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An Interactive Fuzzy Satisficing Method for Multiobjective Stochastic Integer Programming with Simple Recourse

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ABSTRACT

This paper considers multiobjective integer programming problems involving random variables in constraints. Using the concept of simple recourse, the formulated multiobjective stochastic simple recourse problems are transformed into deterministic ones. For solving transformed deterministic problems efficiently, we also introduce genetic algorithms with double strings for nonlinear integer programming problems. Taking into account vagueness of judgments of the decision maker, an interactive fuzzy satisficing method is presented. In the proposed interactive method, after determineing the fuzzy goals of the decision maker, a satisficing solution for the decision maker is derived efficiently by updating the reference membership levels of the decision maker. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

Keywords: Multiobjective Programming; Stochastic Programming; Fuzzy Programming; Interactive Methods; Simple Recourse Model

1. Introduction

In actual decision making situations, we must often make a decision on the basis of imprecise information or uncertain data. For such decision making problems involveing uncertainty, there exist two typical approaches: stochastic programming and fuzzy programming. Stochastic programming, as an optimization method on the basis of the probability theory, has been developing in various ways [1-5], including two stage problem by [6], and chance constrained programming by [7]. Fuzzy mathematical programming representing the vagueness in decision making situations by fuzzy concepts has been studied by many researchers [8-12].

In most practical situations, however, it is natural to consider that the uncertainty in real world decision making problems is often expressed by a fusion of fuzziness and randomness rather than either fuzziness or randomness. For handling not only the decision maker's vague judgments in multiobjective problems but also the randomness of the parameters involved in the objectives and/or constraints, Sakawa and his colleagues incorporated their interactive fuzzy satisficing methods for deterministic problems [9,13] into multiobjective stochastic programming problems. Through the introduction of several stochastic programming models such as expectation optimization [14,15], variance minimization [14], probability maximization [14,16,17] and fractile criterion optimization [14] together with chance constrained programming techniques, they introduced several interactive fuzzy satisficing methods to derive a satisficing solution for a decision maker from Pareto optimal solution sets.

It should be stressed here that in the chance constrained problems, for random data variations, a mathematical model is formulated such that the violation of the constraints is permitted up to specified probability levels. Compared with this, in a two-stage [18] or multistage [19] model including a simple recourse model as a special case, a shortage or an excess arising from the violation of the constraints is penalized, and then the expectation of the amount of the penalties for the constraint violation is minimized [20,21].

Furthermore, it is often found that in real-world decision making situations, decision variables in a multiobjective stochastic programming problem are not continuous but rather discrete. From this observation, we discuss interactive fuzzy multiobjective stochastic integer programming which is a natural extension of multiobjective stochastic programming with continuous variables. To deal with practical sizes of multiobjective stochastic nonlinear integer programming problems formulated for decision making problems in the real-world, we employ genetic algorithms to derive a satisficing solution to the decision maker.

Under these circumstances, in this paper, we consider multiobjective integer programming problems involving random variables in constraints. The main contribution of this paper is to provide a novel decision making methodology including a new model, solution concept and solution algorithm to deal with more realistic problems in the real world, by simultaneously considering various concepts such as fuzziness, randomness, integer decision variables and interactive fuzzy programming, while most of previous papers dealt with either of the concepts or a part of them. Using the concept of simple recourse [20], the formulated multiobjective stochastic simple recourse problems are transformed into deterministic ones. For solving transformed deterministic problems efficiently, genetic algorithms with double strings for nonlinear integer programming problems are introduced. Assuming that a decision maker has a fuzzy goal for each of the objective functions, we present an interactive fuzzy satisficing method to derive a satisficing solution for the decision maker by updating the reference membership levels for fuzzy goals represented by the membership functions. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed method.

