

Probability Analysis for the Damage of Gravity Dam

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Abstract

Damage reliability analysis is an emerging field of structural engineering which is very significant in structures of great importance like arch dams, large concrete gravity dams etc. The research objective is to design and construct an improved method for damage reliability analysis for concrete gravity dam. Firstly, pseudo excitation method and Mazar damage model were used to analyze how to calculate damage expected value excited by random seismic loading and deterministic static load on the condition that initial elastic modulus was deterministic. Moreover, response surface method was improved from the aspects of the regression of sample points, the selection of experimental points, the determined method of weight matrix and the calculation method of checking point respectively. Then, the above method was used to analyze guarantee rate of damage expected value excited by random seismic loading and deterministic static load on the condition that initial elastic modulus was random. Finally, a test example was given to verify and analyze the convergence and stability of this method. Compared with other conventional algorithm, this method has some strong points: this algorithm has good convergence and stability and greatly enhances calculation efficiency and the storage efficiency. From what has been analyzed, we find that damage expected value is insensitive to the randomness of initial elastic modulus so we can neglect the randomness of initial elastic modulus in some extent when we calculate damage expected value.

Keywords: Gravity Dam, Damage, Probability Analysis, Pseudo Excitation Method, Mazar Damage Model, Response Surface Method

1. Introduction

The theory and methods of reliability analysis have been developed significantly during the last twenty years and have been documented in an increasing number of publications. These improvements in structure reliability theory and the attainment of more accurate quantification of the uncertainties associated with structural loads and resistances have stimulated the interest in the structure reliability analysis. Although from a theoretical point of view the field has reached a stage where the developed methodologies are becoming widespread, the quantitative assessment and classification of the reliability is still a complex and difficult task. In order to assess the reliability, a rigorous series of tests has to be carried out.

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As early as 1986, Ross B. Corotis [1] developed the analysis of effects of parameter uncertainty on the response of vibratory systems to random excitation. Benaroya H, Rehak M [2] and Spanos PD, Ghanem RG [3] extended the stochastic finite element method (SFEM) to incorporate the uncertainties in structural parameters. Leger [4] presented guidelines for dam-safety assessment based on the gravity method. Compared with the rigid-body limiting equilibrium method, the FEM used in the calculation of deep anti-sliding measures did not require the assumption of any slide plane. Wang Fei-Yue and Xu Zhi-Sheng [5] studied the stability of tailing dams. Their studies showed that both the fuzziness and the randomness of dam failure need to be considered. A comprehensive review of studies on fuzzy reliability was presented by Ross Reinhard Viert [6], who combined fuzzy set theory and reliability research to evaluate the risk in civil engineers. Motivated by the development of various re-

liability theories, Enrique Castillo and Carmen Castillo [7] gave the main results that allowed a sensitivity analysis to be performed in a general optimization problem, including sensitivities of the objective function, the primal and the dual variables with respect to data. Using a numerical procedure, Junho Song and Won-Hee Kang [8] proposed matrix-based system reliability (MSR) method to compute the probabilities of general system events efficiently by simple matrix operations. Kwai-Sang Chin and Ying-Ming Wang [9] used failure mode and effects analysis (FMEA) to evaluate a system for possible ways in which failures could occur.

The basic purpose of structural reliability analysis is to obtain the probabilistic responses of structural systems with uncertain design parameters, such as loadings, material parameters (strength, elastic modulus, Poisson's ratio, etc.), and shape dimensions. Among the methods available for such problems, the response surface method is a powerful tool [10]. The response surface method, was originally proposed by Box and Wilson [11] as a statistical tool, to find the operating conditions of a chemical process at which some response was optimized. Subsequent generalizations developed this method. Khuri, Cornell [12] and Myers, Montgomery [13] all introduced the response surface method in their books. Wong [14,15] and Faravelli [16,17] and Jiang [18] improved the method to fit the indeterminate coefficients of response surface. Bucher [19] and Rajashekhar [20] researched the convergence and stability of the response surface method. Guan [21] evaluated the effect of response surface parameter variation on structural reliability. Gupta [22] used the response surface method to study the extremes of Von Mises stress in nonlinear structures under Gaussian excitations.

