Journal of Computer and Communications, 2014, 2, 23-28
Published Online May 2014 in SciRes. http://www.scirp.org/journal/jcc
http://dx.doi.org/10.4236/jcc.2014.27004

Enhanced Bilinear Approach for Sensor Network Self-Localization Using Noisy TOF Measurements

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Received March 2014

Abstract

This paper develops a new algorithm for sensor network self-localization, which is an enhanced version of the existing Crocco's method in [11]. The algorithm explores the noisy time of flight (TOF) measurements that quantify the distances between sensor nodes to be localized and sources also at unknown positions. The newly proposed technique first obtains rough estimates of the sensor node and source positions, and then it refines the estimates via a least squares estimator (LSE). The LSE takes into account the geometrical constraints introduced by the desired global coordinate system to improve performance. Simulations show that the new technique offers superior localization accuracy over the original Crocco's algorithm under small measurement noise condition.

Keywords

Self-Localization, Time of Flight (TOF), Global Coordinate System, Least Squares Estimation

1. Introduction

In general terms, a wireless sensor network (WSN) refers to a self-organizing network of a large number of inexpensive and small-size sensor nodes [1]. It has found diverse applications in but not limited to surveillance, industrial process control [2] and environment monitoring [3]. The availability of the location information of the sensor nodes is crucial for many application scenarios where the sensor data collected by the WSN would become useless without proper location stamps. In this work, we shall consider the problem of the WSN self-localization using the signal of opportunities from sources at completely unknown positions.

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How to cite this paper: Gao, X., Yang, L. and Peng, L. (2014) Enhanced Bilinear Approach for Sensor Network Self-Localization Using Noisy TOF Measurements. *Journal of Computer and Communications*, **2**, 23-28.
http://dx.doi.org/10.4236/jcc.2014.27004

Among the existing self-localization techniques, cluster-based [4] and multi-dimensional scaling (MDS)-based algorithms [5] are perhaps the most widely applied approaches. The idea behind the cluster-based method is to first partition the whole WSN into clusters and to localize the nodes within each cluster with respect to a local coordinate system. After an optional location refinement stage, the algorithm reconciles multiple localized clusters under a global coordinate system. On the other hand, MDS is a classic data analysis technique that extracts a geometric structure from obtained distance-like data. It achieves the relative self-localization of a WSN via applying the singular value decomposition (SVD) to a matrix whose entries are defined in terms of the squared Euclidean distances between sensor nodes. The MDS output can be reconciled to a unique global coordinate system in order to remove the unknown translation and rotation inherently introduced by the numerical realization of SVD. Due to the simplicity of the MDS technique, many variants such as MDS-MAP have been proposed in literature. The MDS-based method has been successfully applied in microphone array self-localization using range measurements obtained in a diffusive noise environment [6] [7].

The methods surveyed above assume the availability of the range measurements between sensor nodes to be localized. However, this assumption may be violated e.g., in large-scale networks due to the limitation on the sensor node communication radii and/or under harsh environments where the direct communication between sensor nodes may be jammed. An alternative technique that can address the above issue is to make use of the signals of opportunities from emitting sources [8]. Specifically, the time of flight (TOF) for the source signal to propagate to the sensors may be measured and converted to the source-sensor range measurements through multiplying them with the signal propagation speed. Under the condition that the probability density function (PDF) of the TOF measurement noise is known *a priori*, a maximum likelihood estimator (MLE) that is able to find the relative positions of the sensor nodes and the sources can be established, given that the number of TOF measurements is sufficient [8]. Even for the scenario where the noise PDF is not available, a possibly suboptimal least squares estimator (LS) can be constructed [9].

Due to the nonlinear relation between the measured TOF and the positions of the sensor nodes and the sources, it is non-trivial to solve the MLE and the LSE to accomplish the WSN self-localization task. The iterative Taylor-series (TS) method may be applied but it requires good initial solution guesses to avoid the local convergence or the divergence problem. Guevara *et al*. assumed the nodes are in a plane and just exploited the distance between the nodes and mobile node [10]. This method may be not available when the nodes are not in a plane. In addition, due to lack of inter-node distances, location accuracy may be lower. To invoke the use of linear estimation techniques in the WSN self-localization using sources at unknown positions, Crocco *et al*. recently proposed a bilinear approach that first obtains rough estimates of the source and sensor positions using MDS and then refines them using an LSE. In particular, they developed a closed-form solution for the WSN self-localization problem when one source and one sensor are co-located at the same position.

