Technical Report

Model for Mathematical Analysis and Predicting the Shrinkage-Induced Final Length of Fired Clay Products

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Abstract: Model for mathematical analysis and predicting the shrinkage-induced final length of fired clay has been derived. Different clays were sorted, prepared, molded into shape, dried in air and fired in the furnace to a temperature of 1200^0C. Initial and final fired lengths measured were used for calculating the fractional volume shrinkage (using conventional equation). Fractional volume shrinkage was also calculated in terms of the fractional fired shrinkage. The derived model;

$$L_2 = L_1\left[- \alpha^3+3\alpha^2 - 3\alpha + 1\right]^{1/3}$$

was found to be constituted by three parameters, initial length, L_1 final length L_2 and fired shrinkage α. The model-predicted final fired length L_2 was found to depend on the values of the initial length and fired shrinkage. The validity of the model was found to stem directly on the expression $(L_2/L_1)^3 = [- \alpha^3+3\alpha^2 - 3\alpha + 1]$ where both sides of the expression are correspondingly almost equal to 1. The maximum deviation of the model-predicted fired length L_2 from the corresponding experimental values is less than 9% which is within the acceptable range of deviation limit for experimental results. It was also found that the cube of the ratio of final fired length to initial dried length is equal to 1-fractional volume shrinkage due to firing. [Report and Opinion 2010;2(6):28-33]. (ISSN:1553-9873).

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1. Introduction

Report from studies (Reed,1988) indicates that apart from firing proceeding in three stages, preliminary reactions occur which include binder burnout, elimination of gaseous product of decomposition and oxidation, sintering as well as cooling which may include thermal and chemical annealing.

Following research works (Barsoum,1997;Viewey and Larrly,1978;Keey, 1978) carried out to investigate shrinkage of clay during drying, porosity was found to influence the swelling and shrinkage behaviour of clay products of different geometry.

Reed (1988) further reported that drying occurs in three stages; increasing rate, constant and decreasing rate. The researcher pointed out that during the increasing rate; evaporation rate is higher than evaporating surface hence more water is lost. At constant rate, the evaporation rate and evaporation surface are constant. Further investigation revealed that shrinkage occurs at this stage.

Keey (1978) in a similar study also suggested that at this stage, free water is removed between the particles thereby decreasing the inter-particle separation and eventually resulting in shrinkage.

During the decreasing rate, particles make contacts as water is removed, hence causing shrinkage to cease.

A model for calculating the volume shrinkage resulting from the initial air-drying of wet clay has been derived (Nwoye,2008) . The model;

$$\theta = \gamma^3 - 3\gamma^2 + 3\gamma \qquad (1)$$

calculates the volume shrinkage when the value of dried shrinkage γ, experienced during air-drying of wet clays is known. The model was found to be third-order polynomial in nature. Olokoro clay was found to have the highest shrinkage during the air drying condition, followed by Ukpor clay while Otamiri clay has the lowest shrinkage. The result of the investigations reveals that volume shrinkage increases with increase in dried shrinkage until maximum volume shrinkage was reached, hence a direct relationship.

Derivation of model for the evaluation of overall volume shrinkage in molded clay products (from initial air-drying stage to completion of firing at a temperature of 1200^0C) has been carried out (Nwoye et al.,2008) . It was observed that the overall volume shrinkage values predicted by the model were in

agreement with those calculated using conventional equations. The model;

$$S_T = \alpha^3+\gamma^3-3(\alpha^2+\gamma^2)+3(\alpha+\gamma) \quad (2)$$

was found to be dependent on direct values of the dried γ and fired shrinkage α for its precision. It was also discovered that overall volume shrinkage increases with increase in dried and fired shrinkages until overall volume shrinkage reaches maximum.

