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The Some Properties of Skew Polynomial Rings

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Abstract

This paper mainly studies some properties of skew polynomial ring related to Morita invariance, Armendariz and (quasi)-Baer. First, we show that skew polynomial ring has no Morita invariance by the counterexample. Then we prove a necessary condition that skew polynomial ring constitutes Armendariz ring. We lastly investigate that condition of skew polynomial ring is a (quasi)-Baer ring, and verify that the conditions is necessary, but not sufficient by example and counterexample.

Keywords

Skew Polynomial Ring, (Quasi)-Baer Ring, Armendariz Ring, Morita Context Ring, Morita Invariance, Nozero Divisor Ring

1. Introduction

Throughout this paper every ring is an associative with identity unless otherwise stated. Given a ring *R*, $R[x]$, $R[x;\sigma]$, $r_R(X)$, $l_R(X)$, $\mathrm{Mat}_n(R)$ and $\mathbb{Z}_n$ denote the polynomial ring with an indeterminate *x* over *R*, the skew polynomial ring over *R*, the right annihilator of nonempty subset *X* of ring *R*, the left annihilator of nonempty subset *X* of ring *R*, and the $n \times n$ matrix ring over *R*, the ring of integers modulo *n*, respectively. A ring $R[x;\sigma]$ is called *Skew polynomial ring* if σ is an endomorphism over *R*; operations are usual addition and multiplication defined by $xa = \sigma(a)x, a \in R$. In [1], that skew polynomial ring has no Morita invariance. A ring *R* is called *Armendariz ring* if $f(x)g(x)=0$ implies $a_i b_j = 0$, where $0 \le i \le m$, $0 \le j \le n$ for any $f(x)=\sum_{i=0}^{m} a_i x^i$, $g(x)=\sum_{j=0}^{n} b_j x^j \in R[x]$ in [2]. If *R* is a semiprime ring, then skew polynomial ring $R[x;\sigma]$ is a quasi-Armendariz ring by [3]. G. F. Birkenmeier first introduced the concept of Baer ring, and

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proved that Baer ring is quasi-Baer ring, but converse is not hold, and right principally quasi-Baer ring has Morita invariance by [4]. Q.J. Song gave the condition that iterated skew polynomial ring constitutes (quasi)-Baer ring by [5]. We will show that skew polynomial ring has no Morita invariance by the counterexample, and the condition that skew polynomial ring has properties of Armendarizand (quasi)-Baer, and verify that the condition is necessary, but not sufficient by example and counterexample.

2. Preliminary

Definition 2.1. [6] Let *R* and *S* be rings, then *R* and *S* are *Morita equivalent* if there exists projective module W_R, such that $S \cong \operatorname{End} W_R$. *Morita invariance* is the invariant property under Morita equivalent rings.

Lemma 2.2. [6] The ring *R* and *S* are Morita equivalent, if and only if there exists an integer *n* and idempotent $e \in \operatorname{Mat}_n(R)$, such that $S \cong e\operatorname{Mat}_n(R)e$ and $\operatorname{Mat}_n(R)e\operatorname{Mat}_n(R) = \operatorname{Mat}_n(R)$.

Definition 2.3. [7] A ring *R* is called (*quasi*)-*Baer ring* if the right annihilator of (resp. right ideal) nonempty subset of *R* is generated by an idempotent as a right ideal.

Lemma 2.4. Suppose that *R* is a ring has no zero divisor and σ is a monomorphism over *R*, then skew polynomial ring $R[x;\sigma]$ has no zero divisor.

Proof. For any $f(x) = \sum_{i=0}^{m} a_i x^i$, $g(x) = \sum_{j=0}^{n} b_j x^j \in R[x]$, if

$$f(x)\cdot g(x) = \sum_{i+j=0}^{n+m} a_i\sigma^i(b_j)x^{i+j} = a_0b_0 + (a_1\sigma(b_0) + a_0b_1)x + (a_0b_2 + a_1\sigma(b_1) + a_2\sigma^2(b_0))x^2 + \cdots + a_n\sigma^n(b_m)x^{n+m} = 0$$

