American Journal of Computational Mathematics, 2015, 5, 304-316
Published Online September 2015 in SciRes. http://www.scirp.org/journal/ajcm
http://dx.doi.org/10.4236/ajcm.2015.53028

Peristaltic Pumping of a Conducting Sisko Fluid through Porous Medium with Heat and Mass Transfer

Nabit Tawfiq Mohamed El-Dabe, Ahmed Younis Ghaly, Sallam Nagy Sallam, Khaled Elagamy, Yasmeen Mohamed Younis

Department of Mathematics, Faculty of Education, Ain Shams University, Cairo, Egypt
Email: mastermath2003@gmail.com

Received 2 July 2015; accepted 4 September 2015; published 7 September 2015

Abstract

The mathematical model is presented for the flow of peristaltic pumping of a conducting non-Newtonian fluid obeying Sisko model through a porous medium under the effect of magnetic field with heat and mass transfer. The solutions of the system of equations which represent this motion are obtained analytically using perturbation technique after considering the approximation of long wave length. The formula of the velocity with temperature and concentration of the fluid is obtained as a function of the physical parameters of the problem. The effects of these parameters on these solutions are discussed numerically and illustrated graphically through some graphs.

Keywords

Peristaltic, Sisko Fluid, MHD, Heat and Mass Transfer, Chemical Reaction

1. Introduction

Peristalsis is a form of fluid transport induced by a progressive wave of area contraction or expansion along the walls of distensible duct containing a liquid or mixture. A peristaltic pump is a device for pumping fluids, generally from a region of lower to higher pressure, by means of a contraction wave traveling along a tube like structure. Shapiro *et al*. [1] explained the basic principles and brought out clearly the significances of the various parameters governing the flow. The non-Newtonian effects in peristaltic motion were included in Kaimal [2]. Later several mathematical and experimental models have been developed to understand the fluid mechanical aspects of peristaltic motion. A large body of work already exists on mathematical and experimental models containing a Newtonian or non-Newtonian fluid in a channel.

How to cite this paper: El-Dabe, N.T.M., Ghaly, A.Y., Sallam, S.N., Elagamy, K. and Younis, Y.M. (2015) Peristaltic Pumping of a Conducting Sisko Fluid through Porous Medium with Heat and Mass Transfer. *American Journal of Computational Mathematics*, **5**, 304-316. http://dx.doi.org/10.4236/ajcm.2015.53028

Peristalsis also have industrial and biological applications like sanitary fluid transport blood pumps in heart lungs machines and peristaltic transport of toxic liquid is used in nuclear industries. Some recent investigations made to discuss the mechanism of peristalsis include the works. Radhakrishnamacharya and Srinivasulu [3] studied the influence of wall properties on peristaltic transport with heat transfer. Mekheimer and Abd Elmaboud [4] analyzed the influence of heat transfer and magnetic field on peristaltic transport of Newtonian fluid in a vertical annulus. Hayat *et al*. [5] studied the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. Krishna Kumari *et al*. [6] studied the peristaltic pumping of a magnetohydrodynamic Casson fluid in an inclined channel. Ravi Kumar *et al*. [7] considered power-law fluid in the study of peristaltic transport.

The effect of porous medium on the motion of the fluid has been studied by many authors. Elshehawey *et al*. [8] studied the effect of porous medium on peristaltic motion of a Newtonian fluid. Eldabe [9] studied magnetohydrodynamic flow through a porous medium fluid at a rear stagnation point. Eldabe *et al*. [8] studied MHD flow and heat transfer in a viscoelastic incompressible fluid confined between a horizontal stretching sheet and a parallel porous wall. Elshehawey *et al*. [9] studied the peristaltic motion of a Generalized Newtonian fluid through a porous medium. El-Dabe *et al*. [10] studied the magnetohydrodynamic flow and heat transfer for a peristaltic motion of Carreau fluid through a porous medium.

The study of the influence of mass and heat transfer on Newtonian and non-Newtonian fluids has become important in the last few years. This importance is due to a number of industrial processes. Examples are food-processing, biochemical operations and transport in polymers. Eldabe *et al*. [11] studied the heat and mass transfer in hydromagnetic flow of the non-Newtonian fluid with heat source over an accelerating surface through a porous medium. Srinivas and Kothandapani [12] dealt with peristalsis and heat transfer. Hayat *et al*. [13]-[15] analyzed the peristaltic mechanism with heat transfer. Eldabe *et al*. [16] studied the effect of couple stresses on the MHD of a non-Newtonian unsteady flow between two parallel porous plates. The main aim of this study is to investigate the problem of the peristaltic flow of a conducting Sisko fluid in a porous channel with heat and mass transfer; the system of non-linear partial differential equations which describe this motion with heat and mass transfer subjected to the appropriate boundary conditions is solved analytically by using perturbation method. The expressions of the velocity, the temperature and the concentration are determined. The effects of different parameters on these expressions are discussed through graphs.