2. Multiobjective Stochastic Integer Programming with Simple Recourse

We consider a simple recourse model for the multiobjective stochastic integer programming problems

$$\left.\begin{array}{ll} \text{minimize} & z_1(\boldsymbol{x}) = c_1\boldsymbol{x} \\ & \cdots \\ \text{minimize} & z_k(\boldsymbol{x}) = c_k\boldsymbol{x} \\ \text{subject to} & A\boldsymbol{x} = \bar{\boldsymbol{b}} \\ & x_j \in \{0,1,\cdots,v_j\}, j = 1,2,\cdots,n, \end{array}\right\} \quad (1)$$

where $\boldsymbol{x}$ is an $\boldsymbol{n}$ dimensional integer decision variable column vector, $\boldsymbol{c}_l$, $\boldsymbol{l}$ = 1, 2, ⋯, $\boldsymbol{k}$ are $\boldsymbol{n}$ dimensional coefficient row vectors, $\boldsymbol{A}$ is an $\boldsymbol{m} \times \boldsymbol{n}$ coefficient matrix, and $\bar{\boldsymbol{b}}$ is an $\boldsymbol{m}$ dimensional random variable column vector. It should be noted here that, in a simple recourse model, the random variables are involved only in the right-hand side of the constraints.

To understand an idea of the formulation in the simple recourse model, consider a decision problem of a manufacturing company where the decision variables actually make sense only if they are integer values. Suppose that the company makes m types of products which requires $\boldsymbol{n}$ kinds of working processes, and a decision maker (DM) in the company desires to optimize the total profit and the total production cost simultaneously.

Let $\boldsymbol{x} = (x_1, \cdots, x_n)$ denote activity levels for the $\boldsymbol{n}$ kinds of working processes which are integer decision variables, and then $\boldsymbol{Tx}$ denotes the amount of products, where $\boldsymbol{T}$ is an $\boldsymbol{m} \times \boldsymbol{n}$ matrix transforming the $\boldsymbol{n}$ kinds of activity levels of the working processes into the $\boldsymbol{m}$ types of products. The total profit is expressed as $\boldsymbol{c}_1\boldsymbol{Tx}$, where an m dimensional coefficient vector $\boldsymbol{c}_1$ is a vector of unit product profits for the $\boldsymbol{m}$ types of products, and the total production cost is represented by $\boldsymbol{c}_2\boldsymbol{x}$, where an n dimensional coefficient vector $\boldsymbol{c}_2$ is a vector of unit costs for the $\boldsymbol{n}$ kinds of activity levels of the working processes. Assume that the demand coefficients $\bar{\boldsymbol{d}} = (\bar{d}_1, \cdots, \bar{d}_m)^T$ for the $\boldsymbol{m}$ products are uncertain, and they are represented by random variables. The demand constraints are expressed as $\boldsymbol{Tx} + \boldsymbol{Iy}^+ - \boldsymbol{Iy}^- = \bar{\boldsymbol{d}}$, where $\boldsymbol{y}^+$ and $\boldsymbol{y}^-$ represent the errors for estimating the demands, and $\boldsymbol{I}$ is the $\boldsymbol{m}$ dimensional identity matrix. These two objectives are optimized under the demand constraints $\boldsymbol{Tx} + \boldsymbol{Iy}^+ - \boldsymbol{Iy}^- = \bar{\boldsymbol{d}}$ together with the ordinary constraints $\boldsymbol{Bx} \le \boldsymbol{e}$ without uncertainty for the activity levels such as the capacity, budget, technology, etc. The constraints $\boldsymbol{Ax} = \bar{\boldsymbol{b}}$ in (1) can be interpreted as the combined form of the demand constraints with random variables and the ordinary constraints without uncertainty.

Let us return to a general case of the recourse model. It is assumed that, in this model, the DM must make a decision before the realized values of the random variables involved in (1) are observed, and the penalty of the violation of the constraints is incorporated into the objective function in order to consider the loss caused by random date variations.

To be more specific, by expressing the difference between $\boldsymbol{A}x$ and $\bar{\boldsymbol{b}}$ in (1) as two vectors

$$\boldsymbol{y}^+ = (y_1^+, \cdots, y_m^+)^T$$

and

$$\boldsymbol{y}^- = (y_1^-, \cdots, y_m^-)^T,$$

the expectation of a recourse for the lth objective function is represented by

$$R_l(\boldsymbol{x}) = E\left[\min_{y^+, y^-}\left(\boldsymbol{q}_l^+\boldsymbol{y}^+ + \boldsymbol{q}_l^-\boldsymbol{y}^-\right) \middle| \boldsymbol{y}^+ - \boldsymbol{y}^- = \boldsymbol{b}(\omega) - \boldsymbol{A}x\right] \quad (2)$$

