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Load Static Models for Conservation Voltage Reduction in the Presence of Harmonics

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Abstract

The Conservation Voltage Reduction (CVR) is a technique that aims to achieve the decrease of power consumption as a result of voltage reduction. The customer is supplied with the lowest possible voltage level compatible with the stipulated level by the regulatory agency. International Standards ANSI C84.1-2006 and IEEE std 1250-1995 specify the range of supply voltage to electronics equipment from 0.9 to 1.05 pu of nominal voltage. To analyse the CVR effect in distribution systems with different load characteristics (residential, commercial, industrial or a combination of these), mathematical load models are used. Typically, these equipment/load models are used to analyse load aggregation without any consideration of its nonlinearity characteristics. Aiming to analyse the nonlinear characteristics and its consequences, this paper presents a discussion of the neglected variables as well as the results of a set of measurements of nonlinear loads. Different mathematical models are applied to obtain them for each load. Using these models the load aggregation is evaluated. It is presented that although the models show adequate results for individual loads, the same does not occur for aggregated models if the harmonic contribution is not considered. Consequently, to apply the load model in CVR it is necessary to consider the harmonics presence and the model has to be done using only the fundamental frequency data. The discussion about the causes is done and the models are compared with the measurements.

Keywords

CVR Load Model in the Presence of Harmonics, Conservation Voltage Reduction, Load Model Aggregation in the Presence of Harmonics

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1. Introduction

Conservation Voltage Reduction (CVR) is a method of energy reduction consumption resulting from a reduction of feeder voltage [1]. According to Schneider [1] while in North America there are numerous CVR systems deployed in distribution feeders, the majority of the results published are based on empirical field measurements. Consequently, it is not possible to extrapolate the results of this technology on other distribution feeders. To obtain a better evaluation, it is necessary to use load models and analyse the combination of these models in simulation processes.

The load model for CVR analysis is a mathematical representation of the relationship between the active and reactive power consumed and load feed voltage; in other words, it describes the load behavior when it is connected on power grid considering the voltage variation. Such models can be done without thermal cycles or static [1] [2] where the power at each instant, *t*, is expressed as function of voltage and frequency at the same instant $P_L = f_P(V, f)$, $Q_L = f_Q(V, f)$; or dynamic, where the power is expressed as function of voltage and frequency both at past and present time [2].

Loads without thermal cycles consume energy in a time-invariant manner, with the exception of voltage variations. Specifically, there is no control feedback loop. As an example, a light bulb will consume energy when turned on, as a function of voltage in a fixed manner. In contrast, a load with a thermal cycle, such as a water heater, will have a varying duty cycle dependent on the supply voltage, Schneider [1].

The load model, according to Price *et al.* [2], may bring benefits in planning studies, in electrical power systems operation, resulting in, for example, reduction of system modifications and equipment investments, and system preventing emergencies due to better operating limits.

Some researchers found in literature demonstrate the importance of load model application on Conservation Voltage Reduction (CVR) [1]-[11]. In 1992, a research developed by Electric Power Research Institute (EPRI) [3] presented tests in order to verify the reduction voltage effects in the efficiency of loads existing in residential, commercial and industrial sectors. Such tests were divided in two parts. In the first part, data were collected to model energy consumption as a function of supply voltage. In the second part, the collected data were used to model the loads. In 1989, a field test was performed in Snohomish [4] in four circuits connected to three substations whose loads were predominantly residential and commercial, where there were alternating cycles with nominal voltage and low voltage every 24 hours. In each substation, power meters were installed to collect active power, reactive power and voltage in each phase of the test circuit, where the data were collected every 15 minutes. The test resulted in a reduction of energy consumption. However the result was not considered optimal because each substation at each instant of time had different loads connected. It is evident that the load model is needed for predicting energy saved as a result of applying the CVR method. In 2010, the study described in Schneider [1] presented the CVR application in some feeders, where the static loads were modeled by a polynomial model (ZIP), and the dynamic loads were modeled by an equivalent thermal parameter (ETP) model. The dynamic loads used a more complex model due to the control system in each load that determines the moment when it is energized and for how long.

None of those publications consider the nonlinear characteristics of the load. The contribution of the harmonics in the load behavior has to be considered in the analysis. This paper evaluates three model static load techniques as used in literature, without any consideration of nonlinearities and including their analyses. This paper is divided as following: Firstly three CVR model techniques were presented. These techniques can be applied considering or not the harmonics components presence. Then, in the second part the approach differences were shown. To obtain the model using the measured data, it was presented the parameter estimation technique. It is desired to maintain the physics aspects of the model; then the ellipsoid algorithm (using restrictions) and its application were presented. Finally the used methodology was shown as well as the results. The comparison has been done and the conclusions were shown.

2. End-User Load Modeling—Static Loads

End-user load models are mathematical functions used to describe the behavior of commercial or residential loads considering the active and reactive power as function of both variables: voltage and sometimes frequency. The static models used to represent the load are performed in two main types: by an exponential model and by polynomial models, also known as ZIP models [1] [2] [5] [7].

