exceed the capacity of the vehicle.

$$\sum_{i\in N-\{0\}} q_i \sum_{j\in N} x_{ij}^{k} \leq Q, \forall k \in K$$

4. Vehicles leave customers who are already visited

$$\sum_{i\in N} x_{ih}^{k} - \sum_{j\in N} x_{hj}^{k} = 0, \forall h \in V - \{0\}, \forall k \in K$$

5. Limit value

$$x_{ij}^{k} \in \{0,1\}, \forall i \in N, j \in N, k \in K$$

VRP development occurs with certain constraints. Below are some models that are the development of the VRP.

Capacitated Vehicle Routing Problem (CVRP) is characterized that each vehicle has the same capacity with only a kind of commodity. CVRP can be described as follows: Suppose $G = (V', E)$ is undirected graph where $V' = \{0, 1, ..., N\}$ is a set of $n + 1$ vertices, and E is set of edges. Vertex 0 states depot and the set of vertices $V = V' \setminus \{0\}$ states n customer. Cost d_{ij} is associated with each edge $\{i, j\} \in E$. Goods as much as q_i units supplied from the depot 0 (q_0 is assumed to be 0). A number of m identical vehicles with a Q capacity are placed in depot 0 and should be used to bring the goods to the customer. One route is expressed as the shortest cycle of the graph G passing depot 0 so that the total number of demand of all visited vertices does not exceed the capacity of the vehicles. The research of a method for searching CVRP solution can be seen in [7].

Multi Depot Vehicle Routing Problem (MDVRP) is VRP with more than one depot that serves customers [22]. MDVRP objective function is:

$$Min \sum_{i\epsilon I\cup D} \sum_{d\epsilon I\cup J} \sum_{k\epsilon K} C_{id} X_{id}^{k} = C_{1d} X_{1d}^{1} + \cdots + C_{nd} X_{nd}^{n}$$

$$x_{id}^{k} = \begin{cases} 1, \text{if the vehicle is run from } i \text{ to } d \\ 0, \text{for the others} \end{cases}$$

$$Z_{id} = \begin{cases} 1, \text{if the customer } i \text{ is allocated for depot } d \\ 0, \text{for the others} \end{cases}$$

Z_{id} is the number of groups at each depot.

MDVRP have constraints that each customer is only visited once and only by one vehicle, the total demand of each customer in one route may not exceed the capacity of the vehicle, each vehicle must leave the customers who have been visited, every route is served once, and a customer can be placed on a depot if there is a route from the depot and it passes the customer. The research of the method to find MDVRP solution can be seen in [19], [21] and [22].
The characteristics of another VRP variant solution which have been studied are MDVRP and VRPTW [19]. MTVRP solution characteristics and its application to the distribution problems can be seen in [20].

Research Method

Methodology in the implementation of TSP and VRP variants were;

1. Conducting studies on TSP and VRP variants needed in the field.
2. Identifying necessary and sufficient condition in accordance with the problems and selecting strategies/algorithms in solving the problems in optimizing in industry
3. Developing a computer program that contains the implementation of a number of TSP and VRP variants
4. Internal testing for the functionality aspects using Black Box method and usability aspects with the questionnaire method. Results testing using a standard test data and external test using data taken from the field were also done, and any necessary revision was also conducted.
5. Making program manual use.

External experiment was done in black-box testing, and it was done by the students who did internship (KPL) in the industry, with the real data obtained from the survey. For internal testing, data from TSPLIB was used, as a well-known provider of benchmark data for various cases of TSP and VRP, with solutions that have been recorded and recognized internationally. For other variants that are not available in TSPLIB, data from other sources such as the VRP Web (http: //neo.lcc.uma.es/vrp and http: //www.bernabe.dorronsoro.es/vrp/) were used.