where $\boldsymbol{q}_l^+$ and $\boldsymbol{q}_l^-$ are m dimensional constant row vectors, and $\boldsymbol{b}(\omega)$ is an m dimensional realization vector of $\bar{\boldsymbol{b}}$ for an elementary event ω. Thinking of each element of

$$\boldsymbol{y}^+ = (y_1^+, \cdots, y_m^+)^T$$

and

$$\boldsymbol{y}^- = (y_1^-, \cdots, y_m^-)^T$$

as a shortage and an excess of the left-hand side, respectively, we can regard each element of $\boldsymbol{q}_l^+$ and $\boldsymbol{q}_l^-$ as the cost to compensate the shortage and the cost to dispose the excess, respectively.

Then, for the multiobjective stochastic programming problem, the simple recourse problem is formulated as

$$\left.\begin{array}{ll}\text{minimize} & \boldsymbol{c}_1\boldsymbol{x}+R_1(\boldsymbol{x})\\ & \cdots\\ \text{minimize} & \boldsymbol{c}_k\boldsymbol{x}+R_k(\boldsymbol{x})\\ \text{subject to} & x_j\in\{0,1,\ldots,v_j\},\ j=1,2,\cdots,n,\end{array}\right\}\quad(3)$$

Because $\boldsymbol{q}_l^+$ and $\boldsymbol{q}_l^-$ are interpreted as penalty coefficients for shortages and excesses, it is quite natural to assume that $\boldsymbol{q}_l^+\geq 0$ and $\boldsymbol{q}_l^-\geq 0$, and then, it is evident that, for all i = 1, ⋯, m, the complementary relations

$$\widehat{y}_i^+>0\Rightarrow\widehat{y}_i^-=0,$$

$$\widehat{y}_i^->0\Rightarrow\widehat{y}_i^+=0$$

should be satisfied for an optimal solution. With this observation in mind, we have

$$\widehat{y}_i^+=b_i(\omega)-\sum_{j=1}^n a_{ij}x_j,\ \widehat{y}_i^-=0\text{ if }b_i(\omega)\geq\sum_{j=1}^n a_{ij}x_j,$$

$$\widehat{y}_i^+=0,\ \widehat{y}_i^-=\sum_{j=1}^n a_{ij}x_j-b_i(\omega)\text{ if }b_i(\omega)<\sum_{j=1}^n a_{ij}x_j.$$

Recalling that $\bar{\boldsymbol{b}}_i,\boldsymbol{i}=1,2,\cdots,\boldsymbol{m}$ are mutually independent, (2) can be explicitly calculated as

$$\begin{aligned}R(x)&=E\left[\min_{y^+,y^-}\left(q_l^+y^++q_l^-y^-\right)\mid y^+-y^-=b(\omega)-Ax\right]\\&=\sum_{i=1}^m q_{li}^+\int_{\sum_{j=1}^n a_{ij}x_j}^{+\infty}\left(b_i-\sum_{j=1}^n a_{ij}x_j\right)\mathrm{d}F_i(b_i)\\&\quad+\sum_{i=1}^m q_{li}^-\int_{-\infty}^{\sum_{j=1}^n a_{ij}x_j}\left(\sum_{j=1}^n a_{ij}x_j-b_i\right)\mathrm{d}F_i(b_i)\\&=\sum_{i=1}^m q_{li}^+E\left[\bar{b}_i\right]-\sum_{i=1}^m\left(q_{li}^++q_{li}^-\right)\int_{-\infty}^{\sum_{j=1}^n a_{ij}x_j}b_iF_i(b_i)\\&\quad-\sum_{i=1}^m q_{li}^+\sum_{j=1}^n a_{ij}x_j\\&\quad+\sum_{j=1}^n\left(q_{li}^++q_{li}^-\right)\left(\sum_{j=1}^n a_{ij}x_j\right)F_i\left(\sum_{j=1}^n a_{ij}x_j\right),\end{aligned}\quad(4)$$

where $\boldsymbol{F_i}$ is the probability distribution function of $\bar{\boldsymbol{b}}_i$. Then, (3) can be rewritten as