then all coefficients of the skew polynomial are zero. Since σ is a monomorphism and *R* has no zero divisor, so $\sigma^i(b_j) = 0$ implies $b_j = 0$, $a_0b_0 = 0$ implies $a_0 = 0$ or $b_0 = 0$. Case 1. If $a_0 = 0$, $b_0 \neq 0$, then $\sigma^i(b_0) \neq 0$. Since $a_1\sigma(b_0) + a_0b_1 = 0$, so $a_1 = 0$. Simlarly, we show $a_i = 0$ $(0 \le i \le n)$, thus $f(x) = 0$. Case 2. If $b_0 = 0$, $a_0 \neq 0$, then $\sigma^i(b_0) = 0$. Because $a_1\sigma(b_0) + a_0b_1 = 0$, so $b_1 = 0$, $\sigma^i(b_1) = 0$. Similarly, $b_j = 0$ $(0 \le j \le m)$, thus $g(x) = 0$. Case 3. If $a_0 = b_0 = 0$, $a_0b_2 + a_1\sigma(b_1) + a_2\sigma^2(b_0) = 0$, so $\sigma^i(b_0) = 0$, $a_1\sigma(b_1) = 0$. Similarly, $a_i = 0$ or $b_j = 0$ for $0 \le i \le m$, $0 \le j \le n$, then $f(x) = 0$ or $g(x) = 0$. Therefore $R[x;\sigma]$ is a ring has no divisor of zero.

Definition 2.5. [8] A ring R is called a reversible, if $ab = 0$ implies $ba = 0$ for any $a, b \in R$.

Proposition 2.6. [9] Every reduced ring is a reversible ring, but the converse does not hold.

Proposition 2.7. Let *R* be a reduced ring, then the coefficients of right annihilator of any polynomial over $R[x]$ are the right annihilator of all coefficients of the polynomial.

Proof. For any $f(x) = \sum_{i=0}^{n} a_i x^i$, $g(x) = \sum_{j=0}^{m} b_j x^j \in R[x]$, let $f(x)\cdot g(x) = 0$, then

$f(x)\cdot g(x) = \left(\sum_{i=0}^{n} a_i x^i\right)\left(\sum_{j=0}^{m} b_j x^j\right) = \sum_{i+j=0}^{n+m} a_i\sigma^i(b_j)x^{i+j} = 0$, so $a_0b_0 = 0$, $a_0b_1 + a_1b_0 = 0$, $a_0b_2 + a_1b_1 + a_2b_0 = 0$, $\cdots$, $a_nb_n = 0$ and $b_0 \in r_R(a_0)$. Because *R* be a reduced ring, hence *R* be a reversible ring, so $b_0a_0 = 0$. Because $a_0b_1 + a_1b_0 = 0$, then $b_0a_0b_1 + b_0a_1b_0 = 0$, so $b_0a_1b_0 = 0$, so $a_1b_0 = 0$, thus $b_0 \in r_R(a_1)$ and $b_0a_1 = 0$, so $a_0b_1 = 0$, so $b_1 \in r_R(a_0)$ and $b_1a_0 = 0$. Since $a_0b_2 + a_1b_1 + a_2b_0 = 0$, we have $a_0b_2a_0 + a_1b_1a_0 + a_2b_0a_0 = 0$, then $a_0b_2a_0 = 0$, so $b_2a_0 = 0$, so $b_2 \in r_R(a_0)$ and $a_0b_2 = 0$, so $a_1b_1 + a_2b_0 = 0$, so $a_1b_1a_1 = 0$, hence $a_1b_1 = 0$, so $b_1 \in r_R(a_1)$, $b_1a_1 = 0$, so $a_2b_0 = 0$, thus $b_0 \in r_R(a_2)$, Similarly, we have $b_i \in r_R(a_j)$, which $i = 0,1,2,\cdots,n$, $j = 0,1,\cdots,m$. Therefore the coefficients of right annihilator of any polynomial over $R[x]$ are the right annihilator of all coefficients of the polynomial.

Proposition 2.8. Let R be a reduced ring, then the idempotent of ring R is the idempotent of $R[x]$.