Results and Discussion

To ease the calculation, a computer program in the form of an application of TSP problems solving and its variants and VRP and its variants is made. In general TSP problem, the method implemented is the *Sequential Insertion Heuristic, Nearest Neighbor Heuristic,* and *Cheapest Link.*

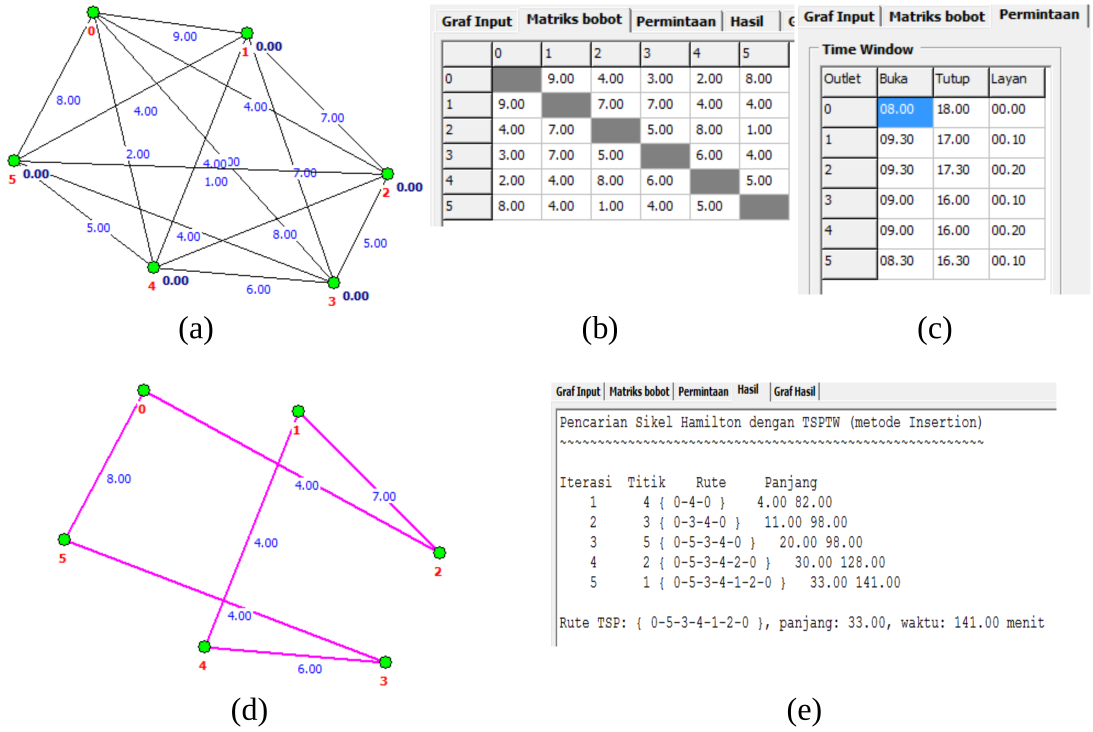
TSP variants implemented is TSP *Multiple* (MTSP) and *TSP with Time Window* (TSPTW). In VRP problem, the variants implemented are CVRP, MDVRP, MTVRP, VRPTW, MFVRP, VRPB, VRPSDP, VRPDP, and SLVRPSDP. The method used to solve the problems in each VRP variant is *Sequential Insertion* because this method can still provide optimal results. Applications are created with Delphi language, and it is run on a Windows-based PC.

Input data are placed in an interactive form (using a *mouse* and *keyboard),* and the application input are:

- Vertices of graph (as a *customer* or *depot,* or as *linehaul* or *backhaul customer* in VRPB case)
- Weight between the edges (the weight can be considered as the distance between points)
- The other parameters which are required: for TSPTW it is *time window* data of each point (load time, closing time, and length of service), for the MTSP is the number of salesmen and the starting point of the route, and for some other variants of VRP is a request (delivery or pickup) at each customer, the number of vehicles and each capacity, and the service time of each customer.

All manual input can be recorded in a data file to be used in other time.

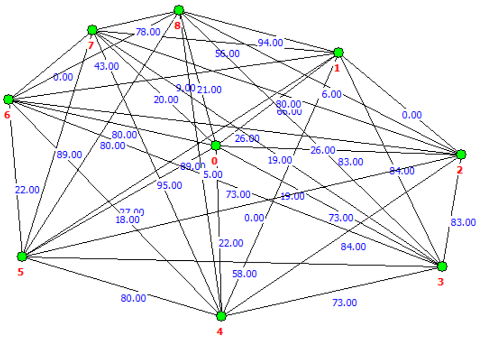
Here are the examples of some problems using programs created:



	0	1	2	3	4	5
0		9.00	4.00	3.00	2.00	8.00
1	9.00		7.00	7.00	4.00	4.00
2	4.00	7.00		5.00	8.00	1.00
3	3.00	7.00	5.00		6.00	4.00
4	2.00	4.00	8.00	6.00		5.00
5	8.00	4.00	1.00	4.00	5.00	

Outlet	Buka	Tutup	Layan
0	08.00	18.00	00.00
1	09.30	17.00	00.10
2	09.30	17.30	00.20
3	09.00	16.00	00.10
4	09.00	16.00	00.20
5	08.30	16.30	00.10

Figure 1: Problems of TSPTW in 6 vertices. (a) initial Graph, (b) Distance table, (c) Time window, (d) resulted route, (e) stages of completion and the final result



(a)

Graf Input | Matriks bobot | Hasil | Graf Hasil

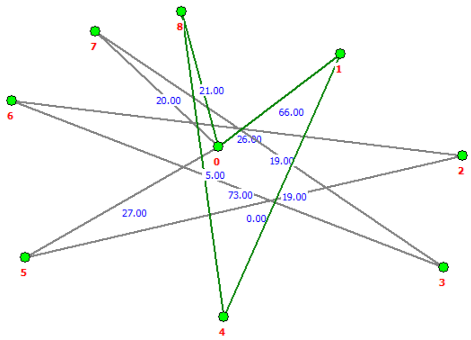
	0	1	2	3	4	5	6	7	8
0		66.00	83.00	73.00	22.00	27.00	80.00	20.00	21.00
1	66.00		0.00	84.00	19.00	89.00	9.00	56.00	94.00
2	83.00	0.00		83.00	84.00	0.00	26.00	80.00	6.00
3	73.00	84.00	83.00		73.00	58.00	73.00	19.00	26.00
4	22.00	19.00	84.00	73.00		80.00	18.00	95.00	5.00
5	27.00	89.00	0.00	58.00	80.00		22.00	89.00	80.00
6	80.00	9.00	26.00	73.00	18.00	22.00		0.00	43.00
7	20.00	56.00	80.00	19.00	95.00	89.00	0.00		78.00
8	21.00	94.00	6.00	26.00	5.00	80.00	43.00	78.00	

(b)

Graf Input | Matriks bobot | Hasil | Graf Hasil

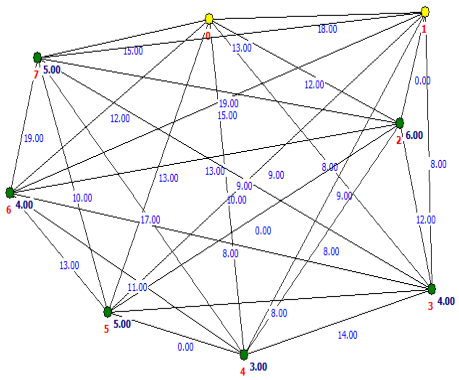
```
Pengelompokan customer pada salesman:
Salesman 0: { 2,3,5,6,7 }
Salesman 1: { 1,4,8 }

Pembentukan rute:
Salesman 0:
Rute awal: { 0-0 }, jarak:  0.00
- Titik: 7, Rute: { 0-7-0 }, jarak: 40.00
- Titik: 3, Rute: { 0-3-7-0 }, jarak: 112.00
- Titik: 5, Rute: { 0-5-3-7-0 }, jarak: 124.00
- Titik: 6, Rute: { 0-5-6-3-7-0 }, jarak: 161.00
- Titik: 2, Rute: { 0-5-2-6-3-7-0 }, jarak: 165.00

Salesman 1:
Rute awal: { 0-0 }, jarak:  0.00
- Titik: 8, Rute: { 0-8-0 }, jarak: 42.00
- Titik: 4, Rute: { 0-4-8-0 }, jarak: 48.00
- Titik: 1, Rute: { 0-1-4-8-0 }, jarak: 111.00

Rute akhir:
Salesman 0:
- Rute : { { 0-5-2-6-3-7-0 } }, jarak: 165.00
Salesman 1:
- Rute : { { 0-1-4-8-0 } }, jarak: 111.00
```

(c)



(d)

Figure 2: Problems of MTSP in 9 vertices and 2 salesmen. (a) initial Graph, (b) Distance table, (c) stages of completion and the final result, (d) resulted route