2.1. Exponential Model—EXP

In exponential models the relationship between the consumed power and the feed voltage is given by an exponential function, as in Equation (1) for the active power and in Equation (2) for the reactive power [2] [5] [10]-[12]:

$$P_i = P_0 \left(\frac{V_i}{V_o} \right)^{\alpha} \tag{1}$$

$$Q_i = Q_0 \left(\frac{V_i}{V_o} \right)^{\beta} \tag{2}$$

where α and β parameters describe the load behavior.

When both coefficients α and β are equal to zero, it means that the load behaves as Constant Power. In case of both α and β are equal to one, the load behaves as Constant Current; and finally if both coefficients are equal to two, the load behaves as Constant Impedance.

2.2. ZIP Model (ZIP)

In polynomial models, ZIP model, the load is represented by following some physical characteristics. Some equipment, such as electronic ballasts, have constant power consumption under voltage variation. On the other hand, others behave as a combination of two or three of the following characteristics: Constant Impedance (*Z*), Constant Current (*I*) and Constant Power (*P*).

The expressions for active and reactive power (P_i, Q_i) of the ZIP model are presented by Equations (3) and (4) respectively. For both Equations, the coefficients have to obey the constraints presented in Equation (5). The relationship between power consumption and voltage magnitude is given by a polynomial Equation [2] [5] [7] [10] [11]:

$$P_i = P_0 \left(P_p + I_p \frac{V_i}{V_o} + Z_p \left(\frac{V_i}{V_o} \right)^2 \right) \tag{3}$$

$$Q_i = Q_0 \left(P_q + I_q \frac{V_i}{V_o} + Z_q \left(\frac{V_i}{V_o} \right)^2 \right) \tag{4}$$

$$\begin{cases} Z_p + I_p + P_p = 1 \\ Z_q + I_q + P_q = 1 \end{cases} \tag{5}$$

where:

P_i : Active power consumption of the i^{th} load.
Q_i : Reactive power consumption of the i^{th} load.
P_o : Active power consumption at nominal voltage.
Q_o : Reactive power consumption at nominal voltage.
V_i : Actual terminal voltage.
V_o : Nominal terminal voltage.
$Z_{p,q}$: Fraction of load that is Constant Impedance.
$I_{p,q}$: Fraction of load that is Constant Current.
$P_{p,q}$: Fraction of load that is Constant Power.

2.3. Modified ZIP Model—(ZIP_mod)

Schneider *et al.* [1] proposed a new ZIP model where, besides the information about the Constant Impedance, Constant Current and Constant Power, there is also information about the phase angle of each one of these components. In this case, both active and reactive Equations are coupled. The active and reactive power calculus is given in Equations (6) and (7) respectively, obeying the constraint in Equation (8):

$$P_i = \left(\frac{V_i}{V_o}\right)^2 S_n Z_{\%} \cos\left(Z_\theta\right) + \frac{V_i}{V_o} S_n I_{\%} \cos\left(I_\theta\right) + S_n P_{\%} \cos\left(P_\theta\right) \tag{6}$$

$$Q_i = \left(\frac{V_i}{V_o}\right)^2 S_n Z_{\%} \sin\left(Z_\theta\right) + \frac{V_i}{V_o} S_n I_{\%} \sin\left(I_\theta\right) + S_n P_{\%} \sin\left(P_\theta\right) \tag{7}$$

$$Z_{\%} + I_{\%} + P_{\%} = 1 \tag{8}$$

where:

P_i : Real power consumption of the i^{th} load.
Q_i : Reactive power consumption of the i^{th} load.
V_i : Actual terminal voltage.
V_o : Nominal terminal voltage.
S_n : Apparent power consumption at nominal voltage.
$Z_{\%}$: Fraction of load that is *Constant Impedance*.
$I_{\%}$: Fraction of load that is *Constant Current*.
$P_{\%}$: Fraction of load that is *Constant Power*.
Z_θ : Phase angle of the *Constant Impedance* component.
I_θ : Phase angle of the *Constant Current* component.
P_θ : Phase angle of the *Constant Power* component.

In any ZIP model, when the load is represented as Constant Impedance, it means that the power varies directly with the square of the voltage; in the case of the Constant Current, the power varies directly with the voltage; and, lastly, if the load is modeled as Constant Power, the power does not vary when the voltage varies [2].

3. Analysing the Harmonics Presence and Its Effect in the Model

None in all presented models is mentioned about harmonic components in any reference although its presence affects the data. Once measurements are used to obtain the parameters for each model, and as the components suffer influences from harmonics, it is necessary to discuss how to consider its influence before start using the data.