However, until now, most of reliability methods such as the first order reliability method (FORM) [23], the second-order reliability method (SORM) [24-27], weighted regression method (WRM) and space reduced weighted regression method (SRWRM) [28] can not be used to analyze large structure, because the traditional reliability methods have two aspects of deficiencies: On the one hand, limited state function is usually implicit when we use finite element method (FEM) to analyze structure. It leads to that the implicit limited state function' partial derivatives for basic random variables are difficult to obtain. On the other hand, in order to overcome the above defects, some reliability methods use polynomial response surface function to fit implicit limited state function, but the number of basic random variables is very big when we analyze large structure. And these reliability methods need more experimental points to confirm the indeterminate coefficients of these basic random

variables. It leads to that, during the process, the calculation efficiency and the storage efficiency of these methods are very low. Even, in some large structure, it is impossible to obtain such many experimental points. Therefore, most of reliability methods only can be used to analyze small structures but inability to large structure.

Furthermore, when traditional methods analyze the damage of gravity dam, the seismic load is considered to deterministic acceleration time course. But, in fact, the acceleration time courses of seismic load are different although they have the same power spectrum density. So the traditional method ignores the randomness of seismic load. In addition, most of traditional probability analysis methods only consider the randomness of load but not consider the randomness of parameters of model at the same time. So these methods for gravity dam probability analysis are not complete.

In this paper, we analyze not only the randomness of load but also the randomness of parameters of gravity dam model for the influence of tension damage factor of elements in gravity dam model. The research route of this paper is as follow: to begin with, based on pseudo excitation method and Mazar damage model, we calculate damage expected value excited by random seismic loading and deterministic static load on the condition that initial elastic modulus is deterministic. In addition, we establish the improved response surface method based on weighted regression which can be used to analyze large structure. And then we use this method to analyze guarantee rate of damage expected value excited by random seismic loading and deterministic static load on the condition that initial elastic modulus is random. Finally, we give a test example to verify and analyze the convergence and stability of this paper's method.

2. Probabilistic Approach to Evaluate Gravity Dam Damage Excited by Random Seismic Load and Deterministic Static Load under the Condition of Deterministic Initial Elastic Modulus

The seismic load is considered to deterministic acceleration time course when we analyze the damage of gravity dam by the conventional method. However, in fact, the acceleration time courses of seismic load are different although they have the same power spectrum density. So the conventional method ignores the randomness of seismic load. In the method of this paper, we analyze the randomness of seismic load directly from power spectrum density.

Firstly, we analyze the element strains of dam model excited by deterministic static load. These element strains

are considered to the expected value $E(\varepsilon)$ of element strains.

Then, we begin to analyze square variance $D(\varepsilon)$ of element strains.

According to the vibration equation of gravity dam multi-degree-of-freedom system, we have

$$M\ddot{V} + C\dot{V} + KV = F(t) \tag{1}$$

where \ddot{V} , \dot{V} and V are acceleration, velocity and displacement of nodes in dam model, respectively; K , C and M are stiffness matrix, damping matrix and mass matrix of dam model, respectively.

By mode-superposition method, set seismic load is non-stationary process $F(t)$ as

$$F(t) = J(t)F_s(t) \tag{2}$$

where $F_s(t)$ and $J(t)$ are stationary process and time envelope curve, respectively.

According to pseudo excitation method, construct virtual force as

$$\tilde{F}(t) = J(t)\sqrt{S_f(\omega)}e^{i\omega t} \tag{3}$$

where $\tilde{F}(t)$ and $S_f(\omega)$ are virtual force and power spectrum density of stationary process $F_s(t)$, respectively.

Substituting (3) into (1) and we have

$$\tilde{V}_z = \sum_{j=1}^n \{\phi\}_{zj} H_j(\omega) \gamma_j J(t) \sqrt{S_f(\omega)} e^{i\omega t} \tag{4}$$

where \tilde{V}_z and $\{\phi\}_{zj}$ are fictive motion response and j vibration mode value of node z , respectively;

$H_j(\omega)$ and γ_j are frequency response function and mode shape participation coefficient, respectively.

And we can obtain $H_j(\omega)$ from (1).

Based on random vibration theory, we have

$$S_{V_z}(\omega) = \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\{\phi\}_{zi} \{\phi\}_{zj} H_i(\omega) H_j(\omega)}{\gamma_i \gamma_j J^2(\omega) S_f(\omega)} \right) \tag{5}$$

$$= \tilde{V}_z \times \tilde{V}_z^* = |\tilde{V}_z|^2$$

where $S_{V_z}(\omega)$ is power spectrum density of displacement response of node z

Through (5), we have

$$D(V_z) = \int_0^{+\infty} S_{V_z}(\omega) d\omega \tag{6}$$

Give the strain of element q as

$$[\varepsilon_q] = [B_{ql}][V_l] \tag{7}$$

where $[B_{ql}]$ and $[V_l]$ are strain matrix of element q and displacement vector of nodes in element q .