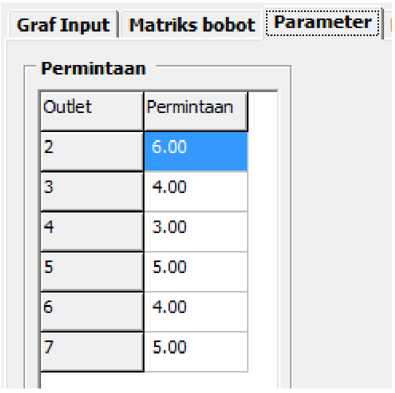
(a)

Graf Input | Matriks bobot | Parameter | Hasil | Graf Hasil

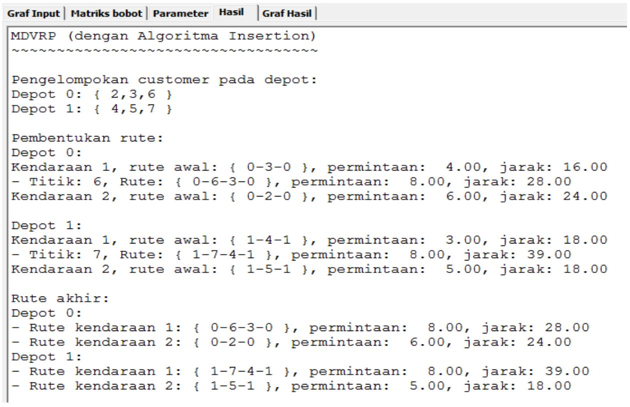
Matriks Bobot

	0	1	2	3	4	5	6	7
0		18.00	12.00	8.00	10.00	13.00	12.00	15.00
1	18.00		0.00	8.00	9.00	9.00	15.00	13.00
2	12.00	0.00		12.00	8.00	0.00	13.00	19.00
3	8.00	8.00	12.00		14.00	8.00	8.00	9.00
4	10.00	9.00	8.00	14.00		0.00	11.00	17.00
5	13.00	9.00	0.00	8.00	0.00		13.00	10.00
6	12.00	15.00	13.00	8.00	11.00	13.00		19.00
7	15.00	13.00	19.00	9.00	17.00	10.00	19.00	

(b)



(c)



(d)

Figure 3: MDVRP Problems. (A) input of vertices, (b) weight matrix, (c) Entries of customer demand on VRP problems, (d) Output in text form.

Functionality test used 17 scenarios to examine whether the program was going well. In general, the testing had proven that the program was successful as it was expected although there were some minor improvements made for example the addition of the ability to accept decimal numbers input. Usabilityaspect test was conducted by 7 student respondents who did KPL with distribution topics with TSP / VRP and using the program which was created. Test results showed in general that 70.23% of respondents agreed that the program made had met the *usability* aspect, and the remaining 19.04% strongly agreed and 0.75% disagreed. General assessment of the *usability* aspect was 3.08 of a scale 1-4. Respondents also gave a number of inputs required for improvement.

For the program testing, the summary of the results is as follows:

Table 1: Summary of test results

Variant	Problem	Description	Result	Best result
TSP	gr17	17 vertices	1943 (starting vertex: 1)	2085
	gr120	120 vertices	8098	6942
	brg180	180 points	4560	
MDVRP	p01	4 depots, 50 customers	665.6; Maximum vehicle: 4	576.87
	p03	5 depots, 75 customers	966.28; Maximum vehicle: 3	641.19
	p04	2 depots, 100 customers	1193.21; Maximum vehicle: 8	1001.59

Problems for TSP were gr17 (17 vertices), gr120 (120 vertices), and brg180 (180 vertices). Problems for VRP were P01 (50 vertices), P03 (75 vertices), and P04 (100 vertices). Results obtained were still below the recorded best results, but the difference was relatively small.

Conclusion

Computer program with Delphi language has been created for the implementation of various TSP and VRP variants. The limited testing or experiment to the standard data shows that the results have not achieved the best results from the data providers (TSPLIB and the VRP web). Functionality testing and *usability* provide input for improvement of application. Further testing or experiment needs to be done for the other variants and the other data sets so that the application program can be used for a wide possible variety of data. The addition of other variants, especially for the VRP, and the route searching method that is proven to provide better and stable results are also needed.

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