2. Basic Equations

The basic equations governing the flow of an incompressible fluid are expressed as follows:

The continuity equation

$$\nabla\cdot\underline{V}^{*}=0, \tag{1}$$

The momentum equation

$$\rho\frac{\mathrm{d}\underline{V}^{*}}{\mathrm{d}t^{*}}=div\,\tau^{*}+\mu_{e}\underline{J}\wedge\underline{B}-\frac{\mu}{k^{*}}\underline{V}^{*}, \tag{2}$$

where $\tau^{*}=-P^{*}I+S^{*}$, and

$$S^{*}=\left[m+\eta\left(\sqrt{\bar{\Pi}}\right)^{n-1}\right]A_{1}^{*},\quad A_{1}^{*}=L^{*}+L^{*\mathrm{T}},\quad L^{*}=grad\,V^{*},\quad \bar{\Pi}=\frac{1}{2}tr\left(A_{1}^{*2}\right), \tag{3}$$

where m,η and n are the material parameters of the fluid. Note that for $n<o$ the fluid describes shear thinning, for $n>o$ the fluid describes shear thicking, for $n=1$ the Newtonian fluid is recovered and for $m=o$ the generalized power-law model can be obtained.

The temperature equation

$$\rho c_{p}\left[\frac{\partial T}{\partial t^{*}}+\left(\underline{V}^{*}\cdot\nabla\right)T\right]=\kappa\nabla^{2}T+\Phi, \tag{4}$$

The concentration equation

$$\left[\frac{\partial C}{\partial t^{*}}+\left(\underline{V}^{*}\cdot\nabla\right)C\right]=D\nabla^{2}C+\frac{D\kappa_{T}}{T_{m}}\nabla^{2}T-A\left(C-C_{2}\right), \tag{5}$$

The dissipation function Φ

$$\Phi = \tau_{ij}^{*} \frac{\partial V_i^{*}}{\partial X_j^{*}}, \tag{6}$$

Maxwell s equations

$$\nabla \times \underline{B} = \mu_e \underline{J}, \tag{7}$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t^{*}},$$

$$\nabla \cdot \underline{B} = 0,$$

Ohm's equation

$$\underline{J} = \sigma\left(\underline{E} + \underline{V}^{*} \times \underline{B}\right) \tag{8}$$

where $\underline{V}^{*}$ is the velocity vector, ρ is the density, $\left(\frac{\mathrm{d}}{\mathrm{d}t^{*}}\right)$ is the material time derivative, k^{*} is the permeability, $\underline{J}$ is the current density, σ is the electrical conductivity, μ_e is the magnetic permeability, $E^{*} = 0$ is the electric field and τ^{*} is the Cauchy stress, T and C are temperature and concentration of the fluid, κ is the thermal conductivity, c_p is the specific heat capacity at constant pressure, D is the coefficient of mass diffusivity, κ_T is the thermal diffusion ratio, T_m is the mean fluid temperature and A is the reaction rate constant.

3. Mathematical Formulation

Consider the two-dimensional motion of an incompressible Sisko fluid in an infinite channel of width a, see **Figure 1**. In the upper wall we assume an infinite sinusoidal wave train moving ahead with constant velocity c along it while the lower plate is fixed at $Y^{*} = 0$. The wavy surface is defined as

$$Y^{*} = H^{*}\left(X^{*}, t^{*}\right) = a + b\mathrm{Sin}\left[\frac{2\pi}{\lambda}\left(X^{*} - ct^{*}\right)\right], \tag{9}$$

where b is the wave amplitude, λ is the wave length and t^{*} is the time.

Now, Equations (2)-(6) can be written in two-dimensional $\left(X^{*}, Y^{*}\right)$ as follows:

$$\frac{\partial U^{*}}{\partial X^{*}} + \frac{\partial V^{*}}{\partial Y^{*}} = 0, \tag{10}$$

$$\rho\left(\frac{\partial}{\partial t^{*}} + U^{*}\frac{\partial}{\partial X^{*}} + V^{*}\frac{\partial}{\partial Y^{*}}\right)U^{*} = -\frac{\partial P^{*}}{\partial X^{*}} + \frac{\partial S^{*}_{X^{*}X^{*}}}{\partial X^{*}} + \frac{\partial S^{*}_{X^{*}Y^{*}}}{\partial Y^{*}} - \frac{\mu}{k^{*}}U^{*} - \sigma B_o^2 U^{*}, \tag{11}$$

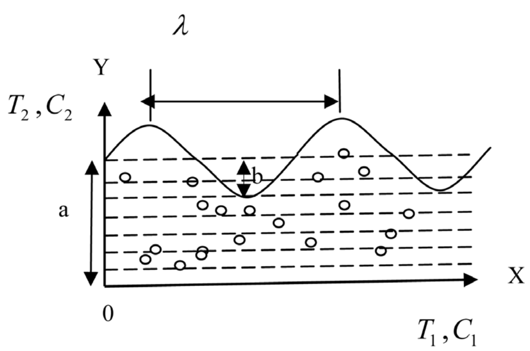

Figure 1. Sketch of the problem.

$$\rho\left(\frac{\partial}{\partial t^*}+U^*\frac{\partial}{\partial X^*}+V^*\frac{\partial}{\partial Y^*}\right)V^*=-\frac{\partial P^*}{\partial Y^*}+\frac{\partial S^*_{X^*Y^*}}{\partial X^*}+\frac{\partial S^*_{Y^*Y^*}}{\partial Y^*}-\frac{\mu}{k^*}V^*, \tag{12}$$

$$\rho c_p\left[\frac{\partial T}{\partial t^*}+U^*\frac{\partial T}{\partial X^*}+V^*\frac{\partial T}{\partial Y^*}\right]=\kappa\left(\frac{\partial^2 T}{\partial X^{*2}}+\frac{\partial^2 T}{\partial Y^{*2}}\right)+S^*_{X^*X^*}\frac{\partial U^*}{\partial X^*}+S^*_{X^*Y^*}\left(\frac{\partial V^*}{\partial X^*}+\frac{\partial U^*}{\partial Y^*}\right)+S^*_{Y^*Y^*}\frac{\partial V^*}{\partial Y^*}, \tag{13}$$