3.1. For Sinusoidal Voltage and Current

The apparent power measured by the equipment gives us two parcels called P and Q. For sinusoidal source supplying a linear load, these parcels are [13]:

$$S^2 = (VI)^2 = (V_1 I_1)^2 = P_1^2 + Q_1^2 = P_M^2 + Q_M^2 \tag{9}$$

where:

V: RMS voltage.
V_1: Fundamental voltage.
V_1: Fundamental current.
S: Apparent power.
P_1: Fundamental active power.
Q_1: Fundamental reactive power.
P_M: Measured active power.
Q_M: Measured reactive power.

3.2. For Nonsinusoidal Voltage and Current

Considering nonsinusoidal source supplying a nonlinear load, however, the apparent power assumes a different value including the harmonics effect. The measured active power P_M now has also the contribution of the harmonic active power P_H. Similarly the measured reactive power Q_M is not only composed by Q_1 but also by a sum of other parcels, namely D_V, D_I and D_H. This measured power should no longer be called reactive power, but nonactive power, [13]. The demonstration is presented below:

$$\begin{aligned} S^2 &= (VI)^2 = \left(V_1^2 + V_H^2\right)\left(I_1^2 + I_H^2\right) = \left(V_1 I_1\right)^2 + \left(V_1 I_H\right)^2 + \left(V_H I_1\right)^2 + \left(V_H I_H\right)^2 \\ S^2 &= P_1^2 + Q_1^2 + \left(V_1 I_H\right)^2 + \left(V_H I_1\right)^2 + P_H^2 + D_H^2 \\ S^2 &= P_1^2 + Q_1^2 + D_V^2 + D_I^2 + P_H^2 + D_H^2 \\ S^2 &= P^2 + N^2 \end{aligned} \tag{10}$$

consequently,

$$P_M = P = P_1 + P_H \tag{11}$$

$$Q_M^2 = N^2 = Q_1^2 + D_V^2 + D_I^2 + D_H^2 \tag{12}$$

where:

V_1: Fundamental voltage.
V_H: Harmonic voltage.
I_1: Fundamental current.
I_H: Harmonic current.
S: Apparent power.
P_1: Fundamental active power.
Q_1: Fundamental reactive power.
P_M: Measured active power.
Q_M: Measured reactive power.
D_V: Voltage distortion power.
D_I: Current distortion power.
P_H: Harmonic active power.
D_H: Harmonic distortion power.
N: Nonactive power.

4. Parameter Estimation

The parameter estimation is a procedure which uses some samples from measurements to calculate one or more unknown parameters. The used measurements (samples) in the estimation process are subject to errors, so that the estimated parameters also have associated errors [14]. Equation (13) shows the value of the samples obtained by a measurement device. The measured value is close to the true value, but with the difference of an error.

$$Z^{meas} = Z^{true} + \eta \tag{13}$$

where Z^{meas} is the value obtained by measurement equipment; Z^{true} is the true value of the measurement; and η is the random measurement error that serves to model the uncertainty in the measurements.

The statistical criteria used in the parameter estimation may be quoted the maximum likelihood, where the goal is to maximize the probability that the estimated state variable is the actual value of the state variable vector [14]. Such estimator assumes that the probability density function (PDF) is known. Other criteria may also be used, for example, the weighted least-squares criterion that aims to minimize the sum of the weighted squares of the estimated measure deviations. In this case, it is not required that the PDF of samples or measurement errors be known, but it is assumed to have Gaussian normal distribution [14].

In the maximum likelihood criterion, it is necessary to estimate the variable x that maximizes the likelihood of measuring Z^{meas} [14]. According to the author, the maximum likelihood estimate of the unknown parameter is thus expressed as the parameter value that minimizes the sum of squares of the difference between each measured value and the actual measured value, with each squared difference weighted by the variance of the meter error, as in (14):

$$\min_x J(x) = \sum_{i=1}^{N} \frac{\left[Z^{meas} - f_i(x)\right]^2}{\sigma_i^2} \tag{14}$$

where:

$f_i(x)$: function used to calculate the value being measured by the i^{th} measurement.
$J(x)$: measurement residual.
N: number of independent measurements.
σ_i^2: variance of the i^{th} measurement.
Z_i^{meas}: i^{th} measurement.
In one matrix approach, this Equation can be written as in (15):

$$\min_x J(x) = \left[Z^{meas} - f(x)\right]^T [R]^{-1} \left[Z^{meas} - f(x)\right] \quad (15)$$

where: R: covariance matrix of the measurements, as can be seen in (16).

$$[R] = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_{Nm}^2 \end{bmatrix} \quad (16)$$

5. Ellipsoid Algorithm

This algorithm consists in finding a solution to the optimization problem through a succession of increasingly smaller ellipsoids, starting from the initial ellipsoid containing the optimal point [15].