So we can write the square variance $D(\varepsilon)$ of element strains as

$$[D(\varepsilon_q)] = [B_{ql}^2][D(V_l)] \tag{8}$$

Through assuming element strains obey normal distribution, we can obtain probability distribution of element strains.

Regard Mazar damage model which is only related to strain as damage constitutive equation of concrete and write the tension damage factor of element as

$$\begin{cases} \Omega_T = 0 & \varepsilon \leq \varepsilon_f \\ \Omega_T = 1 - \frac{\varepsilon_f(1-A_T)}{\varepsilon} - \frac{A_T}{\exp(B_T(\varepsilon - \varepsilon_f))} & \varepsilon > \varepsilon_f \end{cases} \tag{9}$$

where ε_f is damage threshold and $0.7 \leq A_T \leq 1$, $10^4 \leq B_T \leq 10^5$, $0.5 \times 10^{-4} \leq \varepsilon_f \leq 1.5 \times 10^{-4}$.

In which, we only consider element strain at x direction and give the tension damage expected value through probability distribution of element strains and the tension damage factor of element as

$$E(\Omega_T) = \int_{-\infty}^{+\infty} \Omega_T(\varepsilon) N(E(\varepsilon), D(\varepsilon)) d\varepsilon$$

$$= \int_{-\infty}^{+\infty} \Omega_T(\varepsilon) \frac{1}{\sqrt{2\pi D(\varepsilon)}} \exp\left(-\frac{(\varepsilon - E(\varepsilon))^2}{2D(\varepsilon)}\right) d\varepsilon \tag{10}$$

where $E(\Omega_T)$ and $N(\cdot, \cdot)$ are tension damage expected value and normal distribution function, respectively.

Substituting (10) into (11) and we have

$$M\ddot{V} + C\dot{V} + (I - E(\Omega_T))KV = F(t) \tag{11}$$

where $E(\Omega_T)$ and I are tension damage expected value matrix and unit matrix, respectively.

We repeat the above steps until reaching a certain convergence criteria. Then we can obtain the iterative process of expected value $E(\varepsilon)$ and square variance $D(\varepsilon)$ of element strains. And finally we can obtain tension damage expected value matrix $E(\Omega_T)$.

3. Probabilistic Approach to Evaluate Gravity Dam Damage Excited by Random Seismic Load and Deterministic Static Load under the Condition of Random Initial Elastic Modulus

We have analyzed the randomness of load. Then we consider the randomness of initial elastic modulus. In this part, we improve response surface method based on weighted regression and make this method can be used in large structure such as gravity dam. It shows that the algorithm has good convergence and stability and greatly enhances calculation efficiency and the storage efficiency compared with other conventional algorithm.

3.1. Implicit Limited State Function $g(\mathbf{x})$ Setting

We write implicit limited state function $g(\mathbf{x})$ as

$$g(\mathbf{x}) = \min_x \left(\mathbf{E}(\Omega_T) \Big|_{E(x)} - \mathbf{E}(\Omega_T) \Big|_x \right) \quad (12)$$

where $\mathbf{E}(\Omega_T) \Big|_{E(x)}$ and $\mathbf{E}(\Omega_T) \Big|_x$ are tension damage expected value matrix under the condition of deterministic and random initial elastic modulus, respectively. And $\min_x(\cdot)$ is the minimum value of matrix elements.

\mathbf{x} is basic random variables vector (in this paper, the basic random variables are random initial elastic modulus of elements of gravity dam model)

3.2. The Establishment of Improved Response Surface Method Based on Weighted Regression

Use second-order polynomial response surface function $\bar{g}(\mathbf{x})$ to fit implicit limited state function $g(\mathbf{x})$, we have

$$\bar{g}(\mathbf{x}) = b_0 + \sum_{j=1}^n b_j x_j + \sum_{j=1}^n c_j x_j^2 \quad (13)$$

where x_j and n are basic random variables and the number of basic random variables, respectively; b_0 , b_j and c_j are indeterminate coefficients.

However, the number of basic random variables is very big when we analyze large structure. It is impossible to obtain the indeterminate coefficients by the traditional response surface method because we can only obtain m sample points which can not reach the number $2n + 1$ to fit second-order polynomial response surface function $\bar{g}(\mathbf{x})$. Thus, we try to use second-order polynomial response surface function $\bar{g}(\mathbf{x})$ to best approximate implicit limited state function $g(\mathbf{x})$ by m sample points.