$$\left[\frac{\partial C}{\partial t^*}+U^*\frac{\partial C}{\partial X^*}+V^*\frac{\partial C}{\partial Y^*}\right]=D\left(\frac{\partial^2 C}{\partial X^{*2}}+\frac{\partial^2 C}{\partial Y^{*2}}\right)+\frac{D\kappa_T}{T_m}\left(\frac{\partial^2 T}{\partial X^{*2}}+\frac{\partial^2 T}{\partial Y^{*2}}\right)-A\left(C-C_2\right), \tag{14}$$

where

$$S^*_{X^*X^*}=2\left[m+\eta\left\{2\left(\frac{\partial U^*}{\partial X^*}\right)^2+\left(\frac{\partial U^*}{\partial Y^*}+\frac{\partial V^*}{\partial X^*}\right)^2+2\left(\frac{\partial V^*}{\partial Y^*}\right)^2\right\}^{\frac{n-1}{2}}\right]\frac{\partial U^*}{\partial X^*}, \tag{15}$$

$$S^*_{X^*Y^*}=\left[m+\eta\left\{2\left(\frac{\partial U^*}{\partial X^*}\right)^2+\left(\frac{\partial U^*}{\partial Y^*}+\frac{\partial V^*}{\partial X^*}\right)^2+2\left(\frac{\partial V^*}{\partial Y^*}\right)^2\right\}^{\frac{n-1}{2}}\right]\left(\frac{\partial U^*}{\partial Y^*}+\frac{\partial V^*}{\partial X^*}\right), \tag{16}$$

$$S^*_{Y^*Y^*}=2\left[m+\eta\left\{2\left(\frac{\partial U^*}{\partial X^*}\right)^2+\left(\frac{\partial U^*}{\partial Y^*}+\frac{\partial V^*}{\partial X^*}\right)^2+2\left(\frac{\partial V^*}{\partial Y^*}\right)^2\right\}^{\frac{n-1}{2}}\right]\frac{\partial V^*}{\partial Y^*}, \tag{17}$$

$$\Phi=S^*_{X^*X^*}\frac{\partial U^*}{\partial X^*}+S^*_{X^*Y^*}\left(\frac{\partial V^*}{\partial X^*}+\frac{\partial U^*}{\partial Y^*}\right)+S^*_{Y^*Y^*}\frac{\partial V^*}{\partial Y^*}, \tag{18}$$

Subjected to the following appropriate boundary conditions:

$$\begin{aligned}&\frac{\partial U^*}{\partial Y^*}=0, T=T_1, C=C_1 \quad \text{at } Y^*=0,\\ &U^*=0, T=T_2, C=C_2 \quad \text{at } Y^*=h^*=a+b\,\text{Sin}\left[\frac{2\pi}{\lambda}\left(X^*-ct^*\right)\right]\end{aligned} \tag{19}$$

Choose the wave frame $\left(x^*,y^*\right)$ moving with speed *c*, where

$$x^*=X^*-ct^*,\ y^*=Y^*,\ u^*=U^*-c,\ v^*=V^*,\ p^*\left(x^*,y^*\right)=P^*\left(X^*,Y^*,t^*\right). \tag{20}$$

In which $\left(u^*,v^*\right)$ are components of the velocity in the moving coordinates system.
Then, the Equations (10)-(19) can be written as:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \tag{21}$$

$$\rho\left(u^*\frac{\partial}{\partial x^*}+v^*\frac{\partial}{\partial y^*}\right)u^*=-\frac{\partial p^*}{\partial x^*}+\frac{\partial s^*_{x^*x^*}}{\partial x^*}+\frac{\partial s^*_{x^*y^*}}{\partial y^*}-\frac{\mu}{k^*}u^*-\sigma B_o^2u^*, \tag{22}$$

$$\rho\left(u^*\frac{\partial}{\partial x^*}+v^*\frac{\partial}{\partial y^*}\right)v^*=-\frac{\partial p^*}{\partial y^*}+\frac{\partial s^*_{x^*y^*}}{\partial x^*}+\frac{\partial s^*_{y^*y^*}}{\partial y^*}-\frac{\mu}{k^*}v^*, \tag{23}$$

$$\rho c_p\left[u^*\frac{\partial T}{\partial x^*}+v^*\frac{\partial T}{\partial y^*}\right]=\kappa\left(\frac{\partial^2 T}{\partial x^{*2}}+\frac{\partial^2 T}{\partial y^{*2}}\right)+s^*_{x^*x^*}\frac{\partial u^*}{\partial x^*}+s^*_{x^*y^*}\left(\frac{\partial v^*}{\partial x^*}+\frac{\partial u^*}{\partial y^*}\right)+s^*_{y^*y^*}\frac{\partial v^*}{\partial y^*}, \tag{24}$$

$$u^* \frac{\partial C}{\partial x^*} + v^* \frac{\partial C}{\partial y^*} = D\left(\frac{\partial^2 C}{\partial x^{*2}} + \frac{\partial^2 C}{\partial y^{*2}}\right) + \frac{D\kappa_T}{T_m}\left(\frac{\partial^2 T}{\partial x^{*2}} + \frac{\partial^2 T}{\partial y^{*2}}\right) - A(C - C_2), \tag{25}$$