Nwoye (2009a) derived a model for calculating the quantity of water lost by evaporation during oven drying of clay at 90^0C. The model;

$$\gamma = \exp[(\ln t)^{1.0638} - 2.9206] \quad (3)$$

indicated that the quantity of evaporated water, γ during the drying process is dependent on the drying time t, the evaporating surface being constant. It was discovered that the validity of the model is rooted in the expression $(\text{Log}\beta + \ln\gamma)^N = \ln t$.

In processing a bioceramic material, Nwoye (2009b) derived a model for predictive analysis of the quantity of water evaporated during the primary-stage processing of a bioceramic material sourced from kaolin .The model;

$$\alpha = e^{(\ln t/2.1992)} \quad (4)$$

predicts the quantity of water α, evaporated at 110^0C, during the drying process being dependent on the drying time t, where the evaporating surface is constant. The validity of the model was discovered to be rooted on the expression $(\ln t/\ln\alpha)^N = \text{Log}\beta$ where both sides of the expression are correspondingly approximately equal to 3. The respective deviation of the model-predicted quantity of evaporated water from the corresponding experimental value was found to be less than 22% which is quite within the acceptable deviation range of experimental results.

Nwoye et al., (2009) derived a model for quantifying the extent and magnitude of water evaporated during time dependent drying of clay. The model;

$$\gamma = \exp((\ln t/2.9206)^{1.4}) \quad (5)$$

reveals that the quantity of evaporated water γ during the drying process (at 90^0C) is dependent on the drying time, t the evaporating surface being constant. It was found that the validity of the model is rooted in the expression $\ln\gamma = (\ln t/\text{Log}\beta)^N$ where both sides of the expression are correspondingly almost equal.

Following the tendency for water to be absorbed in a hot-humid environment, Nwoye and Mbuka (2009) derived a model for prediction of the quantity of absorbed water in clay materials exposed to hot-humid environment. These clay materials were prepared using different grain sizes; <100μm, 100-300μm, 300-1000 μm and their respective mixtures. The derived model;

$$\beta = \left(\left(\frac{\gamma}{\alpha\,(S)^{0.995}}\right)\right) \quad (6)$$

was found to be dependent on the bulk density, apparent porosity and the shrinkage sustained on the clay body at any point in time under the hot-humid condition. The validity of the model is rooted on the expression; $S = (\gamma/\alpha\beta)^{1.005}$ where both sides of the expression are correspondingly almost equal. The maximum deviation of the model-predicted quantity of absorbed water from the corresponding experimental values is 8% which is within the acceptable range of deviation limit for experimental results.

In processing clay for production of brick, a model was derived (Nwoye,2009) for predicting the quantity of water evaporated during drying of clay at a temperature range 80-110^0C. The model;

$$E = \exp[0.3424(\text{Log}T)^{2.3529}] \quad (7)$$

shows that the quantity of evaporated water during the drying process is dependent on the drying temperature, the evaporating surface being constant. The validity of the model was found to be rooted in the expression $(\ln E \times \text{Log}\,\beta)^N = \text{Log}\,T$ since both sides of the expression are correspondingly approximately equal to 2. The maximum deviation of the model-predicted quantity of evaporated water from the corresponding experimental value was found to be less than 20% which is quite within the acceptable deviation range of experimental results, hence depicting the usefulness of the model. Water evaporation per unit rise in the drying temperature evaluated from experimental and model-predicted results are 0.078 and 0.0502g/^{0}C respectively, indicating proximate agreement.

The present work is to derive a model for mathematical analysis and predicting of the shrinkage-induced final length of fired clay products.

2. Materials and Methods

Experimental processes and the respective methodologies involving the clay preparation, molding and firing are detailed in previous report (Nwoye,2009). The volume shrinkages based on length were evaluated using the conventional equation (Cooke,1988) while volume shrinkages based on fired shrinkage were evaluated using model from Nwoye (2009).