Given the constrained optimization problem described in (17) [12], the convergence is achieved if the objective function and the constraints are convex and if $x \in R^n$ [16]. The algorithm starts with an ellipsoid E_o centered at the x_o point as in (18):

$$x^* = \arg\min f(x)$$
$$\text{subject to} \begin{cases} q_j(x) \le 0, & j = 1, \cdots, r \\ h_l(x) \le 0, & l = 1, \cdots, s \\ -h_l(x) \le 0, & l = 1, \cdots, s \end{cases} \quad (17)$$

$$E_0 \triangleq \left\{ x \middle| (x - x_0)^T Q_0^{-1} (x - x_0) \le 1 \right\} \quad (18)$$

where:
$q(\cdot)$: depicts the constraints of inequality,
$h(\cdot)$: represents equality constraints described as two inequality constraints, and
$f(\cdot)$: objective function.

The ellipsoid algorithm can be described by recursive Equations (19), (20) and (21), which generate a sequence of x_k points [12].

$$x_{k+1} = x_k - \frac{\beta_1 Q_k m_k}{\left(m_k^T Q_k m_k\right)^{\frac{1}{2}}} \quad (19)$$

$$Q_{k+1} = Q_k - \frac{\beta_3 (Q_k m_k)(Q_k m_k)^T}{m_k Q_k m_k} \quad (20)$$

$$\beta_1 = \frac{1}{n+1},\ \beta_2 = \frac{n^2}{n^2+1},\ \beta_3 = \frac{2}{n+1} \quad (21)$$

where m is the gradient or sub-gradient of the function or of the most violated constraint, according to the rule described in (22) and (23) [12].

$$g_{\max} = \max g_i(x) \quad (22)$$

$$m(\cdot)=\begin{cases}\nabla g(x) & \text{if } g_{\max}\geq 0\\ \nabla f(x) & \text{if } g_{\max}<0\end{cases} \quad (23)$$

Figure 1 illustrates the flowchart of the ellipsoid algorithm. Firstly, the variables that define i. the region which is contained in the solution (Q) (initial ellipsoid), ii. initial optimal solution (x_{opt}) and its objective function value (f_{opt}), and iii. the algorithm parameters (for example the tolerance for calculate the gradient numerically (tol)) are initialized. After that, the variable receives the value of the largest constraint function at x. Then the objective function value at the point x is calculated. If r is negative, the algorithm calculates the objective function gradient at point x (f_x) and checks whether the value of f_k is smaller than the optimal value achieved so far. If f_k is smaller than f_{opt} (optimal function value), x_{opt} and f_{opt} receive the value of x_k and f_k, respectively, otherwise the algorithm goes to the next step. If r is bigger than or equal to zero, the algorithm calculates the gradient of the most violated constraint. In the next step, the algorithm computes the variables to generate the next ellipsoid based on the computed gradient vector. This process is repeated until the convergence is achieved.

6. Method to Obtain the Model

The data measurements were performed three times for each load and at different times. During the experiments, the voltage has been changed in steps. An energy analyzer was used to collect data. Every 5 minutes the input voltage was increased by five volts starting the measurement at 110 V and ending at 130 V. The nominal voltage is 127 V. Samples were collected every second. For each load, it was used at least 4500 measurements. These data were used in the following models.

For the exponential model, the optimization problem is given as in Equation (24) [6]:

$$\begin{aligned}&\min_x J(x)=\left[Z^{meas}-f(x)\right]^T[R]^{-1}\left[Z^{meas}-f(x)\right]\\&\text{subject to}\left\{\alpha\geq 0\right.\end{aligned} \quad (24)$$

where: $x=\left[\alpha \quad P_0\right]^T$ and $f(x)$ is the function described in (1). The same formulation is presented for the exponential reactive power model.

For ZIP model [2] the parameters are estimated according to the optimization problem presented in Equation (25) [6]:

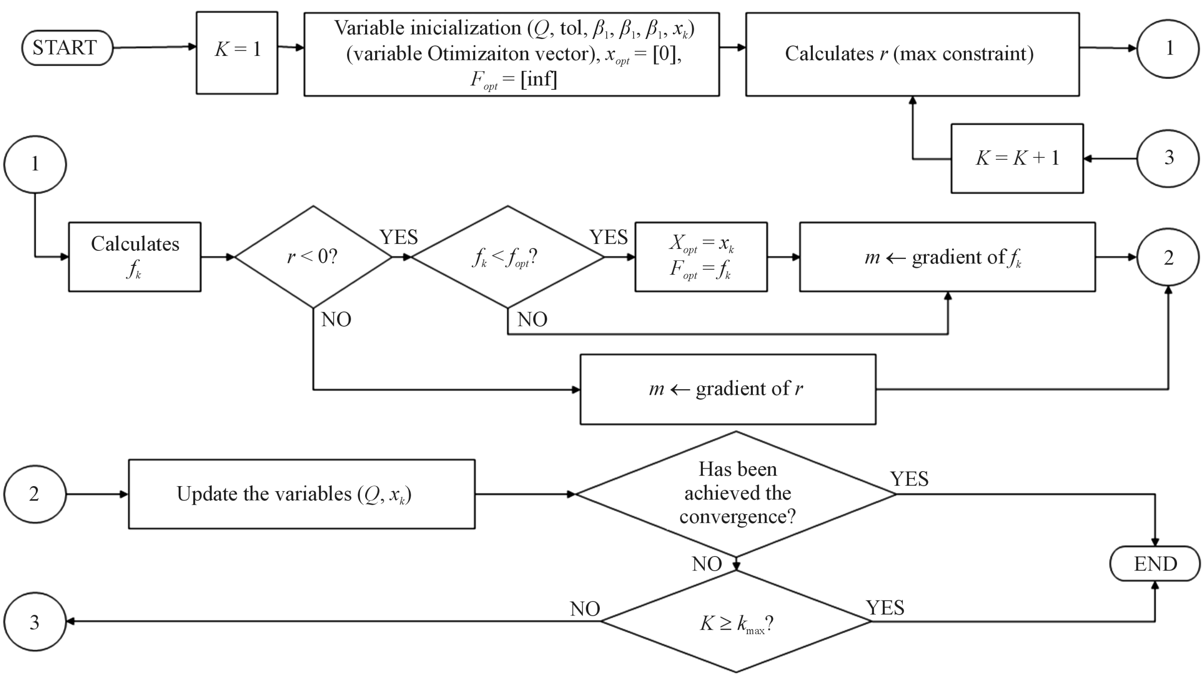


Figure 1. Flowchart of the ellipsoid algorithm.

$$\min_x J(x) = \left[Z^{meas} - f(x)\right]^T [R]^{-1} \left[Z^{meas} - f(x)\right]$$
$$\text{subject to} \begin{cases} Z_p + I_P + P_P = 1 \\ 0 \le Z_p \le 1 \\ 0 \le I_p \le 1 \\ 0 \le P_p \le 1 \end{cases} \quad (25)$$

where:

$$x = \begin{bmatrix} P_p & I_p & Z_p & P_0 \end{bmatrix}^T;$$

P_0 is the active power at nominal voltage and $f(x)$ is the function described in (3).

The same formulation is presented for the ZIP reactive power model [2].

For the ZIP_mod model [1], the estimated parameters must satisfy simultaneously the Equations of active and reactive power, so that the optimization problem is defined as in Equation (26):

$$\min_x \alpha J_1(x) + (1-\alpha) J_2(x)$$
$$\text{subject to} \begin{cases} P_{\%} + I_{\%} + Z_{\%} = 1 \\ 0 \le P_{\%} \le 1 \\ 0 \le I_{\%} \le 1 \\ 0 \le Z_{\%} \le 1 \\ -\pi \le P_\theta \le \pi \\ -\pi \le I_\theta \le \pi \\ -\pi \le Z_\theta \le \pi \\ S_n \ge 0 \end{cases} \quad (26)$$

where: $\alpha = 0,5$ in order to minimize both active and reactive power functions at the same intensity; $J_1(x)$ is the Equation concerning the active power with $f_1(x)$ given according to Equation (6); $J_2(x)$ is the Equation related to reactive power with $f_2(x)$ as in Equation (7); and S_n is the apparent power consumption at nominal voltage.

The model proposes to minimize the active and reactive power functions simultaneously through a weighted sum, that is, both power functions are minimized with the same weight. Often the objective functions (active and reactive power) have different magnitude order. Therefore, one function can be minimized faster than the other. To solve this problem, the optimal objective function was calculated for the active power (J_1^*) and reactive (J_2^*) separately using the constraints of Equation (26), and then, the optimal values obtained were used to calculate the objective function of the problem in Equation (27):

$$J = \lambda \frac{J_1}{J_1^*} + (1-\lambda) \frac{J_2}{J_2^*} \quad (27)$$

This paper uses the Ellipsoid optimization technique for parameter estimation obeying the model constraints described on Equations (24), (25) and (26).

7. Methodology

Knowing load mathematical models, it is possible to obtain a model of a group of loads by simply adding the single models, that is, the procedure used for linear loads (superposition theorem). In order to analyse the effect of the harmonics in the load models and in their aggregation, an experiment is carried out with six different loads, and the analysis is divided in two phases.

1) During the first phase, each equipment has been tested for different voltage values in the range of 0.9 and 1.05 pu. The measurements have been made repeatedly and the variables S, S_1, P, P_1, N, Q_1 have been registered. Using the two groups of variables (S_1, P_1, Q_1 and S, P, N) the mathematical models have been obtained by following the procedure previously mentioned. Consequently, two representations (i. only the fundamental fre-

quency components (S_1, P_1, Q_1) and ii. harmonics contributions (S, P, N)) have been calculated for each of the three models (Exponential, ZIP and ZIP_mod) and for each equipment.

2) In the second phase, six pieces of equipment were connected simultaneously and the measurements for the same variables shown in phase 1 have been collected. Six models have been adjusted, two representations for the two groups of variables (i. only the fundamental frequency components (S_1, P_1, Q_1) and ii. harmonics contributions (S, P, N)) for each method (Exponential, ZIP and ZIP_mod).