$$\left.\begin{array}{ll}\text{minimize} & z_1^R\boldsymbol{x}\\ & \cdots\\ \text{minimize} & z_k^R(\boldsymbol{x})\\ \text{subject to} & x_j\in\{0,1,\cdots,v_j\},\ j=1,2,\cdots,n,\end{array}\right\}\quad(5)$$

where

$$\begin{aligned}z_l^R(\mathbf{x})&=\sum_{i=1}^m q_{li}^+E\left[\bar{b}_i\right]+\sum_{j=1}^n\left(c_{lj}-\sum_{i=1}^m a_{lj}q_{li}^+\right)x_j\\&\quad+\sum_{i=1}^m\left(q_{li}^++q_{li}^-\right)\left\{\left(\sum_{j=1}^n a_{ij}x_j\right)F_i\left(\sum_{j=1}^n a_{ij}x_j\right)\right.\\&\quad\left.-\int_{-\infty}^{\sum_{j=1}^n a_{ij}x_j}b_i\mathrm{d}F_i(b_i)\right\}\end{aligned}$$

3. Fuzzy Goals

In order to consider the imprecise nature of the DM's judgments for each objective function $z_l^R(\boldsymbol{x})$ in (5), by introducing the fuzzy goals such as "$z_l^R(\boldsymbol{x})$ should be substantially less than or equal to a certain value," (5) can be interpreted as

$$\left.\begin{array}{ll}\text{minimize} & \mu_1\left(z_1^R(\boldsymbol{x})\right)\\ & \cdots\\ \text{minimize} & \mu_k\left(z_k^R(\boldsymbol{x})\right)\\ \text{subject to} & x_j\in\{0,1,\cdots,v_j\},\ j=1,2,\cdots,n,\end{array}\right\}\quad(6)$$

where μ_l is a membership function to quantify a fuzzy goal for the lth objective function in (5) as shown in **Figure 1**.

It should be noted here that (6) is regarded as a multiobjective decision making problem, and that there rarely exists a complete optimal solution that simultaneously optimizes all the objective functions. By directly extending Pareto optimality in ordinary multiobjective programming problems, Sakawa et al. defined M-Pareto optimality on the basis of membership function values as a reasonable solution concept for the fuzzy multiobjective decision making problem [9,13].

Definition 1 (M-Pareto optimal solution). *A point* $\boldsymbol{x}^*\in X$ *is said to be an M-Pareto optimal solution if and only if there does not exist another* $\boldsymbol{x}\in X$ *such that*

$$\mu_l\left(z_l(\boldsymbol{x})\right)\geq\mu_l\left(z_l\left(\boldsymbol{x}^*\right)\right)$$

for all $l\in\{1,\cdots,k\}$ *and*

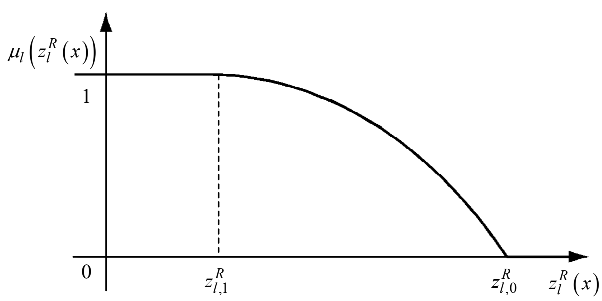

Figure 1. Example of a membership function $\mu_l\left(z_l^R(x)\right)$.

$$\mu_j\left(z_j\left(\boldsymbol{x}\right)\right) \neq \mu_j\left(z_j\left(\boldsymbol{x}^*\right)\right)$$

for at least one $j \in \{1, \cdots, k\}$, *where X denotes the feasible region of the problem.*

To help the DM specify the membership functions, it is recommended to calculate the individual minima of $z_l^R(\boldsymbol{x})$ by solving the nonlinear integer programming problems

$$\left.\begin{array}{ll}\text{minimize} & z_l^R(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in X^{\text{int}}\end{array}\right\}, l = 1, 2, \cdots, k \quad (7)$$

where X^{int} denotes the feasible region of (6).