where

$$s^*_{x^*x^*} = 2\left[m + \eta\left\{2\left(\frac{\partial u^*}{\partial x^*}\right)^2 + \left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)^2 + 2\left(\frac{\partial v^*}{\partial y^*}\right)^2\right\}^{\frac{n-1}{2}}\right]\frac{\partial u^*}{\partial x^*}, \tag{26}$$

$$s^*_{x^*y^*} = \left[m + \eta\left\{2\left(\frac{\partial u^*}{\partial x^*}\right)^2 + \left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)^2 + 2\left(\frac{\partial v^*}{\partial y^*}\right)^2\right\}^{\frac{n-1}{2}}\right]\left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right), \tag{27}$$

$$s^*_{y^*y^*} = 2\left[m + \eta\left\{2\left(\frac{\partial u^*}{\partial x^*}\right)^2 + \left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)^2 + 2\left(\frac{\partial v^*}{\partial y^*}\right)^2\right\}^{\frac{n-1}{2}}\right]\frac{\partial v^*}{\partial y^*}, \tag{28}$$

The boundary conditions are:

$$\begin{aligned} &\frac{\partial u^*}{\partial y^*} = 0, T = T_1, C = C_1 \quad \text{at } y^* = 0, \\ &u^* = -c, T = T_2, C = C_2 \quad \text{at } y^* = h^* = a + b\,\text{Sin}\left(\frac{2\pi}{\lambda}x^*\right) \end{aligned} \tag{29}$$

In order to simplify the governing equations of the motion, we may introduce the following dimensionless transformations:

$$x = \frac{2\pi}{\lambda}x^*,\ y = \frac{y^*}{a},\ u = \frac{u^*}{c},\ v = \frac{v^*}{c},\ s = \frac{a}{\mu c}s^*,\ p = \frac{2\pi a^2}{c\lambda\mu}p^*,\ h = \frac{h^*}{a},\ \phi = \frac{C - C_2}{C_1 - C_2},\ \theta = \frac{T - T_2}{T_1 - T_2}, \tag{30}$$

where, T_1 and C_1 are the temperature and the concentration of the fluid at the lower wall of the channel, T_2 and C_2 are the temperature and the concentration of the fluid at the upper wall of the channel.

Substituting (30) into Equations (21)-(29) we obtain the following non-dimensional equations after dropping the star mark:

$$\delta\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{31}$$

$$R_e\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)u = -\frac{\partial p}{\partial x} + \delta\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{xy}}{\partial y} - \frac{1}{D_a}u - Mu, \tag{32}$$

$$\delta R_e\left(\delta u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)v = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial s_{xy}}{\partial x} + \delta\frac{\partial s_{yy}}{\partial y} - \frac{\delta}{D_a}v, \tag{33}$$

$$R_e\left[\delta u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y}\right] = \frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + E_c\left(\delta s_{xx}\frac{\partial u}{\partial x} + s_{xy}\left(\delta\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + s_{yy}\frac{\partial v}{\partial y}\right), \tag{34}$$

$$R_e\left[\delta u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y}\right] = \frac{1}{S_c}\left(\delta^2\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + S_r\left(\delta^2\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) - R_c\phi, \tag{35}$$

$$s_{xx} = 2\left\{1 + \eta\left[2\delta^2\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \delta\frac{\partial v}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{n-1}{2}}\right\}\frac{\partial u}{\partial x}, \tag{36}$$

$$s_{xy}=2\left\{1+\eta\left[2\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{n-1}{2}}\right\}\left(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right), \tag{37}$$

$$s_{yy}=2\left\{1+\eta\left[2\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2\right]^{\frac{n-1}{2}}\right\}\frac{\partial v}{\partial y}, \tag{38}$$

The boundary conditions are:

$$\begin{aligned}&\frac{\partial u}{\partial y}=0,\ \theta=1,\ \phi=1\quad \text{at } y=0,\\ &u=-1,\ \theta=0,\ \phi=0\quad \text{at } y=h\end{aligned} \tag{39}$$

where, the dimensionless parameters are defined by:

$\delta=\frac{2\pi a}{\lambda}$ is the Wave number, $R_e=\frac{\rho ca}{\mu}$ is the Reynolds number, $K=\frac{k^*}{a^2}$ is the Darcy number, $M=\frac{\sigma\mu_e^2H^2a^2}{\mu}$ is the Magnetic field parameter, $\eta^*=\frac{l}{m\left(\frac{a}{c}\right)^{n-1}}$ is the Non-Newtonian parameter, $P_r=\frac{\nu\rho c_p}{\kappa}$ is the Prandtle number, $E_c=\frac{C^2}{c_p\left(T_1-T_2\right)}$ is the Eckert number, $S_c=\frac{\nu}{D}$ is the Schmidt number, $S_r=\frac{D\kappa_T\left(T_1-T_2\right)}{T_m\nu\left(C_1-C_2\right)}$ is the Soret number and $R_c=\frac{A\left(C_1-C_2\right)}{\nu}$ is the chemical reaction parameter.