8. Results

Following the methodology, the experiments have been done and the results are presented in this section. **Table 1** contains the modeled loads and their rated power. Among the modeled loads, there are resistive loads, electronics and motors.

8.1. Phase 1—Models Obtained for Each Load

Figure 2(a) is presented to illustrate how the active powers P and P_1 varies when the voltage changes in steps. They are almost the same, as the active harmonic power P_H is very low. Otherwise **Figure 2(b)** shows a big difference between Q_M (that in fact is N) and Q_1. **Figure 3** presents similar results for Liquid Crystal Display measurements. In this case the difference between active powers P and P_1 is more evident. Using these data, the models, as a function of voltage, Exp, ZIP and ZIP_mod, are obtained.

Table 2 presents the parameters α and β for the exponential model expressed by Equation (7) and (8) for all modeled loads, considering the measured data (P, N) and the filtered data (P_1, Q_1). As the active power suffers little influence of harmonics, the calculated parameters (α, Po) are almost equal, even for models considering or not harmonics contribution. Otherwise, the nonactive power is strongly influenced by the presence of harmonics.

Table 1. Modeled loads.

Load	Abbreviation
Compact Fluorescent Light (16 W)	CFL
Incandescent Light Bulb (100 W)	Inc.
Venti-Delta Fan (45 W)	Fan
LG LCD—Liquid Crystal Display (30 W)	LCD
Personal Computer (100W)	PC
CCE Television (Cathod Ray Tube) (75 W - MAX)	TV

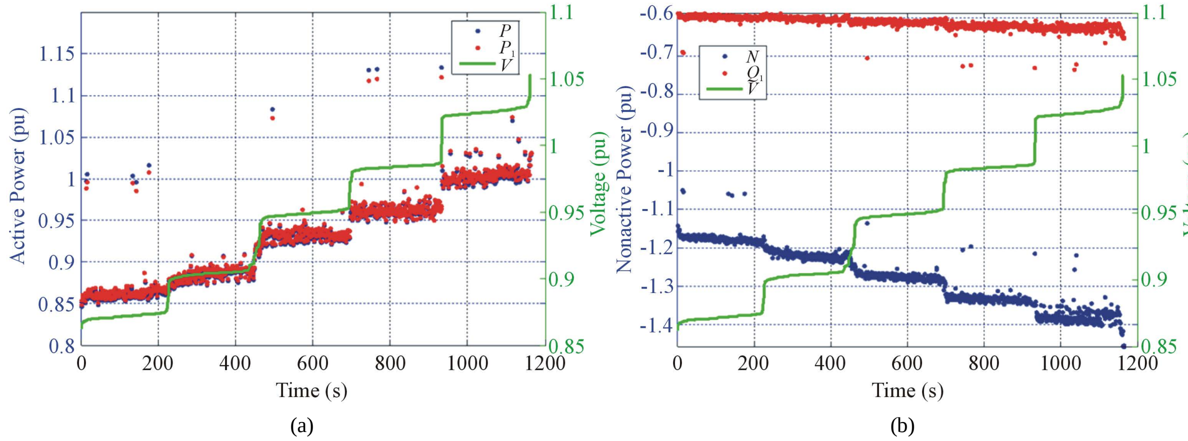
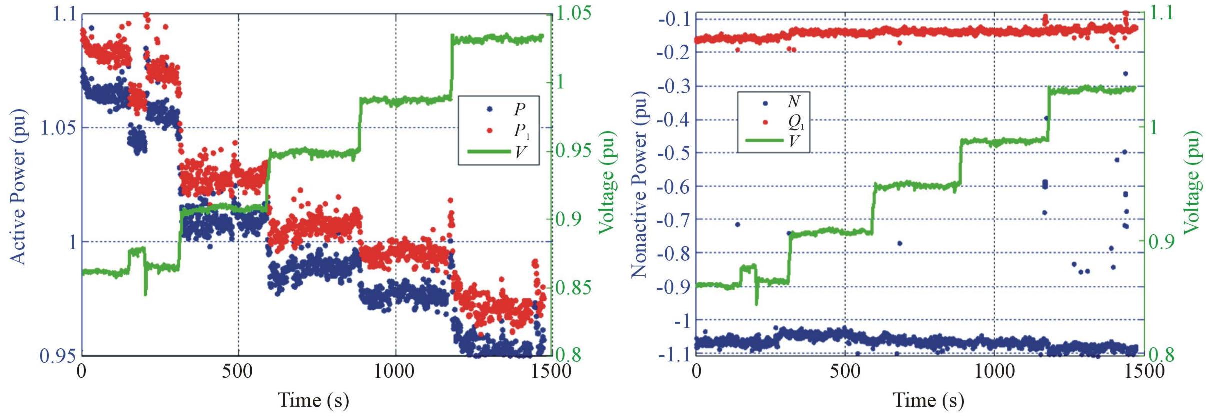


Figure 2. Compact fluorescent measurements. (a) Active power (P_1, P_m) and voltage variation; (b) Nonactive power (Q_1, N) and voltage variation.