In order to find a candidate for the satisficing solution, the DM specifies the reference membership levels $\hat{\mu}_l, l = 1, \cdots, k,$ and then by solving the augmented minimax problem

$$\left.\begin{array}{ll}\text{minimize} & \max\limits_{1 \le l \le k}\left\{\hat{\mu}_l - \mu_l\left(z_l^R(\boldsymbol{x})\right)\right. \\ & \left. + \rho\sum\limits_{l=1}^{k}\left(\hat{\mu}_l - \mu_l\left(z_l^R(\boldsymbol{x})\right)\right)\right\} \\ \text{subject to} & x_j \in \{0, 1, \cdots, v_j\}, j = 1, \cdots, n,\end{array}\right\} \quad (8)$$

an M-Pareto optimal solution corresponding to $\hat{\mu}_l, l = 1, \cdots, k$ is obtained, and (8) is equivalently expressed by

$$\left.\begin{array}{ll}\text{minimize} & v \\ \text{subject to} & \mu_1 - \mu_1\left(z_1^R(\boldsymbol{x})\right) + \rho\sum_{i=1}^{k}\left(\mu_i - \mu_i\left(z_l^R(\boldsymbol{x})\right)\right) \le v \\ & \cdots \\ & \mu_k - \mu_k\left(z_k^R(\boldsymbol{x})\right) + \rho\sum\limits_{i=1}^{k}\left(\mu_i - \mu_i\left(z_l^R(\boldsymbol{x})\right)\right) \le v \\ & x_j \in \{0, 1, \cdots, v_j\}, j = 1, \cdots, n,\end{array}\right\} \quad (9)$$

where ρ is a sufficiently small positive number.

Observing that (7) and (9) are nonlinear integer programming problems, we cannot directly apply GADSLPRRSU [11] for solving them.

4. Genetic Algorithms for Nonlinear Integer Programming

For solving linear integer programming problems on the framework of genetic algorithms, Sakawa proposed GADSLPRRSU [11]. GADSLPRRSU is an abbreviation for genetic algorithms with double strings based on linear programming relaxation and reference solution updating. This method includes three key ideas: double strings (DS), linear programming relaxation (LPR), and reference solution updating (RSU). Unfortunately, however, due to nonlinearity, we cannot directly apply GADSLPRRSU for solving (7) and (9). However, we can introduce the revised GADSLPRRSU where GENOCOPIII [22,23] is employed for solving a nonlinear continuous relaxation problem.

As an efficient approximate solution method, the revised GADSLPRRSU are designed for nonlinear integer programming problems formulated as:

$$\left.\begin{array}{ll}\text{minimize} & f(\boldsymbol{x}) \\ \text{subject to} & g_i(\boldsymbol{x}) \le 0, i = 1, 2, \cdots, m \\ & x_j \in \{0, 1, \cdots, v_j\}, j = 1, 2, \cdots, n,\end{array}\right\} \quad (10)$$

where $\boldsymbol{x}$ is an n dimensional integer decision variable column vector. Furthermore, $f(\cdot)$ and $g_i(\cdot), i = 1, 2, \cdots, m$ may be nonlinear.

Quite similar to genetic algorithms with double (GADS) [11], an individual is represented by a double string shown in **Figure 2**. In **Figure 2**, for a certain j, $s(j) \in \{1, 2, \cdots, n\}$ represents an index of a decision variable $\boldsymbol{x}_{s(j)}$ in the solution space, while $\boldsymbol{y}_{s(j)}, j = 1, 2, \cdots, n$ does the integer value among $\{0, 1, \cdots, v_j\}$ of the $s(j)$ th decision variable $\boldsymbol{x}_{s(j)}$.

Now we can summarize the computational procedures of the revised GADSLPRRSU as follows.

Computational procedures of the revised GADSLPRRSU

Step 0: Determine values of the parameters used in the genetic algorithm.

Set the generation counter t at 0.

Step 1: Generate the initial population consisting of N individuals based on the information of the optimal solution to the continuous relaxation problem.

Step 2: Decode each individual in the current population and calculate its fitness based on the corresponding solution.

Step 3: If the termination condition is fulfilled, stop. Otherwise, let $t := t + 1$.

Step 4: Apply reproduction operator using elitist expected value selection after linear scaling.

Step 5: Apply crossover operator, called PMX (Partially Matched Crossover) for double string.