Select m ($m < 2n + 1$) experimental points \mathbf{x}_i ($i = 1, 2, \dots, m$), and calculate implicit limit state function value $g(\mathbf{x}_i)$ which corresponds to the experimental points $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{in})^T$, and then obtain the sample vector $\mathbf{y} = [g(\mathbf{x}_1), g(\mathbf{x}_2), \dots, g(\mathbf{x}_m)]^T$.

Set $\mathbf{b} = [b_0, b_1, \dots, b_n, c_1, \dots, c_n]^T$ as the solution vector which is to be determined, and use m experimental points \mathbf{x}_i to compose experimental matrix \mathbf{A} as

$$\mathbf{A} = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1n} & x_{11}^2 & x_{12}^2 & \dots & x_{1n}^2 \\ 1 & x_{21} & x_{22} & \dots & x_{2n} & x_{21}^2 & x_{22}^2 & \dots & x_{2n}^2 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & x_{m1} & x_{m2} & \dots & x_{mn} & x_{m1}^2 & x_{m2}^2 & \dots & x_{mn}^2 \end{bmatrix} \quad (14)$$

By singular value decomposition of experimental matrix \mathbf{A} , we have

$$\mathbf{A} = \mathbf{U} \begin{bmatrix} \boldsymbol{\Sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{V}^H \quad (15)$$

where $\boldsymbol{\Sigma}$ is $m \times m$ diagonal matrix; \mathbf{U} and \mathbf{V} are m -order and $(2n + 1)$ -order unitary matrix, respectively.

Give the solution vector \mathbf{b} as

$$\mathbf{b} = \mathbf{V} \begin{bmatrix} (\boldsymbol{\Sigma})^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{U}^H \mathbf{y} \quad (16)$$

Set weight matrix \mathbf{M} is $m \times m$ diagonal matrix which gives m experimental points \mathbf{x}_i weight value.

$$\mathbf{M} = \begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_m \end{bmatrix}_{m \times m} \quad (17)$$

Rewrite the solution vector \mathbf{b} as

$$\mathbf{b} = \mathbf{V} \begin{bmatrix} (\mathbf{M}\boldsymbol{\Sigma})^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} (\mathbf{M}\mathbf{U})^H \mathbf{y} \quad (1)$$

3.3. The Establishment of Weight Matrix \mathbf{M}

We want to achieve two goals when using second-order polynomial response surface function $\bar{g}(\mathbf{x})$ to approximate implicit limited state function $g(\mathbf{x})$. The first goal is that we want the value of implicit limited state function $g(\mathbf{x})$ to be zero. The second goal is that we want to approximate implicit limited state function $g(\mathbf{x})$ around checking point \mathbf{x}_0 . Thus, we establish weight matrix \mathbf{M} as

$$g_{best}^1 = \min_{i=1}^m |g(\mathbf{x}_i)| \quad (19)$$

$$g_{best}^2 = \max_{i=1}^m \|\mathbf{x}_i - \mathbf{x}_1\|_2 \quad (20)$$

$$w_i = \alpha \frac{g_{best}^1}{|g(\mathbf{x}_i)|} + \beta \frac{g_{best}^2 - \|\mathbf{x}_i - \mathbf{x}_1\|_2}{g_{best}^2} \quad (\alpha + \beta = 1) \quad (21)$$

$$\mathbf{M} = \text{diag}(w_i) \quad (22)$$

where \mathbf{x}_1 is center point.

3.4. The m Experimental Points \mathbf{x}_i Selection in the Initial Iterative Step

In the initial iterative step, we select m experimental points \mathbf{x}_i as

$$\mathbf{x}_1^0 = (u_1, u_2, \dots, u_n)^T \tag{23}$$

$$\mathbf{x}_i^0 = (u_1 + r_{i1}\sigma_1, u_2 + r_{i2}\sigma_2, \dots, u_j + r_{ij}\sigma_j, \dots, u_n + r_{in}\sigma_n)^T$$

$$i = 2, \dots, m \quad j = 1, \dots, n \tag{24}$$

where u_j and σ_j are expected value and mean square deviation of basic random variables x_j , respectively; r_{ij} is random number in the interval $[-v, v]$ where v is deviation factor.

3.5. Deviation Factor Adjusting

Deviation factor has an influence on convergence speed in iterative procedure. Thus, in order to improve convergence speed, we adjust deviation factor in each iterative step as

$$v^k = v^{k-1} \frac{\|\mathbf{x}_1^k - \mathbf{x}_0^k\|_2}{\|\mathbf{x}_1^{k-1} - \mathbf{x}_0^{k-1}\|_2} \tag{25}$$

where k is the iterative step number.