(a) (b)

Figure 3. Liquid crystal display measurements. (a) Active power (P_1, P) and voltage variation; (b) Nonactive power (Q_1, N) and voltage variation.

Table 2. Exponential model parameters for active and nonactive power.

Load	Exponential model parameters (V_o = 127V)							
	Reference parameter P		Reference parameter P_1		Reference parameter N		Reference parameter Q_1	
	α	P_o	α	P_o	β	Q_o	β	Q_o
CFL	0.0100	15.64	0.0007	15.64	0.9957	−21.85	0.2684	−10.37
Inc	1.3424	102.24	1.3419	102.16	0.0000	0.00	0.0001	0.00
TV	0.0015	49.45	0.0001	50.21	0.0456	−49.10	0.0001	−2.48
Fan	1.7390	137.57	1.7358	137.33	1.4688	59.91	1.8514	47.64
LCD	0.0002	30.71	0.0004	31.11	0.5800	−23.09	0.0000	−3.11
PC	0.0002	90.79	0.0016	92.09	1.1222	74.13	0.0001	7.33

Table 3 presents the parameters for active and nonactive power Equation considering the ZIP model. It can be seen that there are some subtle differences in active Equation parameters, mainly for the nonlinear loads like CFL, LCD and TV. On the other hand, significant differences are observed for nonactive power parameters. In fact the model parameters for nonactive power are completely different.

For Modified ZIP model, the results are presented in a different way. As the Equations of active and reactive power are coupled, the results are presented in **Table 4**, each one for different reference data S_1, P_1, Q_1 and S, P, N respectively. It can be seen that the results are strongly affected by the presence of harmonics.

8.2. Phase 2—Load Aggregation

In the second phase the modeled loads, used in phase I, are turned on in the same time, and measurements of total active and nonactive power are done. Using these data, the models for the group of equipments data are obtained (Collective Model). **Table 5**, **Table 6** and **Table 7** present the obtained models.

Finally, the model obtained by summing up the individual models is calculated (Sum of Models) by applying superposition theorem used in Equation (28).

$$P_{\text{Total}} = \sum_{i=1}^{n} P_i , \quad Q_{\text{Total}} = \sum_{i=1}^{n} Q_i \tag{28}$$

Table 8 presents a comparison among the obtained models outputs (Collective Model and Sum of Models), considering or not harmonics components (i. only the fundamental frequency components (S1, P1, Q1) and ii. harmonics contributions (S, P, N)), for two different voltage levels (110 and 130V). The line of the measured

Table 3. ZIP model parameters for active and nonactive power.

Load	ZIP model parameters (V_o = 127V)							
	Reference parameter P				Reference parameter P_1			
	P_p	I_p	Z_p	P_o (W)	P_p	I_p	Z_p	P_o (W)
CFL	0.90	0.08	0.01	15.55	0.93	0.07	0.01	15.65
Inc	0.30	0.00	0.70	102.22	0.30	0.00	0.70	102.14
TV	0.93	0.05	0.02	49.53	0.85	0.13	0.02	51.23
Fan	0.12	0.00	0.88	137.53	0.12	0.00	0.88	137.29
LCD	0.95	0.02	0.03	30.84	0.85	0.09	0.06	31.27
PC	0.98	0.00	0.01	91.00	0.97	0.02	0.02	92.70
Load	Reference parameter N				Reference parameter Q_1			
	P_q	I_q	Z_q	Q_o (Var)	P_q	I_q	Z_q	Q_o (Var)
CFL	0.00	1.00	0.00	−21.85	0.72	0.28	0.00	−10.37
Inc	-	-	-	0.00	-	-	-	0.00
TV	0.29	0.58	0.13	−45.55	0.72	0.08	0.21	−1.95
Fan	0.25	0.00	0.75	59.89	0.07	0.00	0.93	47.62
LCD	0.68	0.00	0.32	−23.10	0.56	0.38	0.06	−3.22
PC	0.00	0.88	0.12	74.11	0.86	0.12	0.02	7.19

Table 4. Modified ZIP model parameters for active and nonactive power.