2. 1 Model Formulation

Results of the experiment previously carried out (Nwoye,2009) were used for the model derivation.

$$\beta = \alpha^3 - 3\alpha^2 + 3\alpha \quad (8)$$

$$Vs = 1 - \left[\left(1 - \left(\frac{L_1 - L_2}{L_1} \right) \right)^3 \right] \quad (9)$$

Studies carried out on equation (8) (Nwoye,2009) and the conventional equation (Cooke,1988) in equation (9) indicates that;

$$Vs = \beta \quad (10)$$

Results from equation (10) as shown in Tables 1, 2 and 3 indicate that;

$$1 - \left[\left(1 - \left(\frac{L_1 - L_2}{L_1} \right) \right)^3 \right] = \alpha^3 - 3\alpha^2 + 3\alpha \quad (11)$$

Where

L_1 = Dried length of sample after air-drying (mm)

L_2 = Fired Length (mm)

Vs = Fractional volume shrinkage due to firing

And

β= Fractional volume shrinkage in terms of fired shrinkage during firing (just after air-drying) to1200^{0}C

α=Fractional fired shrinkage during firing (just after air-drying) to1200^{0}C

$$1 - \left[\left(1 - \left(\frac{L_1 - L_2}{L_1} \right) \right)^3 \right] = \alpha^3 - 3\alpha^2 + 3\alpha \quad (12)$$

$$- \left[\left(1 - \left(\frac{L_1 - L_2}{L_1} \right) \right)^3 \right] = \alpha^3 - 3\alpha^2 + 3\alpha - 1 \quad (13)$$

$$\left[\left(1 - \left(\frac{L_1 - L_2}{L_1} \right) \right)^3 \right] = - [\alpha^3 - 3\alpha^2 + 3\alpha - 1] \quad (14)$$

$$\left(1 - \left(\frac{L_1 - L_2}{L_1} \right) \right)^3 = - \alpha^3 + 3\alpha^2 - 3\alpha + 1 \quad (15)$$

$$\left(1 - \left(\frac{L_1 - L_2}{L_1} \right) \right) = [- \alpha^3 + 3\alpha^2 - 3\alpha + 1]^{1/3} \quad (16)$$

$$- \left(\frac{L_1 - L_2}{L_1} \right) = [- \alpha^3 + 3\alpha^2 - 3\alpha + 1]^{1/3} - 1 \quad (17)$$

$$\frac{L_2 - L_1}{L_1} = [- \alpha^3 + 3\alpha^2 - 3\alpha + 1]^{1/3} - 1 \quad (18)$$

$$\frac{L_2}{L_1} - \frac{L_1}{L_1} = [- \alpha^3 + 3\alpha^2 - 3\alpha + 1]^{1/3} - 1 \quad (19)$$

$$\frac{L_2}{L_1} - 1 = [- \alpha^3 + 3\alpha^2 - 3\alpha + 1]^{1/3} - 1 \quad (20)$$

$$\frac{L_2}{L_1} = [- \alpha^3 + 3\alpha^2 - 3\alpha + 1]^{1/3} \quad (21)$$

$$L_2 = L_1 [- \alpha^3 + 3\alpha^2 - 3\alpha + 1]^{1/3} \quad (22)$$

Equation (22) is the derived model.

Table 1: Variation of shrinkages with dried and fired lengths (Olokoro Clay)

L_1(exp)	L_2(exp)	(α)	β = Vs
64.40	63.40	0.0943	0.2571
64.52	63.38	0.0946	0.2578
64.63	63.40	0.0943	0.2571
64.38	63.42	0.0940	0.2563
64.49	63.52	0.0926	0.2529

Table 2: Variation of shrinkages with dried and fired lengths (Ukpor Clay)

L_1(exp)	L_2(exp)	(α)	β = Vs
65.00	64.20	0.0829	0.2287
64.99	63.99	0.0859	0.2362
65.02	64.12	0.0840	0.2314
64.80	64.00	0.0857	0.2357
64.70	63.70	0.0900	0.2464

Table 3: Variation of shrinkages with dried and fired lengths (Otamiri Clay)

L_1(exp)	L_2(exp)	(α)	β = Vs
65.80	64.80	0.0743	0.2060
65.08	64.98	0.0717	0.2000
65.60	64.75	0.0750	0.2085
65.99	64.99	0.0716	0.1998
65.82	64.67	0.0761	0.2114

3. Boundary and initial conditions

Consider a rectangular shaped clay product of length 70mm and width 30mm, exposed to air for drying, while it was in wet condition and thereafter fired to a temperature of 1200^{0}C. Initially atmospheric levels of oxygen are assumed. Atmospheric pressure was assumed to be acting on the clay samples during air-drying and firing. The grain size of clay particles used is 100μm, and air-drying duration; 48hours.