Proof. For any $f(x) = a_0 + a_1x + \cdots + a_nx^n \in R[x]$, if $f^2(x) = f(x)$, we have

$$\begin{aligned}
&(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n)^2 \\
&= a_0^2 + (a_0a_1 + a_1a_0)x + (a_0a_2 + a_1a_1 + a_2a_0)x^2 + \cdots + (a_0a_n + a_1a_{n-1}x + a_2a_{n-2}x^2 + \cdots + a_na_0)x^n \\
&\quad + (a_1a_n + a_2a_{n-1} + a_3a_{n-2} + \cdots + a_na_1)x^{n+1} + \cdots + (a_{n-1}a_n + a_na_{n-1})x^{2n-1} + (a_na_n)x^{2n} \\
&= a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n
\end{aligned}$$

then $\sum_{i+j=n+1}^{n+m} a_i\sigma^i(b_j)x^{i+j} = 0$. Since R is a reduced ring and $a_n^2 = 0$, $a_{n-2}a_n + (a_{n-1})^2 + a_na_{n-2} = 0$, so $a_n = 0$,

$a_{n-1}=0$. Similarly, we have $a_{n-2}=\cdots=a_2=0$, and because $a_0a_2+a_1^2+a_2a_0=0$, so $a_1=0$. Thus $f(x)=a_0$, which $a_0^2=a_0\in R$ is the idempotent of $R[x]$.

3. Main Results

The property of skew polynomial ring relation to Morita invariance , we have the following counterexample.

Example 3.1. Suppose that a ring Z_2 and σ is an endomorphism over Z_2, define the usual addition and multiplication by $xa=\sigma(a)x$ for any $a\in\mathrm{Z}_2$, then $\mathrm{Z}_2[x;\sigma]$ is a skew polynomial ring, but has no Morita invariance.

In fact, clearly, $\mathrm{Z}_2[x;\sigma]$ is a skew polynomial ring. Consider a ring $\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])=\begin{pmatrix} f_1(x) & f_2(x)\\ f_3(x) & f_4(x)\end{pmatrix}$, which $f_1(x)=\sum_{i=0}^{n}a_ix^i$, $f_2(x)=\sum_{i=0}^{n}b_ix^i$, $f_3(x)=\sum_{i=0}^{n}c_ix^i$, $f_4(x)=\sum_{i=0}^{n}d_ix^i\in\mathrm{Z}_2[x;\sigma]$, we have all idempotents of $\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])$ are $\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $\begin{pmatrix}0&0\\0&0\end{pmatrix}$, $\begin{pmatrix}0&0\\0&1\end{pmatrix}$, $\begin{pmatrix}1&0\\0&0\end{pmatrix}$, $\begin{pmatrix}1&1\\0&0\end{pmatrix}$, $\begin{pmatrix}1&0\\1&0\end{pmatrix}$. Suppose that $f_i(x)\neq 0,\ i=(1,2,3,4)$.

Case 1. If $e=\begin{pmatrix}1&0\\0&1\end{pmatrix}$, then

$$\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])e\,\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])=\begin{pmatrix} f_1^2(x)+f_2(x)f_3(x) & f_1(x)f_2(x)+f_2(x)f_4(x)\\ f_3(x)f_1(x)+f_4(x)f_3(x) & f_3(x)f_2(x)+f_4^2(x)\end{pmatrix}$$
$$\neq\begin{pmatrix} f_1(x) & f_2(x)\\ f_3(x) & f_4(x)\end{pmatrix}=\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])$$

Case 2. If $e=\begin{pmatrix}0&0\\0&0\end{pmatrix}$, then $\begin{pmatrix}0&0\\0&0\end{pmatrix}\neq\begin{pmatrix} f_1(x) & f_2(x)\\ f_3(x) & f_4(x)\end{pmatrix}$. Case 3. If $e=\begin{pmatrix}0&0\\0&1\end{pmatrix}$, then $\begin{pmatrix} f_1(x) & f_2(x)\\ f_3(x) & f_4(x)\end{pmatrix}\neq\begin{pmatrix} f_2(x)f_3(x) & f_2(x)f_4(x)\\ f_4(x)f_3(x) & f_4^2(x)\end{pmatrix}$. Similarly, we show that $\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])e\,\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])\neq\mathrm{Mat}_2(\mathrm{Z}_2[x;\sigma])$ for all idempotente. Clearly, the condition of $\mathrm{Mat}_n(\mathrm{Z}_2[x;\sigma])e\,\mathrm{Mat}_n(\mathrm{Z}_2[x;\sigma])=\mathrm{Mat}_n(\mathrm{Z}_2[x;\sigma])$ is not true for any integer n, so $\mathrm{Z}_2[x;\sigma]$ and any ring S are not Morita equivalent by lemma 2.2, therefore $\mathrm{Z}_2[x;\sigma]$ has no Morita invariance.