One of the major drawbacks of the Crocco's closed-form method in [11] is its sensitivity to the TOF measurement noise, as indicated in their simulation results. In fact, in the development of the closed-form solution, [11] imposed constraints that a source and a sensor are co-located at the origin, and they are coplanar with another two sensors, one assumed to be positioned on the x-axis and the other on x-y plane. These constraints are introduced to remove the translation and rotation ambiguity inherent in the relative localization problems. However, when solving for the refined estimates of the source and sensor positions, these constraints were not taken into account explicitly. They were indeed utilized after the application of the LSE to reconcile the self-localization result under a global coordinate system.

In this work, we propose an enhanced bilinear approach for the WSN self-localization task using TOF measurements from the sources at unknown positions. The new method uses the same bilinear approach as in the Crocco's algorithm to obtain the rough source position and sensor position estimates. They are later refined via an LSE that incorporates the geometrical constraints introduced by the desired global coordinate system. In this way, significantly improved self-localization accuracy can be attained, due to the utilization of additional information on the source and sensor nodes contained in the constraints. The good performance of the proposed technique, in comparison to the original Crocco's algorithm, is illustrated via computer simulations.

The rest of the paper is structured as follows. Section 2 formulates the problem. Section 3 describes the proposed algorithm and section 4 gives the simulation results. Finally, Section 5 concludes the paper.

2. Problem Formulation

Consider that N sensors and M sources are placed in a unit cube. Let $\mathbf{X}$ and $\mathbf{A}$ be the sensor location and the source location matrices defined as

$$\mathbf{X}=\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_N & y_N & z_N \end{bmatrix}, \mathbf{A}=\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ \vdots & \vdots & \vdots \\ a_M & b_M & c_M \end{bmatrix}. \tag{1}$$

(x_i, y_i, z_i) and (a_j, b_j, c_j) are the coordinates of sensor *i* and source *j*, respectively. The signal TOF from source *j* is measured at sensor *i* and it is equal to, after being multiplied with the signal propagation speed *c*,

$$d_{i,j}=d_{i,j}^o+n_{i,j} \tag{2}$$

where $d_{i,j}=ct_{i,j}$, $t_{i,j}$ is the noisy TOF, and $d_{i,j}^o=\sqrt{(x_i-a_j)^2+(y_i-b_j)^2+(z_i-c_j)^2}$ is the true source-sensor distance. $n_{i,j}$ is the measurement noises modeled as independently and identically distributed (i.i.d.) Gaussian random variables with zero mean and standard deviation σ. Collecting $d_{i,j}$ yields a distance matrix **D** with $[\mathbf{D}]_{i,j}=d_{i,j}$.

To obtain a unique global coordinate system, we introduce the following geometric constraints [11]

$$x_1=y_1=z_1=a_1=b_1=c_1=y_2=z_2=z_3=0 \quad x_2>0, y_3>0, z_4>0 \tag{3}$$

In other words, it is assumed that both the first sensor node and the first source are co-located at the origin (to remove the translation ambiguity and to enable the development of a closed-form solution to the WSN self-localization), the second sensor lies on the positive *x*-axis and the third sensor is coplanar with the first and the second sensor nodes (to eliminate the rotation ambiguity). The fourth sensor has a positive *z*-coordinate so that we no longer have the reflection ambiguity.

The task of WSN self-localization is to estimate (x_i, y_i, z_i) and (a_j, b_j, c_j) from the distance matrix **D** and the geometric constraints given in (3).

3. Algorithm

The new algorithm first follows the bilinear approach proposed in [11] to obtain rough estimates of the sensor and the source positions. In particular, we evaluate $\tilde{d}_{i,j}=d_{i,j}^2-d_{1,j}^2-d_{i,1}^2+d_{1,1}^2$ and collect them in a matrix $\tilde{\mathbf{D}}$, where $[\tilde{\mathbf{D}}]_{i,j}=\tilde{d}_{i,j}$. Note from (2) that the true value of $\tilde{d}_{i,j}$, denoted by $\tilde{d}_{i,j}^o$, is equal to

$$-2(x_i-x_1)(a_j-a_1)-2(y_i-y_1)(b_j-b_1)-2(z_i-z_1)(c_j-c_1)=\tilde{d}_{i,j}^o \tag{4}$$

In other words, we have that under noiseless condition

$$-2\tilde{\mathbf{X}}\tilde{\mathbf{A}}^T=\tilde{\mathbf{D}}$$

where

$$\tilde{\mathbf{X}}=\begin{bmatrix} x_2 & 0 & 0 \\ x_3 & y_3 & 0 \\ \vdots & \vdots & \vdots \\ x_N & y_N & z_N \end{bmatrix}, \tilde{\mathbf{A}}=\begin{bmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ \vdots & \vdots & \vdots \\ a_M & b_M & c_M \end{bmatrix} \tag{5}$$

and Equations (1) and (3) have been used.