Load	ZIP_mod (V_o = 127V)						
	Reference parameters S_1, P_1 and Q_1						
	P%	I%	Z%	P^θ	I^θ	Z^θ	Sn
CFL	0.90	0.00	0.09	5.79	6.13	3.97	21.03
Inc	0.0000	0.3550	0.6450	0.00	0.00	0.00	102.38
TV	0.8315	0.0143	0.1541	6.05	1.86	1.36	60.26
Fan	0.0415	0.3287	0.6299	3.79	0.33	0.36	157.96
LCD	0.7300	0.2655	0.0045	6.00	0.46	0.65	33.56
PC	0.5121	0.4203	0.0676	0.30	5.95	2.42	112.49
Load	Reference parameters S, P and N						
	P%	I%	Z%	P^θ	I^θ	Z^θ	Sn
CFL	0.48	0.29	0.23	5.88	5.14	4.28	33.16
Inc	0.2529	0.1165	0.6306	0.00	0.00	0.00	103.41
TV	0.8064	0.0020	0.1916	5.38	1.95	1.66	102.76
Fan	0.0017	0.3033	0.6950	5.94	0.74	0.27	153.65
LCD	0.8334	0.0476	0.1190	5.77	5.48	4.63	40.85
PC	0.5871	0.1544	0.2585	0.27	0.63	1.96	152.63

Table 5. Exponential model parameters for active and nonactive power.

Exponential model parameters (V_o = 127V)							
Reference parameter P		Reference parameter P_1		Reference parameter N		Reference parameter Q_1	
α	P_o	α	P_o	β	Q_o	β	Q_o
0.8122	443.22	0.8089	445.40	2.2265	129.85	2.2334	38.04

Table 6. ZIP model parameters for active and nonactive power.

ZIP model parameters (V_o = 127V)							
Reference parameter P				Reference parameter P_1			
P_p	I_p	Z_p	P_o (W)	P_p	I_p	Z_p	P_o (W)
0.57	0.00	0.43	443.11	0.57	0.00	0.43	445.29
Reference parameter N				Reference parameter Q_1			
P_q	I_q	Z_q	Q_o (Var)	P_q	I_q	Z_q	Q_o (Var)
0.21	0.15	0.64	131.43	0.14	0.28	0.58	37.07

Table 7. Modified ZIP model parameters for active and nonactive power.

ZIP_mod model parameters (V_o = 127V)						
Reference parameters S_1, P_1 and Q_1						
$P\%$	$I\%$	$Z\%$	P^θ	I^θ	Z^θ	Sn
0.0615	0.7927	0.1458	0.61	6.17	2.21	592.76
Reference parameters S, P and N						
$P\%$	$I\%$	$Z\%$	P^θ	I^θ	Z^θ	Sn
0.4998	0.0313	0.4689	6.24	0.02	0.65	489.70

Table 8. Comparison among models for two different voltages and for different references.

	V_i = 110V				V_i = 130V			
	P_m or P	P_1	Q_M, N	Q_1	P_m	P_1	Q_m or N	Q_1
Measured values	396.47	398.45	93.81	28.33	454.78	456.84	140.34	40.82
Exponential model output (Equation (1) and (2) with the data of Table 2)								
Collective model[a]	394.40	396.52	25.88	27.60	451.71	453.89	37.54	40.08
Sum of models[b]	378.00	380.28	−19.74	28.28	435.35	437.47	−11.04	41.06
ZIP model output (Equation (3) and (4) with the data of Table 3)								
Collective model[a]	395.18	397.30	107.80	30.32	452.29	454.47	135.91	38.34
Sum of models[b]	377.30	379.55	30.98	28.79	436.12	439.95	45.88	41.29
Modified ZIP model output (Equation (6) and (7) with the data of Table 4)								
Collective model[a]	395.29	395.88	94.61	27.57	452.22	454.12	135.85	40.00
Sum of models[b]	380.41	377.59	20.79	27.18	435.52	441.11	48.08	42.28

[a]These values were obtained using the model built using data for all equipment turned on simultaneously; [b]These values were obtained using the model built using the sum of individual models.

values presents the reference data. It can be observed that the difference between obtained models (Exp, ZIP and ZIP_mod) of active power for both voltage levels is almost irrelevant, considering or not harmonics (P_m or P_1). Otherwise for nonactive power the model presents very significant differences. For fundamental frequency component (Q_1) only, the results present similar values. Nonetheless, when the harmonic components are considered (Q_m, N), the results are completely different, since the superposition theorem cannot be applied.

9. Conclusion

This paper demonstrated the application of optimization technique for the treatment of ellipsoid constraints of three static load models widely used in the literature. This technique was used in order to obtain real models for the loads, so that the result achieved can produce physical meaning. All the mathematical formulations, as well as the methodology used to achieve the results are available in the initial sections.

For most of the loads, the three models showed similar results, except for electronic loads such as compact fluorescent lamp, monitor, computer and television. Such loads showed a reduction in power consumption with the increase in voltage level. Thus, it was evident that among the three physical models presented, only the ZIP_mod can faithfully represent the load, once the other models do not show curves with negative slope.

The literature presents these models and suggests using the superposition theorem to obtain the model which represents a group of loads by simple summing up the individual models. This paper shows that the results of the obtained models, which do not filter the data (use only the fundamental frequency), cannot be used for load aggregation. In other words, to apply this kind of load model it is necessary to work with only the filtered data of the load, otherwise the final model will present very significant errors.

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