By using the following definition of stream function ψ as:

$$u=\frac{\partial\psi}{\partial y},\quad v=-\delta\frac{\partial\psi}{\partial x}. \tag{40}$$

The system of Equations (3.30)-(3.38) can be written as:

$$\delta R_e\left[\left(\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial y}\right]=-\frac{\partial p}{\partial x}+\delta\frac{\partial s_{xx}}{\partial x}+\frac{\partial s_{xy}}{\partial y}-\left(\frac{1}{K}-M\right)\frac{\partial\psi}{\partial y}, \tag{41}$$

$$\delta^3R_e\left[\left(-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}+\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\psi}{\partial x}\right]=-\frac{\partial p}{\partial y}+\delta^2\frac{\partial s_{xy}}{\partial x}+\delta\frac{\partial s_{yy}}{\partial y}+\frac{\delta^2}{K}\frac{\partial\psi}{\partial x}, \tag{42}$$

$$R_e\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)=\frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+E_c\left[\delta s_{xx}\left(\frac{\partial^2\psi}{\partial x\partial y}\right)+s_{xy}\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)-\delta s_{yy}\left(\frac{\partial^2\psi}{\partial x\partial y}\right)\right] \tag{43}$$

$$R_e\delta\left(\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}\right)=\frac{1}{S_c}\left(\delta^2\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+S_r\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-R_c\phi, \tag{44}$$

$$s_{xx}=2\delta\left[1+\eta^*\left[2\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2+2\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right]^{\frac{n-1}{2}}\right]\frac{\partial^2\psi}{\partial x\partial y}, \tag{45}$$

$$s_{xy}=\left[1+\eta^*\left[2\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right)^2+2\delta^2\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right]^{\frac{n-1}{2}}\right]\left(\frac{\partial^2\psi}{\partial y^2}-\delta^2\frac{\partial^2\psi}{\partial x^2}\right), \tag{46}$$

The boundary conditions for the dimensionless stream function in the moving frame are given by:

$$\psi = 0, \frac{\partial^2 \psi}{\partial y^2} = 0, \theta = 1, \phi = 1 \quad \text{at } y = 0,$$
$$\frac{\partial \psi}{\partial y} = -1, \psi = F, \theta = 0, \phi = 0 \quad \text{at } y = h. \tag{47}$$

The dimensionless form of the surface of the peristaltic wall can be written as:

$$h = 1 + \chi \text{Sin} x$$

where $\chi = \dfrac{b}{a}$ is the amplitude ratio or the occlusion. and

$$F = \int_0^h u \mathrm{d}y = \int_0^h \frac{\partial \psi}{\partial y} \mathrm{d}y = \psi(h) - \psi(0) \quad \text{is the total flux number} \tag{48}$$

4. Solution of the Problem

According to long wavelength approximation $(\delta \lll 1)$ [1], Equations (41)-(46) after eliminating the pressure gradient become:

$$\frac{\partial^2 s_{xy}}{\partial y^2} - L^2 \frac{\partial^2 \psi}{\partial y^2} = 0, \tag{49}$$

$$\frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} + E_c S_{xy} \left(\frac{\partial^2 \psi}{\partial y^2} \right) = 0, \tag{50}$$

$$\frac{1}{S_c} \left(\frac{\partial^2 \phi}{\partial y^2} \right) + S_r \left(\frac{\partial^2 \theta}{\partial y^2} \right) - R_c \phi = 0, \tag{51}$$

$$S_{xx} = 0, S_{yy} = 0, \tag{52}$$

$$s_{xy} = \left[1 + \eta \left[\left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 \right]^{\frac{n-1}{2}} \right] \left(\frac{\partial^2 \psi}{\partial y^2} \right), \tag{53}$$

$$s_{xy} = \frac{\partial^2 \psi}{\partial y^2} + \eta \left(\frac{\partial^2 \psi}{\partial y^2} \right)^n, \tag{54}$$

In order to solve the Equations (49)-(51) subjected to the boundary conditions, we suppose the following perturbation for small non-Newtonian parameter η:

$$\psi = \psi_o + \eta \psi_1 + \cdots$$
$$\theta = \theta_o + \eta \theta_1 + \cdots \tag{55}$$
$$\phi = \phi_o + \eta \phi_1 + \cdots$$

Substituting (55) into (49)-(54) and comparing the coefficient of zero and first order of η we get.

4.1. Zero Order System of η

$$\frac{\partial^4 \psi_0}{\partial y^4} - L^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0, \tag{56}$$

$$\frac{1}{P_r} \frac{\partial^2 \theta_0}{\partial y^2} + E_c \left(\frac{\partial^2 \psi_0}{\partial y^2} \right)^2 = 0, \tag{57}$$

$$\frac{1}{S_c}\frac{\partial^2\phi_0}{\partial y^2}+S_r\frac{\partial^2\theta_0}{\partial y^2}-R_c\phi_0=0, \tag{58}$$

With the respective boundary conditions

$$\begin{aligned}&\psi_0=0,\ \frac{\partial^2\psi_0}{\partial y^2}=0,\ \theta_0=1,\ \phi_0=1 \quad \text{at } y=0,\\ &\psi_0=F,\ \frac{\partial\psi_0}{\partial y}=-1,\ \theta_0=0,\ \phi_0=0 \quad \text{at } y=h=1+\phi\text{Sin}x.\end{aligned} \tag{59}$$

