

Comments on “Average Life Prediction Based on Incomplete Data”

Tachen Liang

Wayne State University, Detroit, USA
 Email: aa4156@wayne.edu

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ABSTRACT

We comment on the correctness of the article “Average life prediction based on incomplete data” by [1] (Applied Mathematics, Vol. 2, pp. 93-105).

Keywords: Average Life Prediction; Censored Data

1. Introduction

Tang *et al.* [1] studied average life prediction based on incomplete data assuming the prior distribution being unknown. However, the paper contains serious errors and the concluded results are incorrect. We shall point out the errors in the following. To do so, we first describe the considered statistical model as below.

Suppose that there are n different manufacture units possessing the same technology and regulations. For a known integer $m > 0$, a sample of m components are selected from unit j , and put on life test at time $t = 0$, for each $j = 1, \dots, n$. It is assumed that the lifetime of a component arising from unit j follows a two-parameter exponential distribution having probability density

$$f(x; \theta_j, \mu_j) = \frac{1}{\theta_j} \exp\left(-\frac{x - \mu_j}{\theta_j}\right) I(x > \mu_j).$$

Let X_{j1}, \dots, X_{jm} denote the lifetimes of the m components. The life test experiment will be terminated if one of the m components fails. Thus,

$X_j = \min(X_{j1}, \dots, X_{jm})$ is the lifetime of the ineffective component from unit j . Let $a > 0$ be a pre-specified constant. If $X_j \geq a$, then a second round sample is carried, at which we sample one more component from unit j , and denote its unknown lifetime by Y_j .

Furthermore, it is assumed that (θ_j, μ_j) , $j = 1, \dots, n$, are iid random parameters, and that X_j are possibly censored from the right by a non-negative random censoring variable V_j , where V_1, \dots, V_n are iid, with a common distribution W , and (V_1, \dots, V_n) are independent of (X_1, \dots, X_n) . Thus, X_j may not always be ob-

servable. Instead, one can only observe $Z_j = \min(X_j, V_j)$ and $\delta_j = I(X_j \leq V_j)$. Through the preceding assumptions, $(X_j, Y_j, V_j, Z_j, \delta_j, \theta_j, \mu_j)$, $j = 1, \dots, n$ are iid, $\{V_j, j = 1, \dots, n\}$ and $\{(X_j, Y_j, \theta_j, \mu_j), j = 1, \dots, n\}$ are mutually independent.

Let $S_1 = \frac{1}{n} \sum_{j=1}^n I(X_j \geq a) Y_j$ and

$S_2 = \frac{1}{n} \sum_{j=1}^n I(X_j \geq a)$. $S = S_1/S_2$ is the average life of

the second round sample. Tang *et al.* (2011) attempted to predict $S = S_1/S_2$ based on the data (Z_j, δ_j) , $j = 1, \dots, n$. Let

$$\bar{S}_1 = \frac{1}{n(n-1)} \sum_{i=1}^n \left[\sum_{j \neq i}^n \frac{Z_j \delta_j}{1 - W(Z_j)} \right] \frac{I(Z_i \geq a) \delta_i}{1 - W(Z_i)},$$

$$+ \frac{(m-1)}{n} \sum_{i=1}^n (Z_i - a) \frac{I(Z_i \geq a) \delta_i}{1 - W(Z_i)}$$

$$\bar{S}_2 = \frac{1}{n} \sum_{i=1}^n \frac{I(Z_i \geq a) \delta_i}{1 - W(Z_i)}, \text{ and } \bar{S} = \bar{S}_1/\bar{S}_2.$$

Tang *et al.* [1] proposed using \bar{S} to predict S . Tang *et al.* [1] claimed the following results:

$$E[\bar{S}_j] = E[S_j], \quad j = 1, 2. \quad (1)$$

(see (2.8)-(2.9) of Tang *et al.* [1]).

Based on the identity property of (1), and some additional conditions, Tang *et al.* [1] claimed in their Theorem 1 that $\bar{S} - S \rightarrow 0$ in probability as $n \rightarrow \infty$. They

further apply the identity property of (1) and Theorem 1 to claim their Theorem 2.

We now point out the errors of Tang *et al.* [1] as follows.

1) The sampling scheme is not well defined. Since random censorship model is considered, we may be unable to observe the exact value of X_j . In case that $\delta_j = 0$ and $Z_j < a$, it is possible that $X_j \geq a$ or $X_j < a$. In such a situation, shall we carry the second round of sampling and sample one more component from unit j ? This is not discussed in the paper.