So the skew polynomial ring has no Morita invariance by the counterexample. The following theorem shows that the condition of skew polynomial ring constitutes Armendariz ring.

Theorem 3.2. Let R be a ring that has no zero divisor and σ be a monomorphism over R, then skew polynomial ring $R[x;\sigma]$ is an Armendariz ring.

Proof. Since R has no zero divisor, so $R[x;\sigma]$ has no zero divisor by lemma2.4, then $R[x;\sigma]$ is a reversible ring. For any $f_i(x)$, $g_j(x)\in R[x;\sigma]$, and $l(x)=\sum_{j=0}^{m}g_j(x)y^j$, $h(x)=\sum_{i=0}^{n}f_i(x)y^i\in R[x;\sigma][y]$, if

$$0=h(x)\cdot l(x)=\sum_{i+j=0}^{n+m}f_i(x)\sigma^i(g_j(x))y^{i+j}$$
$$=f_0(x)g_0(x)+\left(f_0(x)g_1(x)+f_1(x)g_0(x)\right)y+\cdots+f_n(x)g_m(x)y^{n+m}$$

then the all coefficients of $R[x;\sigma]$ are zero. Since $f_i(x)g_j(x)=0$, $f_0(x)g_1(x)+f_1(x)g_0(x)=0$, so $g_1(x)f_0(x)=0$, and hence $f_0(x)g_1(x)=0$, $f_1(x)g_0(x)=0$. Because $f_0(x)g_2(x)+f_1(x)g_1(x)+f_2(x)g_0(x)=0$, $f_1(x)g_1(x)+f_2(x)g_0(x)=0$, so $f_1(x)g_1(x)=0$, $f_2(x)g_0(x)=0$, and have $g_2(x)f_0(x)=0$, $f_0(x)g_2(x)=0$. Similarly, $f_i(x)g_j(x)=0$ $(0\le i\le n, 0\le j\le m)$. Thus the skew polynomial ring $R[x;\sigma]$ of no zero divisor is an Armendariz ring.

Next research the necessary and sufficient of this condition by the following example.

Example 3.3. Let $R=\begin{pmatrix} Z_2 & Z_2 \\ 0 & Z_2 \end{pmatrix}$ be a ring with a monomorphism σ defined by $\sigma\left(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\right)=\begin{pmatrix} a & b \\ 0 & -c \end{pmatrix}$. For any $f(x)=\sum_{i=0}^{n} A_i x^i$, $g(x)=\sum_{j=0}^{m} B_j x^j \in R[x]$, define the usual addition and multiplication by $xA=\sigma(A)x$, then $R[x;\sigma]$ is a skew polynomial ring, but $R[x;\sigma]$ is not an Armendariz ring.

In fact, clearly, $R[x;\sigma]$ is a skew polynomial ring. Let $f(x)=\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}+\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}x$, $g\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}+\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}x \in R[x;\sigma]$, if $f(x)\cdot g(x)=0$, then all coefficients of the skew polynomial $R[x;\sigma]$ are zero. But $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\neq 0$, $\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}=\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}\neq 0$, thus $R[x;\sigma]$ is not an Armendariz ring.

It derives from the above example 3.3 that we further verify the condition is necessary. Next we study that skew polynomial ring $R[x;\sigma]$ is a (quasi)-Baer ring under the condition of no zero divisor .

Theorem 3.4. Let *R* be a ring that has no zero divisor and σ is an endomorphism over *R*, then skew polynomial ring $R[x;\sigma]$ is a (quasi)-Baer ring.