4.2. First Order System of η

We shall consider the case of Dielatent fluids when $n > 1$, and we choose ($n = 3$), then we have the following system of first order of η

$$\frac{\partial^4\psi_1}{\partial y^4}-L^2\frac{\partial^2\psi_1}{\partial y^2}=-\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^3, \tag{60}$$

$$\frac{1}{P_r}\frac{\partial^2\theta_1}{\partial y^2}+E_c\left(2\frac{\partial^2\psi_0}{\partial y^2}\frac{\partial^2\psi_1}{\partial y^2}+\left(\frac{\partial^2\psi_0}{\partial y^2}\right)^4\right)=0, \tag{61}$$

$$\frac{1}{S_c}\frac{\partial^2\phi_1}{\partial y^2}+S_r\frac{\partial^2\theta_1}{\partial y^2}-R_c\phi_1=0, \tag{62}$$

with the respective boundary conditions

$$\begin{aligned}&\psi_1=0,\ \frac{\partial^2\psi_1}{\partial y^2}=0,\ \theta_1=0,\ \phi_1=0 \quad \text{at } y=0,\\ &\psi_1=0,\ \frac{\partial\psi_1}{\partial y}=0,\ \theta_1=0,\ \phi_1=0 \quad \text{at } y=h=1+\chi\text{Sin}x.\end{aligned} \tag{63}$$

4.3. Solution for System of Zero Order

$$\psi_0=-y+G\left(\text{Sinh}Ly-yL\text{Cosh}Lh\right), \tag{64}$$

$$\theta_0=-\frac{P_rE_cG^2L^4}{4}\left\{\frac{1}{2L^2}\text{Cosh}2Ly-y^2-\frac{1}{2L^2}-\frac{y}{h}\left(\frac{1}{2L^2}\text{Cosh}2Lh-h^2-\frac{1}{2L^2}\right)\right\}-\frac{y}{h}+1, \tag{65}$$

$$\phi_0=2A\text{Sinh}\sqrt{S_cR_c}\,y+\left(1-\frac{P_rE_cS_rS_cG^2L^4}{2\left(4L^2-S_rS_c\right)}-\frac{P_rE_cG^2L^4}{2}\right)e^{-\sqrt{S_cR_c}\,y}+\frac{P_rE_cS_rS_cG^2L^4}{2\left(2L^2-S_rR_c\right)}\text{Cosh}2Ly+\frac{P_rE_cG^2L^4}{2}. \tag{66}$$

4.4. Solution for System of First Order

$$\begin{aligned}\psi_1=&\ G_1L\left(\text{Cosh}Ly-L\text{Cosh}Lh\right)-\frac{3}{16}G^3L^4\left(-\frac{L}{2}\text{Cosh}3Lh+2L\text{Cosh}Lh\right.\\ &\left.+2L^2h\text{Sinh}Lh-\frac{L}{2}\text{Cosh}3Ly+2L\text{Cosh}Ly+2L^2y\text{Sinh}Ly\right),\end{aligned} \tag{67}$$

$$\begin{aligned}\theta_1=&\ A_1y+B-2P_rE_cGL^2\left\{A_2\left(\frac{\text{Cosh}2Ly}{2L^2}-y^2\right)+\frac{1}{16}G^3L^4\left(\frac{3yL}{4}\text{Sinh}2Ly\right.\right.\\ &\left.\left.-\frac{19}{16}\text{Cosh}2Ly-\frac{5}{64}\text{Cosh}4Ly\right)+\frac{3}{32}G^3L^6y^2\right\},\end{aligned} \tag{68}$$

$$\phi_1 = 2A_1 \text{Sinh}\sqrt{R_c S_c}\, y - 2P_r E_c G L^2 S_r S_c Z e^{-\sqrt{R_c S_c}\, y} + 2P_r E_c G L^2 S_r S_c \left\{ A_2 \left(\frac{2L\text{Cosh}2Ly}{4L^2 - R_c S_c} + \frac{2}{R_c S_c} \right) \right.$$
$$\left. + A_3 \left(\frac{3}{2}(\text{Sinh}2Ly + \text{Cosh}2Ly) + 3Ly\text{Sinh}2Ly - \frac{19}{4}\text{Cosh}2Ly - \frac{5}{4}\text{Sinh}4Ly \right) - \frac{3}{16} G^3 L^6 \right\} \quad (69)$$

5. Results and Discussions

The problem of the peristaltic flow of a Sisko fluid through a porous medium with heat and mass transfer has been discussed. The effects of non-Newtonian dissipation and chemical reaction on the fluid flow have been considered. We obtained the solutions of the momentum, heat and mass equations analytically by using the perturbation technique for small non-Newtonian parameter η after considering the long wave approximations.

Figure 2 illustrates the relation between the value of stream function ψ and the magnetic parameter M, it is clear that ψ increases with increasing the magnetic parameter, while it decreases with increasing the permeability parameter K which seen through **Figure 3**.

In **Figure 4**, the relation between the value of stream function ψ and the amplitude ratio χ has been illustrated. it is clear that ψ decreases with the decrease of χ.

Figure 5, illustrates the relation between the value of temperature θ and the magnetic parameter M, it is clear that increases with increasing the magnetic parameter.

In **Figure 6**, we can observe that the value of temperature increases with increasing the Prandtle number parameter P_r, while in **Figure 7**, the relation between the value of temperature θ and the Eckert number E_c has been illustrated. It is clear that θ increases with the increase of E_c.