2) For the distribution W known case, unfortunately, the claimed identity that $E[\bar{S}_1] = E[S_1]$ is incorrect. The error is pointed out as follows. In (2.8) of [1], it is stated that (see Equation (2)):

In (2) (or (2.8) of Tang *et al.* [1]), the first equality

is not true, where the notation (θ, μ) is abused. The correct computation is given below. Note that for each j , given (θ_j, μ_j) , X_j follows a double exponential distribution having pdf $f(x; \theta_j, \mu_j)$, and $(X_j, V_j, Z_j, \delta_j, \theta_j, \mu_j)$, $j = 1, \dots, n$ are iid. Using the iid property, we can obtain Equation (3)

Note that the expression of (3) is different from that of (2). So, we see that $E[\bar{S}_1] \neq E[S_1]$

This type of computational errors also occur at (5.3), (5.4), (5.7), (5.8) and (5.9) of Tang *et al.* [1], where $E[S_1^2]$, $E[S_1 \bar{S}_1]$ and $E[\bar{S}_1^2]$ were calculated.

Based on the preceding discussion, the correctness of Theorem 1 in Tang *et al.* [1] is doubtful.

3) Tang *et al.* (2011) then applied the result of their Theorem 1 for the case where the distribution W is un-

$$\begin{aligned}
 E[\bar{S}_1] &= \frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j \neq i}^n E \left\{ E \left[\frac{Z_j \delta_j}{1-W(Z_j)} \middle| (\theta, \mu) \right] E \left[\frac{I(Z_i \geq a) \delta_i}{1-W(Z_i)} \middle| (\theta, \mu) \right] \right\} \\
 &\quad + (m-1) E \left\{ E \left[\frac{(Z_1 - a) I(Z_1 \geq a) \delta_1}{1-W(Z_1)} \middle| (\theta, \mu) \right] \right\} \\
 &= E \left\{ \iint_{x \leq v} \frac{x}{1-W(x)} dW(v) l(x|\theta, \mu) dx \iint_{x \leq v} \frac{I(x \geq a)}{1-W(x)} dW(v) l(x|\theta, \mu) dx \right\} \\
 &\quad + (m-1) E \left\{ \iint_{x \leq v} \frac{(x-a) I(x \geq a)}{1-W(x)} dW(v) l(x|\theta, \mu) dx \right\} \\
 &= E_{(\theta, \mu)} \left\{ \left(\mu + \frac{\theta}{m} \right) \exp \left(-\frac{m(a-\mu)}{\theta} \right) + \frac{(m-1)\theta}{m} \exp \left(-\frac{m(a-\mu)}{\theta} \right) \right\} = E[S_1]
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 E[\bar{S}_1] &= \frac{1}{n(n-1)} \sum_{i=1}^n \sum_{j \neq i}^n E \left\{ E \left[\frac{Z_j \delta_j}{1-W(Z_j)} \middle| (\theta_j, \mu_j) \right] E \left[\frac{I(Z_i \geq a) \delta_i}{1-W(Z_i)} \middle| (\theta_i, \mu_i) \right] \right\} \\
 &\quad + (m-1) E \left\{ E \left[\frac{(Z_1 - a) I(Z_1 \geq a) \delta_1}{1-W(Z_1)} \middle| (\theta_1, \mu_1) \right] \right\} \\
 &= E \left\{ \iint_{x \leq v} \frac{x}{1-W(x)} dW(v) l(x|\theta_j, \mu_j) dx \iint_{x \leq v} \frac{I(x \geq a)}{1-W(x)} dW(v) l(x|\theta_i, \mu_i) dx \right\} \\
 &\quad + (m-1) E \left\{ \iint_{x \leq v} \frac{(x-a) I(x \geq a)}{1-W(x)} dW(v) l(x|\theta, \mu) dx \right\} \\
 &= E \left\{ \left(\mu_j + \frac{\theta_j}{m} \right) \exp \left(-\frac{m(a-\mu_i)}{\theta_i} \right) \right\} + E_{(\theta, \mu)} \left\{ \frac{(m-1)\theta}{m} \exp \left(-\frac{m(a-\mu)}{\theta} \right) \right\} \\
 &= E_{(\theta_j, \mu_j)} \left\{ \left(\mu_j + \frac{\theta_j}{m} \right) \right\} E_{(\theta_i, \mu_i)} \left\{ \exp \left(-\frac{m(a-\mu_i)}{\theta_i} \right) \right\} + E_{(\theta, \mu)} \left\{ \frac{(m-1)\theta}{m} \exp \left(-\frac{m(a-\mu)}{\theta} \right) \right\} \\
 &= E_{(\theta, \mu)} \left\{ \left(\mu + \frac{\theta}{m} \right) \right\} E_{(\theta, \mu)} \left\{ \exp \left(-\frac{m(a-\mu)}{\theta} \right) \right\} + E_{(\theta, \mu)} \left\{ \frac{(m-1)\theta}{m} \exp \left(-\frac{m(a-\mu)}{\theta} \right) \right\}
 \end{aligned} \tag{3}$$

known, and claimed their Theorem 2. However, since Theorem 1 is dubious, the correctness of Theorem 2 is also doubtful.

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REFERENCES

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