Proof. For any $f(x)=\sum_{i=0}^{n} a_i x^i$, $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x;\sigma]$, let $f(x)\cdot g(x)=0$. If $f(x)=0$, then $g(x)$ is any polynomial ring, and has $r_{R[x;\sigma]}(f(x))=R=1_{R[x;\sigma]}\cdot R$. If $f(x)\neq 0$, there exists $a_i\neq 0$, such that $0=f(x)\cdot g(x)=\sum_{i+j=0}^{n+m} a_i x^i b_j x^j=\sum_{i+j=0}^{n+m} a_i\sigma^i(b_j)x^{i+j}$, then $a_i\sigma^i(b_j)=0$. And because *R* is a ring has no zero divisor, so $a_i=0$ or $\sigma^i(b_j)=0$. Since arbitrary of a_i and *i*, we have $\sigma^i(b_j)=0$ implies $b_j=0\ (0\le j\le m)$, so $g(x)=0$. Thus the right annihilators set of any nonempty subset X is $r_{R[x;\sigma]}(X)=0=0\cdot R$. So $R[x;\sigma]$ is a Baer ring, and $R[x;\sigma]$ is a quasi-Baer ring by [5].

The following example shows that skew polynomial ring is (quasi)-Baer ring.

Example 3.5. Let $R=Z_2=\{0,1\}$ be a ring with an endomorphism σ defined by $\sigma(a)=-a$, then $R[x;\sigma]$ is a skew polynomial ring, and $R[x;\sigma]$ is a (quasi)-Baer ring.

In fact, clearly, *R* is a field, so *R* is a no zero divisor ring. Therefore the right annihilator of every nonempty subset $X\subset R$ is $r_{R[x;\sigma]}(X)=0=0\cdot R$, then the right ideal generated by the idempotent 0. Thus $R[x;\sigma]$ is a (quasi)-Baer ring clearly by theorem 3.4.

So we proof the condition of no zero divisor is necessary. The following counterexample shows that the condition is not sufficient condition that skew polynomial ring is a (quasi)-Baer ring.

Example 3.6. Suppose that $R=\begin{pmatrix} Z_4 & Z_4 \\ 0 & Z_4 \end{pmatrix}$ be a ring with an endomorphism σ defined by $\sigma\left(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\right)=\begin{pmatrix} a & -b \\ 0 & c \end{pmatrix}$ over *R*, for any $f(x)=\sum_{i=0}^{n} A_i x^i$, $g(x)=\sum_{j=0}^{m} B_j x^j \in R[x]$, define the usual addition and multiplication is defined by $xz=\sigma(z)x$, then $R[x;\sigma]$ is a skew polynomial ring, but is not a (quasi)-Baer ring.

In fact, clearly, $R[x;\sigma]$ is a skew polynomial ring. For any $f(x)=\sum_{i=0}^{n} A_i x^i \in R[x;\sigma]$, if $f^2(x)=f(x)$, then $A_0^2=A_0$, $A_0A_1+A_1\sigma(A_0)=A_1$, $\cdots$, $A_n\sigma^n(A_n)=0$ .We have all idempotents of $R[x;\sigma]$ are $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}+\begin{pmatrix} 0 & b_1 \\ 0 & 0 \end{pmatrix}x+\cdots+\begin{pmatrix} 0 & b_n \\ 0 & 0 \end{pmatrix}x^n$, $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}+\begin{pmatrix} 0 & b_1 \\ 0 & 0 \end{pmatrix}x+\cdots+\begin{pmatrix} 0 & b_n \\ 0 & 0 \end{pmatrix}x^n$. Let $f(x)=\begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}\in R[x;\sigma]$, since

$$r_{R[x;\sigma]}\left(f\left(x\right)\right)=\left\{\begin{pmatrix} a_0 & b_0 \\ 0 & c_0 \end{pmatrix}+\begin{pmatrix} a_1 & b_1 \\ 0 & c_1 \end{pmatrix}x+\cdots+\begin{pmatrix} a_n & b_n \\ 0 & c_n \end{pmatrix}x^n \middle| a_i,b_i,c_i\in Z_4\right\}\neq eR\left[x\right]$$

which $e^2=e$. Thus $R\left[x;\sigma\right]$ is not a (quasi)-Baer ring.

4. Conclusion

Inthis paper, we show that skew polynomial ring has no Morita invariance by the counterexample, and give the condition that skew polynomial ring constitutes Armendariz and (quasi)-Baer ring, and verify that the condition is necessary, but not sufficient.

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