Step 6: Apply mutation based on the information of a solution to the continuous relaxation problem.

Step 7: Apply inversion operator, return to Step 2.

Further details of GADSLPRRSU and the revised GADSLPRRSU can be found in [11,24].

5. Interactive Fuzzy Satisficing Method

We can now construct the interactive algorithm for

$s(1)$	$s(2)$	⋯	$s(n)$
$y_s(1)$	$y_s(2)$	⋯	$y_s(n)$

Figure 2. Double string.

deriving a satisficing solution for the DM where (9) for the updated reference membership levelsis repeatedly solved until the DM is satisfied with an obtained optimal solution.

Interactive Fuzzy Satisficing Method for the Simple Recourse Model with Integer Decision Variables

Step 1: Calculate the individual minima $z_{l,\min}^R$ of $z_l^R(\boldsymbol{x}), l = 1,\cdots,k$ by solving (7) through the revised GADSLPRRSU.

Step 2: Ask the DM to specify the membershipfunctions $\hat{\mu}_l, l = 1,\cdots,k$ taking into account the individual minima obtained in step 1.

Step 3: Set the initial reference membership levels at 1s, which can be viewed as the ideal values, *i.e.* $\hat{\mu}_l, l = 1,\cdots,k$.

Step 4: Solve the augmented minimax problem (9) for the current reference membership levels $\hat{\mu}_l, l = 1,\cdots,k$ by using the revised GADSLPRRSU.

Step 5: The DM is supplied with the corresponding M-Pareto optimal solution $\boldsymbol{x}^*$. If the DM is satisfied with the current membership function values

$$\mu_l\left(z_l^R\left(\boldsymbol{x}^*\right)\right), l = 1,\cdots,k,$$

stop the algorithm. Otherwise, ask the DM to update the reference membership levels $\hat{\mu}_l, l = 1,\cdots,k$, in consideration of the current membership function values, and return to step 4.

Here it should be stressed for the DM that any improvement of one membership function value can be achieved only at the expense of at least one of the other membership function values.

6. Numerical Example

In order to demonstrate the feasibility and efficiency of the interactive fuzzy satisficing method for the simple recourse model, as a numerical example of (1), consider the multiobjective stochastic integer programming problem formulated as

$$\left.\begin{array}{ll} \text{minimize} & z_1(\boldsymbol{x}) = c_1\boldsymbol{x} \\ \text{minimize} & z_2(\boldsymbol{x}) = c_2\boldsymbol{x} \\ \text{minimize} & \\ \text{subject to} & z_3(\boldsymbol{x}) = c_3\boldsymbol{x} \\ & a_1\boldsymbol{x} = \bar{b}_1 \\ & a_2\boldsymbol{x} = \bar{b}_2 \\ & a_3\boldsymbol{x} = \bar{b}_3 \\ & x_j \in \{0,1,\cdots,v_j\}, j = 1,2,\cdots,10, \end{array}\right\} \quad (11)$$

where $\bar{b}_1$, $\bar{b}_2$ and $\bar{b}_3$ are Gaussian random variables $N(230, 12)$, $N(345, 18)$ and $N(437, 22)$, respectively. Each element of coefficient vectors $\boldsymbol{a}_i, i = 1,2,3$ is randomly chosen from among a set of integers $\{1,\cdots,10\}$, and each element of $\bar{\boldsymbol{c}}_1$, $\bar{\boldsymbol{c}}_2$ an d $\bar{\boldsymbol{c}}_3$ is also randomly chosen from among sets of integers $\{-1,\cdots,-10\}$, $\{1,\cdots,10\}$ and $\{-1,\cdots,-10\}$, respectively. These values are shown in **Table 1**. The penalty coefficient row vectors $\boldsymbol{q}_l^+$ and $\boldsymbol{q}_l^-, l = 1,2,3$ for violating the constraints are given in **Table 2**.