3.6. Calculation Method of Checking Point \mathbf{x}_0

Derivation calculus to second-order polynomial response surface function $\bar{g}(\mathbf{x})$ is complex. Thus, we adopt improved method based on Lagrange multiplier rule as follow.

We can obtain the design checking point \mathbf{x}_0 through solving the constrained optimization problem (26) as

$$\begin{cases} \min \beta = f(\mathbf{x}_0) \\ = \sqrt{\left(\frac{x_{01} - u_1}{\sigma_1}\right)^2 + \left(\frac{x_{02} - u_2}{\sigma_2}\right)^2 + \dots + \left(\frac{x_{0n} - u_n}{\sigma_n}\right)^2} \\ \text{s.t. } g(\mathbf{x}_0) = 0 \end{cases} \tag{26}$$

where β is reliability index.

Substitute (13) into the constrained optimization problem (26) and rewrite constrained optimization problem (27) as

$$\begin{cases} \min f(\mathbf{x}_0) \\ = \left(\frac{x_{01} - u_1}{\sigma_1}\right)^2 + \left(\frac{x_{02} - u_2}{\sigma_2}\right)^2 + \dots + \left(\frac{x_{0n} - u_n}{\sigma_n}\right)^2 \\ \text{s.t. } \bar{g}(\mathbf{x}_0) = b_0 + \sum_{j=1}^n b_j x_{0j} + \sum_{j=1}^n c_j x_{0j}^2 = 0 \end{cases} \tag{27}$$

Based on Lagrange multiplier rule, we can rewrite the constrained optimization problem (27) as

$$\begin{cases} f_{x_{01}}(\mathbf{x}_0) + \lambda \bar{g}_{x_{01}}(\mathbf{x}_0) = 0 \\ \dots \\ f_{x_{0n}}(\mathbf{x}_0) + \lambda \bar{g}_{x_{0n}}(\mathbf{x}_0) = 0 \\ \bar{g}(\mathbf{x}_0) = 0 \end{cases} \tag{28}$$

Unfold (28) as

$$\begin{cases} \left[\begin{array}{cccc} \frac{2}{\sigma_1} + 2c_1\lambda & & & \\ & \frac{2}{\sigma_2} + 2c_2\lambda & & \\ & & \dots & \\ & & & \frac{2}{\sigma_n} + 2c_n\lambda \end{array} \right] \begin{bmatrix} x_{01} \\ x_{02} \\ \vdots \\ x_{0n} \end{bmatrix} \\ = \begin{bmatrix} -\lambda b_1 + 2\sigma_1 u_1 \\ -\lambda b_2 + 2\sigma_2 u_2 \\ \vdots \\ -\lambda b_n + 2\sigma_n u_n \end{bmatrix} \\ \bar{g}(\mathbf{x}_0) = 0 \end{cases} \tag{29}$$

Solve (29) as

$$\begin{cases} x_{0,j} = \frac{1}{\frac{2}{\lambda\sigma_j} + 2c_j} \left(-b_j + \frac{2\sigma_j u_j}{\lambda} \right) \\ j = 1, 2, \dots, n \end{cases} \tag{30}$$

From (29) and (30), we find the second-order polynomial $\bar{g}(\mathbf{x}_0)$ is the single-variable λ function. Thus we have

$$\bar{g}(\mathbf{x}_0) = \bar{g}(\lambda) = 0 \tag{31}$$

By Binary Search, we can solve (31) as follow:

First step: Taking two values λ_1, λ_2 to fit the conditions $\bar{g}(\lambda_1) \times \bar{g}(\lambda_2) < 0$, and making $\lambda = \frac{\lambda_1 + \lambda_2}{2}$.

Second step: When $\bar{g}(\lambda_1) \times \bar{g}(\lambda) < 0$, making $\lambda_2 = \lambda$ and $\lambda = \frac{\lambda_1 + \lambda_2}{2}$.

Third step: When $\bar{g}(\lambda_2) \times \bar{g}(\lambda) < 0$, making $\lambda_1 = \lambda$ and $\lambda = \frac{\lambda_1 + \lambda_2}{2}$.

Through above iterative process, we can obtain the value of the variable λ . Substituting λ into (30), we can obtain the value of the design checking point \mathbf{x}_0 .