The boundary conditions were therefore the atmospheric levels of oxygen at the top and bottom of the clay samples, since they were dried and fired under the atmospheric condition. No external force due to compression or tension was applied to the drying and fired clays. Clay products were charged into the kiln for firing immediately after air-drying,

so the drying process was continuous until completion. Dried and fired linear shrinkage were assumed to be dried and fired shrinkage respectively. The sides of the particles and the rectangular shaped clay products were taken to be symmetries.

4. Model validation

In order to establish the validity and precision of the derived model, clay samples from three clay deposits (Olokoro, Ukpor and Otamiri) in south-eastern Nigeria were obtained, prepared and moulded into rectangular shaped solids. The solids were air-dried at room temperature for 48hrs before firing to a temperature of 1200^0C. Furthermore, the lengths L_2 obtained directly from experiment were compared with L_2 from the derived model and extent of deviation determined.

5. Results and Discussion

Result of chemical analysis of the clay materials used as presented in Table 4 shows variations in the Al_2O_3 and SiO_2 content, which affected shrinkage significantly.

Table 4: Result of chemical analysis of clay materials used

Source	Al_2O_3 (%)	Fe_2O_3 (%)	TiO_2 (%)	MgO (%)	CaO (%)	SiO_2 (%)	Na_2O (%)	K_2O (%)	Loss of ignition (%)
Ukpor	31.34	0.63	2.43	0.14	0.06	51.43	0.04	0.10	12.04
Olokoro	29.10	7.95	-	0.75	1.26	45.31	0.05	0.09	11.90
Otamiri	15.56	0.05	1.09	-	0.29	69.45	0.01	0.21	13.01

Comparison of equations (8) and (22) shows that the model in equation (22) is mathematically the same as $L_2 = L_1[(1- \beta)^{1/3}]$ since $\beta = \alpha^3 - 3\alpha^2 + 3\alpha$. It was found that on re-arranging equation (22) for the value of L_1, the model becomes; $L_1 = L_2 [- \alpha^3+3\alpha^2 - 3\alpha + 1]^{-1/3}$. Furthermore equation (22) shows that the ratio $(L_2/L_1)^3 = - \alpha^3+3\alpha^2 - 3\alpha + 1$. This invariably means that the cube of the ratio of final fired length to initial dried length is equal to 1-fractional volume shrinkage due to firing.

Table 5: Variation of $(L_2/L_1)^3$ with $[(1-\beta)^{1/3}]$ (Olokoro Clay)

L_2/L_1	$(L_2/L_1)^3$	$(1-\beta)$
0.9845	0.9542	0.7429
0.9823	0.9479	0.7422
0.9810	0.9441	0.7429
0.9851	0.9560	0.7437
0.9850	0.9557	0.7471

Table 6: Variation of $(L_2/L_1)^3$ with $[(1-\beta)^{1/3}]$ (Ukpor Clay)

L_2/L_1	$(L_2/L_1)^3$	$(1-\beta)$
0.9877	0.9636	0.7713
0.9846	0.9545	0.7638
0.9862	0.9592	0.7686
0.9877	0.9636	0.7643
0.9845	0.9542	0.7536

Table 7: Variation of $(L_2/L_1)^3$ with $[(1-\beta)^{1/3}]$ (Otamiri Clay)