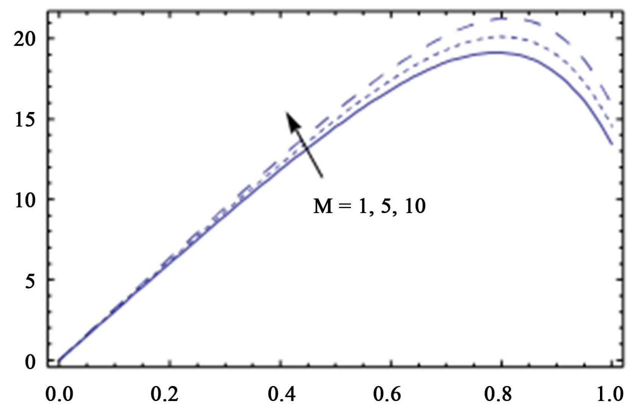

Figure 2. Stream function profiles $\psi(y)$ for some values of M.

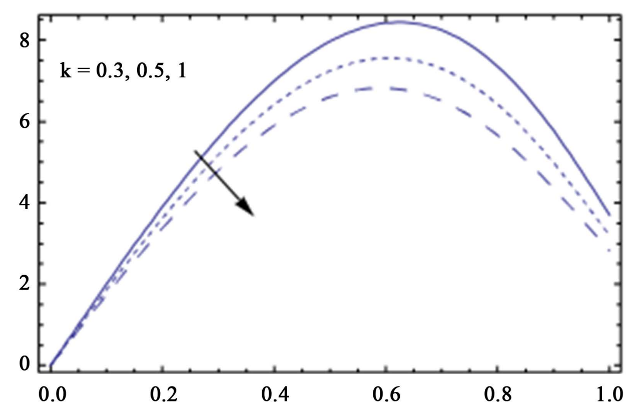

Figure 3. Stream function profiles $\psi(y)$ for some values of K.

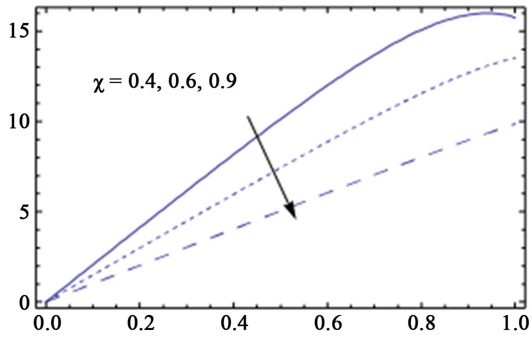

Figure 4. Stream function profiles $\psi(y)$ for some values of χ.

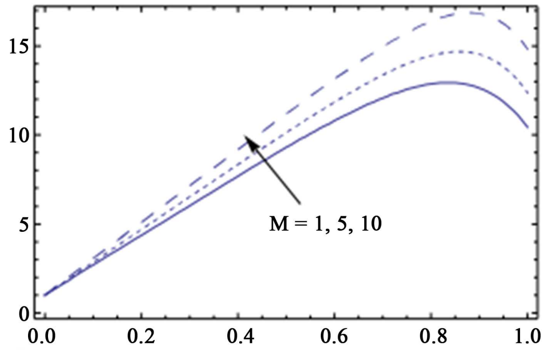

Figure 5. Temperature profiles $\theta(y)$ for some values of M.

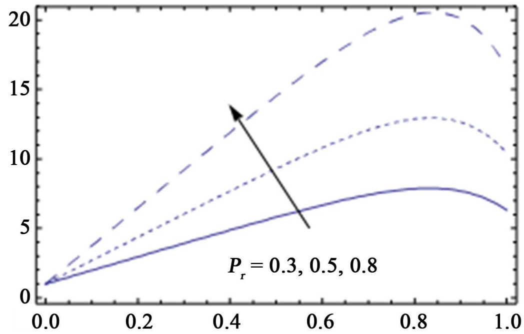

Figure 6. Temperature profiles $\theta(y)$ for some values of P_r.

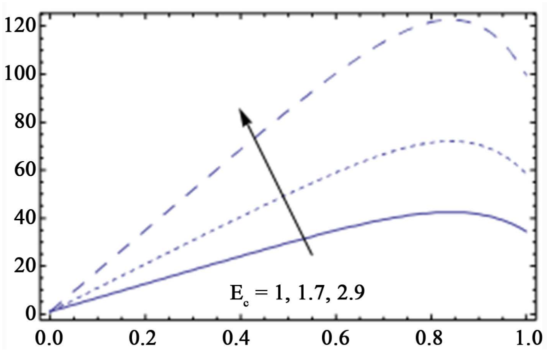

Figure 7. Temperature profiles $\theta(y)$ for some values of E_c.

In **Figure 8**, the relation between the value of concentration ϕ and the magnetic parameter M, it is clear that ϕ decreases with increasing the magnetic parameter. But we can observe from **Figure 9** that ϕ decreases with increasing the Schmidt parameter S_c, also in **Figure 10**, the relation between the value of concentration function ϕ and the Soret value S_r has been illustrated. It is clear that ϕ increases with the increase of R_c as in **Figure 11**.