By using the revised GADSLPRRSU, the individual minima $z_{l,\min}^R$ are calculated as $z_{1,\min}^R = 61.647$, $z_{2,\min}^R = -945.002$ and $z_{3,\min}^R = -766.075$. Taking these values into account, suppose that the DM determines the linear membership functions as

$$\mu_l\left(z_l^R(x)\right) = \begin{cases} 1 & \text{if } z_l^R(\boldsymbol{x}) < z_{l,1}^R \\ \dfrac{z_l^R(\boldsymbol{x}) - z_{l,0}^R}{z_{l,1}^R - z_{l,0}^R} & \text{if } z_{l,1}^R \le z_l^R(\boldsymbol{x}) \le z_{l,0}^R \\ 0 & \text{if } z_{l,0}^R < z_l^R(\boldsymbol{x}) \end{cases} \quad (12)$$

where $z_{1,1}^R$ and $z_{1,0}^R$ are calculated as $z_{1,1}^R = 61.467$, $z_{1,0}^R = 477.837$, $z_{2,1}^R = -945.002$, $z_{2,0}^R = 662.894$, $z_{3,1}^R = -766.075$ and $z_{3,0}^R = 633.461$ by using the Zimmermann method [25].

For the initial reference membership levels $(\bar{\mu}_1, \bar{\mu}_2, \bar{\mu}_3) = (1.00, 1.00, 1.00)$, the corresponding augmentedminimax problem (8) is solved by using the revised GADSLPRRSU, and the DM is supplied with the membership function values of the first iteration shown in **Table 3**.

Assume that the DM is not satisfied with these membership function values, and the DM updates the reference membership levels as $(1.00, 1.00, 0.90)$ for improving the satisfaction levels μ_1 and μ_2 at the expense of μ_3. For the updated reference membership levels, the corresponding augmented minimax problem is solved

Table 1. Value of each element of c_i, i = 1, 2, 3 and a_i, i = 1, 2, 3.

c_1	−8	−1	−2	−7	−3	−5	−1	−4	−10	5
c_2	3	5	2	6	1	1	4	7	2	9
c_3	2	3	−10	4	4	5	−9	1	−8	2
a_1	4	4	1	2	6	1	1	7	5	8
a_2	10	2	6	1	2	2	8	5	2	8
a_3	3	8	8	5	1	9	7	7	3	2

Table 2. Value of each element of q_l^+ and q_l^-, l = 1, 2, 3.

q_1^+	2.0	0.4	0.4	q_1^-	0.2	0.6	0.3
q_2^+	1.0	0.6	1.0	q_2^-	0.5	2.0	3.0
q_3^+	1.2	1.0	0.6	q_3^-	1.4	0.9	1.1

Table 3. Process of interaction.

Iteration	1st	2nd	3rd
$\hat{\mu}_1$	1.000	1.000	0.950
$\hat{\mu}_2$	1.000	1.000	1.000
$\hat{\mu}_3$	1.000	0.900	0.900
$\mu_1\left(z_1^R(\boldsymbol{x})\right)$	0.543	0.583	0.554
$\mu_2\left(z_2^R(\boldsymbol{x})\right)$	0.540	0.570	0.606
$\mu_3\left(z_3^R(\boldsymbol{x})\right)$	0.543	0.474	0.507
$z_1^R(\boldsymbol{x})$	306.716	292.734	302.738
$z_2^R(\boldsymbol{x})$	–292.957	–329.051	–372.403
$z_3^R(\boldsymbol{x})$	–557.129	–530.395	–543.244

again, and the membership function values calculated in the second iteration are shown in **Table 3**.

A similar procedure continues until the DM is satisfied with the membership function values. In this example, we assume that the satisficing solution for the DM is derived in the third interaction.

7. Conclusion

In this paper, we focused on multiobjective integer programming problems with random variables in in the right-hand side of the constraints. Through the use of the simple recourse model, the formulated multiobjective stochastic integer programming problems with simple recourse are transformed into deterministic ones. Assuming that the DM has fuzzy goals for the objective functions, an interactive fuzzy satisficing method for deriving a satisficing solution for the decision maker from among the M-Pareto optimal solution set has been proposed. In the proposed interactive method, after determining the fuzzy goals of the DM, a satisficing solution is derived efficiently by updating the reference membership levels of the DM. Genetic algorithm for nonlinear integer programming, called the revised GADSLPRRSU were introduced for solving transformed deterministic ones efficiently.An illustrative numerical example was provided to demonstrate the feasibility and efficiency of the proposed method. Extensions to other stochastic programming models will be considered elsewhere.

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