L_2/L_1	$(L_2/L_1)^3$	$(1-\beta)$
0.9848	0.9551	0.7940
0.9893	0.9682	0.8000
0.9870	0.9615	0.7915
0.9848	0.9551	0.8002
0.9825	0.9484	0.7886

The validity of the model was found to be rooted on equation (22) where $(L_2/L_1)^3 = - \alpha^3+3\alpha^2 - 3\alpha + 1$. Since $\beta = \alpha^3 - 3\alpha^2 + 3\alpha$, $(L_2/L_1)^3 = - \alpha^3+3\alpha^2 - 3\alpha + 1$ is equivalent to $(L_2/L_1)^3 = [(1-\beta)^{1/3}]$. Based on this mathematical analysis, the validity of the model directly stems on $(L_2/L_1)^3 = [(1-\beta)^{1/3}]$ where both sides of the expression are correspondingly almost equal to 1. Tables 5, 6 and 7 also agree with this equation following the values of $(L_2/L_1)^3$ and $[(1-\beta)^{1/3}]$ evaluated from Tables 1, 2 and 3 as a result of corresponding computational analysis.

Analysis and comparison between the L_2 values reveal deviations of model-predicted L_2 from those of the experimental values. This is believed to be due to the fact that the surface properties of the clay and the physiochemical interactions between the clay and binder, which were expected to have played vital role during the evaporation of water were not considered during the model formulation. This necessitated the introduction of correction factor, to bring the model-predicted L_2 value to that of the corresponding experimental value.

Deviation (Dv) (%) of model-predicted values of L_2 from the experimental values is given by

$$Dv = \left(\frac{L_{2M} - L_{2exp}}{L_{2exp}} \right) \times 100 \quad (23)$$

Correction factor (Cf) is the negative of the deviation i.e

$$Cf = -Dv \quad (24)$$

Therefore

$$Cf = -100 \left(\frac{L_{2M} - L_{2exp}}{L_{2exp}} \right) \quad (25)$$

Where

L_{2M} = Model-Predicted final length (mm)

L_{2exp}= Final length obtained from experiment (Nwoye,2009) (mm)

Introduction of the value of Cf from equation (25) into the model gives exactly the corresponding experimental L_2 value.

Table 8: Variation of model-predicted L_2 with its associated deviation and correction factor (Olokoro Clay)

L_{2M}	Dv (%)	Cf (%)
58.33	-8.00	+8.00
58.42	-7.83	+7.83
58.53	-7.68	+7.68
58.33	-8.03	+8.03
58.52	-7.87	+7.87

Table 9: Variation of model-predicted L_2 with its associated deviation and correction factor (Ukpor Clay)

L_{2M}	Dv (%)	Cf (%)
59.61	-7.15	+7.15
59.41	-7.16	+7.16
59.56	-7.11	+7.11
59.25	-7.42	+7.42
58.88	-7.57	+7.57

Table 10: Variation of model-predicted L_2 with its associated deviation and correction factor (Otamiri Clay)

L_{2M}	Dv (%)	Cf (%)
60.93	-5.97	+5.97
60.97	-6.17	+6.17
60.68	-6.29	+6.29
61.26	-5.74	+5.74
60.81	-5.97	+5.97

Tables 8, 9 and 10 show that the maximum deviation of the model-predicted fired length L_2 from the corresponding experimental values is less than 9% which is within the acceptable range of deviation limit for experimental results.

Conclusion

The model mathematically analyzes and predicts the shrinkage-induced final length L_2 of fired clay. The model-predicted final fired length L_2 is dependent on the values of the initial length and fired shrinkage. The validity of the model was found to stem directly on the expression $(L_2/L_1)^3 = [-\alpha^3+3\alpha^2 - 3\alpha + 1]$ where both sides of the expression are correspondingly almost equal to 1. The maximum deviation of the model-predicted fired length L_2 from the corresponding experimental values is less than 9% which is within the acceptable range of deviation limit for experimental results. The cube of the ratio of final fired length to initial dried length is equal to 1-fractional volume shrinkage due to firing.

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