3.7. The Basic Steps of Improved Response Surface Method Based on Weighted Regression

The basic steps of improved response surface method

based on weighted regression are as follow:

First step: in k iterative step, we obtain m experimental points \mathbf{x}_i through (23), (24). We calculate implicit limit state function value $g(\mathbf{x}_i)$ which corresponds to the experimental points $\mathbf{x}_i = (x_{i1}, x_{i2}, \dots, x_{in})^T$, and then obtain the sample vector $\mathbf{y} = [g(\mathbf{x}_1), g(\mathbf{x}_2), \dots, g(\mathbf{x}_m)]^T$. We obtain weight matrix \mathbf{M} by (19-22), and then obtain solution vector \mathbf{b} by (1).

Second step: We obtain checking point \mathbf{x}_0^k by improved method based on Lagrange multiplier rule, and then we calculate center point \mathbf{x}_1^{k+1} at next iterative step as

$$\mathbf{x}_1^{k+1} = \mathbf{x}_1^k + (\mathbf{x}_0^k - \mathbf{x}_1^k) \frac{g(\mathbf{x}_0^k)}{g(\mathbf{x}_0^k) - g(\mathbf{x}_1^k)} \quad (32)$$

Third step: We obtain deviation factor v^{k+1} by (25) and obtain m experimental points \mathbf{x}_i through (23), (24). If reliability index $\|\beta_k - \beta_{k-1}\| \leq \varepsilon$, we stop iterative procedure. If reliability index $\|\beta_k - \beta_{k-1}\| > \varepsilon$, we return the first step.

3.8. Numerical Example

We give a numerical example in order to verify and analyze the convergence and stability of this method.

Set implicit limited state function $g(x, y) = \exp(0.2x + 6.2) - \exp(0.47y + 5.0)$ where basic random variables x and y obey standard normal dis-

tribution and $\alpha = 0.7, \beta = 0.3$. We use 2 experimental points in this paper's method compared with 3-5 experimental points in other conventional algorithm. We obtain the comparison result when using the same initial deviation factor v^0 as **Table 1** [19]. We obtain the iterative process when initial deviation factor $v^0 = 3.00$ as **Table 2**.

From **Table 1** and **Table 2**, we can obtain the conclusion that this algorithm has good convergence and stability and greatly enhances calculation efficiency and the storage efficiency compared with other conventional algorithm.

4. Numerical Analysis of Gravity Dam Model

The gravity dam is 160 m high. The normal pool level (NPL) is 155 m deep. The level of back of dam is 10 m deep. The elevation of upstream and downstream broken-line sloping surface relative to foundation plane are 80 m and 140 m respectively. The concrete strength of gravity dam is C20. The finite element model of the gravity dam is divided into 2432 elements. The model consisted of 8-node iso parametric plane elements for the dam and foundation. The density of dam is 2450 kg/m³, and Poisson ratio $\lambda = 0.18$. The initial elastic modulus of dam $E = 3.50$ Gpa. The density of rock foundation is 2700 kg/m³, and Poisson ratio is 0.25. The initial elastic modulus of rock foundation $E = 4.00$ Gpa. And parame-

Table 1. Final result of example.

Method	Initial deviation factor v^0	The iterative step number k	Reliability index β	Relative error of reliability index (%)
FOSM			2.3493	0
	2	4	2.0944	10.85
TLM	3	6	1.8421	21.59
	10	60	0.3939	83.23
WRM	2	4	2.3494	0.00
	3	6	2.3508	0.06
	10	8	2.4279	3.35
SRWRM	2	4	2.3496	0.01
	3	6	2.3504	0.05
	10	5	2.4270	3.31
The method of this paper	2	4	2.3492	0.00
	3	6	2.3502	0.04
	10	8	2.3557	0.27

Table 2. Iterative procedure of example (initial deviation factor = 3).

Method	Deviation factor v^k	The iterative step number k	Reliability index β	Relative error of reliability index (%)
The method of this paper	1.8371E-01	1	2.9446	25.33946
	3.3100E-02	2	2.3664	0.72788
	1.8370E-03	3	2.3504	0.04682
	2.5969E-04	4	2.3503	0.04257
	2.6841E-05	5	2.3503	0.04257
	8.2334E-06	6	2.3502	0.03831
	8.2334E-06	7	2.3499	0.02554
	8.2334E-06	8	2.3493	0.00000

ters $\alpha = 0.9, \beta = 0.1$.

Deterministic static load includes gravity load and hydro static and uplift pressure while random load includes seismic load. The power spectrum density of seismic load is given as

$$\left\{ \begin{aligned} S_f(\omega_k) &= \frac{2\xi}{\pi\omega_k} [S_a^T(\omega_k)]^2 \frac{1}{-2 \ln\left(-\frac{\pi}{\omega_k T_d} \ln p\right)} \\ \Delta\omega &= 2\pi/T_d \\ \omega_k &= \Delta\omega k \quad k = 1, 2, 3, \dots, N \\ N &= \frac{T_d}{\Delta t} \end{aligned} \right. \quad (33)$$

where $S_a^T(\omega_k)$ and ξ are target response spectrum and damping ratio, respectively; p ($p \leq 0.15$) and T_d are exceeding response spectrum probability and duration of ground motion, respectively; N and Δt are the number of trigonometric series and time step, respectively.