6. Conclusion and Applications

In this work, we study the peristaltic motion of magneto-hydrodynamics flow with heat and mass transfer for incompressible non-Newtonian fluid through a porous medium. The governing partial differential equations of this problem, subjected to the boundary conditions are solved analytically by using perturbation technique. The

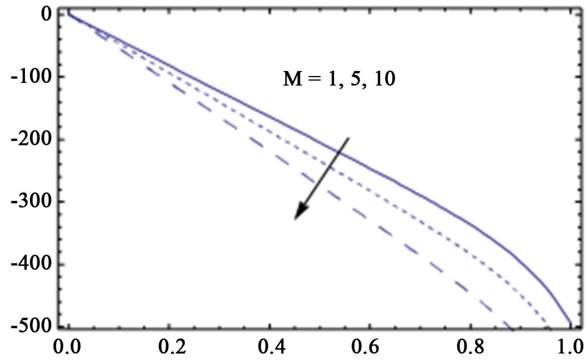

Figure 8. Concentration profiles $\phi(y)$ for some values of M.

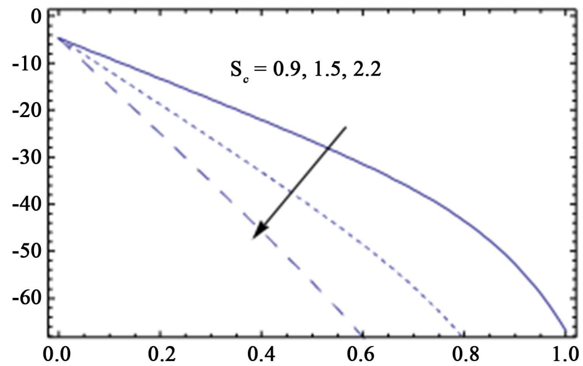

Figure 9. Concentration profiles $\phi(y)$ for some values of S_c.

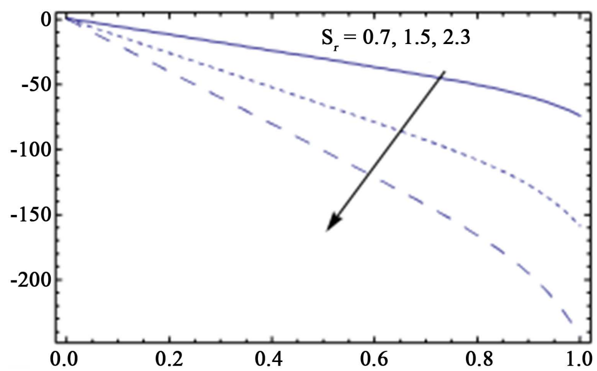

Figure 10. Concentration profiles $\phi(y)$ for some values of S_r.

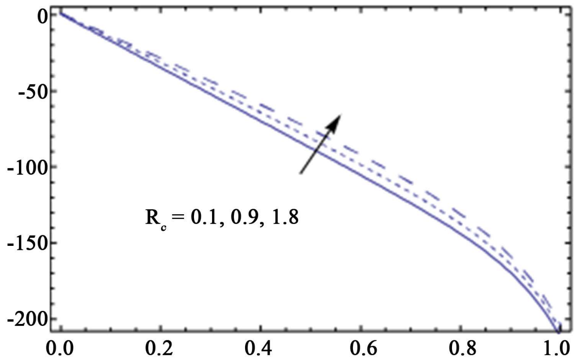

Figure 11. Concentration profiles $\phi(y)$ for some values of R_c.

analytical forms for the stream distribution ψ, the temperature θ and the concentration ϕ are obtained. The effects of various physical parameters of the problem on these formulas are discussed and have been shown graphically. It is seen that the ψ increases with increasing the magnetic parameter M, while it decreases with increasing the permeability parameter K. Also it is clear that θ increases with the increase of E_c,

The study of this phenomenon is very important, because the study of flow through porous medium has many applications. It has an important role in agricultural, extracting pure petrol from crude oil and chemical engineering. There are examples of natural porous media such as wood, filter paper, cotton, leather and plastics. As a good biological example on the porous medium, the human lung galls bladder and the walls of vessels. The peristaltic motion has been found to involve in many biological organs such as esophagus, small and large intestine, stomach, the human ureter, lymphatic vessels and small blood vessels. Also, peristaltic transport occurs in many practical applications involving biomechanical systems such as finger pumps.

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Appendix

$$L^2 = \frac{1}{K} + M,$$

$$G = \frac{F+h}{\text{Sinh}Lh - h L\text{Cosh}Lh},$$

$$G_1 = \frac{3}{16}\frac{hL^4G^3}{\text{Sinh}Lh - hL\text{Cosh}Lh}\left(-\frac{L}{2}\text{Cosh}3Lh + 2L\text{Cosh}Lh + 2L^2\text{Sinh}Lh\right),$$

$$B = \frac{P_r E_c G L^2}{4}\left(G_1 - \frac{15}{128}G^3L^4\right),$$

$$A_1 = \frac{2P_r E_c G L^2}{h}\left(\left(\frac{G_1 L^2}{4} + \frac{3}{16}G^3L^6\right)\left(\frac{\text{Cosh}2Lh}{2L^2} - h^2\right)\right.$$
$$\left. + \frac{1}{16}G^3L^4\left(\frac{3hL}{4}\text{Sinh}2Lh - \frac{19}{16}\text{Cosh}2Lh - \frac{5}{64}\text{Cosh}4Lh\right) + \frac{3}{32}G^3L^6h^2\right) - \frac{B}{h},$$

$$Z = \left(\frac{G_1L^2}{4} + \frac{3}{16}G^3L^6\right)\left(\frac{2L}{4L^2 - R_cS_c} + \frac{2}{R_cS_c}\right) - \frac{9G^3L^6}{32\left(4L^2 - R_cS_c\right)} - \frac{3}{16}G^3L^6.$$