The dam model is shown in **Figure 1**. The probability distributions of all random parameters of each element are shown in **Table 3**. And the expected values of tension damage factors of each element are shown in **Figure 2**. The reliability index, guarantee rate about damage expected value and deviation factor under the condition of random initial elastic modulus iterative procedure are shown in **Figure 3**, **Figure 4** and **Figure 5**. The values of **Figure 3**, **Figure 4** and **Figure 5** are shown in **Table 4**.

In each iterative step of the method of this paper, we use only 10 experimental points to approximate implicit limited state function $g(x)$ while the traditional response surface method needs 4865 experimental points. So the method of this paper saves large storage space and can be accepted in analyzing large structure such as gravity dam.

From what has been analyzed above, we have the conclusion that the damage locations of gravity dam are jetty

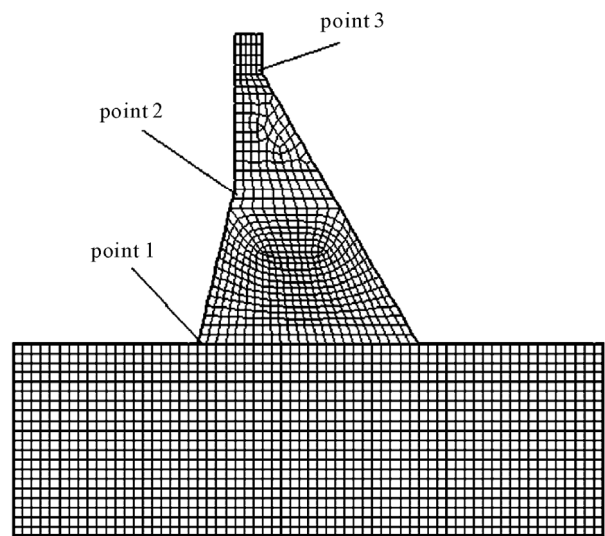


Figure 1. Gravity dam model.

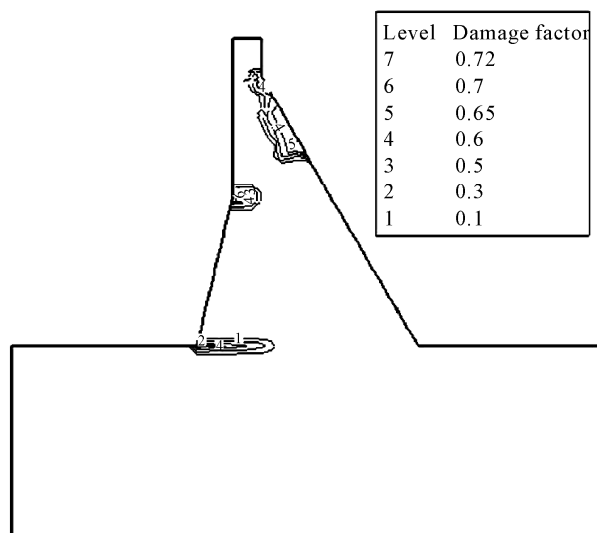


Figure 2. Damage contour map of gravity dam model.

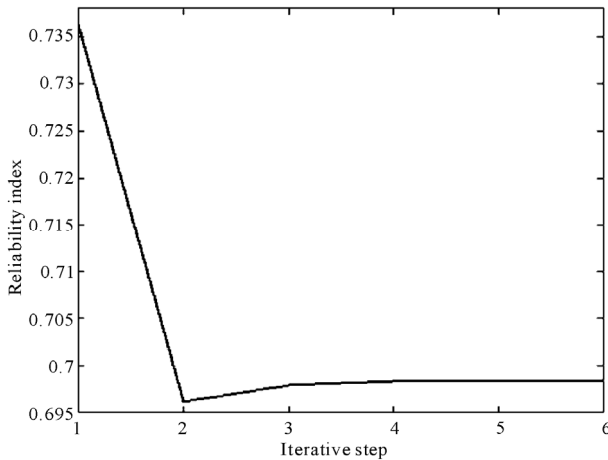


Figure 3. Reliability index iterative procedure.

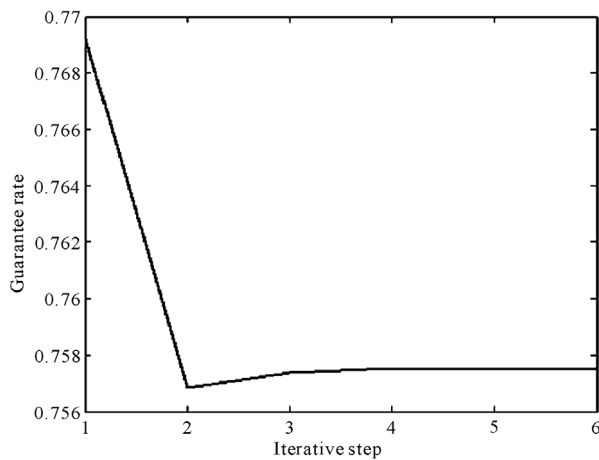


Figure 4. Guarantee rate iterative procedure about damage expected value.

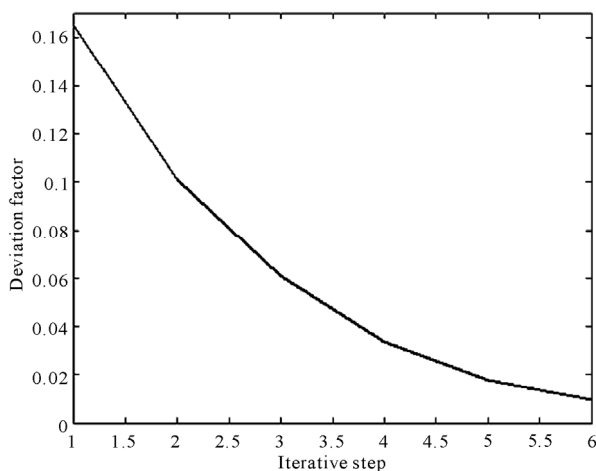


Figure 5. Deviation factor iterative procedure.

head and the heel of gravity dam so we should strengthen these locations above. From **Figure 3**, we have the con-

Table 3. The probability distribution of all random parameters of each element.

Elastic modulus of rock foundation		
probability distribution	expected value (pa)	coefficient of variation
normal distribution	4.00E+10	0.1
Elastic modulus of dam		
probability distribution	expected value (pa)	coefficient of variation
normal distribution	3.50E+10	0.1

clusion that, under the condition of deterministic initial elastic modulus, the convergence rates of the expected values of tension damage factors of each element are fast. They generally turn to be stable at third iterative step. It shows the method of calculating expected values of tension damage factors has good convergence and stability. From **Figure 4**, **Figure 5** and **Table 4**, we have the conclusion that, under the condition of random initial elastic modulus, the convergence rates are also fast. The deviation factor decreases exponentially which shows that the improved response surface method based on weighted regression also has good convergence and stability and greatly enhances calculation efficiency and the storage efficiency. And the method of analyzing large structure such as gravity dam is very applicable. The deviation factor reaches 0.16495 at the first iterative step. It shows that damage expected value is insensitive to the randomness of initial elastic modulus so we can neglect the randomness of initial elastic modulus in some extent when we calculate damage expected value. And we can obtain the guarantee rate about damage expected value is 75.755%.

5. Conclusions

In this paper, we analyze the probability of gravity dam damage. To begin with, based on pseudo excitation method and Mazar damage model, we calculate damage expected value excited by random seismic loading and deterministic static load on the condition that initial elastic modulus is deterministic. Furthermore, we establish the improved response surface method based on weighted regression to analyze guarantee rate of damage expected value excited by random seismic loading and deterministic static load on the condition that initial elastic modulus is random. At last, we give a test example to verify and analyze the convergence and stability of this method. And it shows this algorithm has good convergence and stability and greatly enhances calculation efficiency and the storage efficiency. Through analysis of

Table 4. The iterative procedure of improved response surface method based on weighted regression.

Method	Deviation factor v^k	The iterative step number k	Reliability index β	Guarantee rate about damage expected value $P = \phi(\beta)$
The method of this paper	0.16495	1	0.73619	0.76919
	0.10102	2	0.69619	0.75684
	0.06065	3	0.69786	0.75737
	0.03349	4	0.69835	0.75752
	0.01766	5	0.69832	0.75751
	0.00975	6	0.69845	0.75755

examples, we find that damage expected value is insensitive to the randomness of initial elastic modulus so we can neglect the randomness of initial elastic modulus in some extent when we calculate damage expected value.

6. References

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