With noisy TOF measurements, the singular value decomposition (SVD) of $\tilde{\mathbf{D}}$ can be approximated using $\tilde{\mathbf{D}}\approx\mathbf{UVW}$. By comparing with (5), we have that

$$\tilde{\mathbf{X}}\approx\mathbf{UR}, -2\tilde{\mathbf{A}}^T\approx\mathbf{R}^{-1}\mathbf{VW} \tag{6}$$

As such, rough estimates of the sensor and the source positions can be obtained if we set **R** = **I**. Here, **V** is a 3×3 diagonal matrix whose diagonal entries are the largest three singular values of $\tilde{\mathbf{D}}$. It is obtained by noting from (5) that under the noiseless condition, $\tilde{\mathbf{D}}$ would have a rank of 3. The columns in **U** and the rows in **W** are orthogonal vectors. **R** is referred to as the mixing matrix, which is

$$\mathbf{R} = \begin{bmatrix} r_1 & r_2 & r_3 \\ r_4 & r_5 & r_6 \\ r_7 & r_8 & r_9 \end{bmatrix}. \tag{7}$$

To accomplish the WSN self-localization (*i.e.*, determine $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{A}}$) thus is equivalent to finding the nine elements in $\mathbf{R}$. An LSE is applied to refine the rough sensor and source position estimates. It finds $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{A}}$ via the following least squares problem

$$\min_{\tilde{\mathbf{A}},\tilde{\mathbf{X}}} \sum_{i=2}^{N} \sum_{j=2}^{M} [x_i^2 + y_i^2 + z_i^2 - 2x_i a_j - 2y_i b_j - 2z_i c_j - d_{i,j}^2 + d_{1,j}^2]^2 \tag{8}$$

Putting (7) into (6), substituting the result back into the cost function above and applying some algebraic manipulations lead to

$$\begin{aligned} \hat{\mathbf{R}} = \arg\min_{\mathbf{R}} \sum_{i=1}^{N-1} \sum_{j=1}^{M-1} [&u_{i1}^2 (r_1^2 + r_2^2 + r_3^2) + u_{i2}^2 (r_4^2 + r_5^2 + r_6^2) \\ &+ u_{i3}^2 (r_7^2 + r_8^2 + r_9^2) + 2u_{i1}u_{i2}(r_1 r_4 + r_2 r_5 + r_3 r_6) + 2u_{i1}u_{i3}(r_1 r_7 \\ &+ r_2 r_8 + r_3 r_9) + 2u_{i2}u_{i3}(r_4 r_7 + r_5 r_8 + r_6 r_9) + u_{i1} v_{11} w_{1j} + u_{i2} v_{22} w_{2j} \\ &+ u_{i3} v_{33} w_{3j} - d_{i,j}^2 + d_{1,j}^2]^2 \end{aligned} \tag{9}$$

The proposed self-localization technique from [11] simply sets r_4, r_7, r_8 to be zeros to find $\mathbf{R}$. The self-localization results are then reconciled under the global coordinate system determined by (3). In this work, we first note from (5) that under the geometric constraints, the y-coordinate of sensor 2and the z-coordinates of sensors 2 and 3 are all zeros. As a result, the elements in $\mathbf{R}$ must satisfy

$$r_8 = -\frac{u_{11} r_2 + u_{12} r_5}{u_{13}}, r_6 = -\frac{u_{11} u_{23} - u_{13} u_{21}}{u_{12} u_{23} - u_{13} u_{22}} r_3, r_9 = -\frac{u_{11} u_{22} - u_{12} u_{21}}{u_{13} u_{22} - u_{12} u_{23}} r_3. \tag{10}$$

Note that with (6), the desired global coordinate system is incorporated explicitly in the process of determining $\mathbf{R}$, or equivalently, the self-localization solution.

To find the other six elements in $\mathbf{R}$, we define

$$\begin{aligned} \mathbf{p}^T &= \mathbf{1}_{1\times(M-1)} \otimes \mathbf{S}^T \\ \mathbf{S}^T &= [\mathbf{s}_1 \quad \mathbf{s}_2 \quad \cdots \quad \mathbf{s}_{N-1}]_{6\times(N-1)} \\ \mathbf{f}^T &= [r_1^2 + r_2^2 + r_3^2 \quad r_4^2 + r_5^2 + r_6^2 \\ &\quad r_7^2 + r_8^2 + r_9^2 \quad r_1 r_4 + r_2 r_5 + r_3 r_6 \\ &\quad r_1 r_7 + r_2 r_8 + r_3 r_9 \quad r_4 r_7 + r_5 r_8 + r_6 r_9]_{1\times 6} \\ \mathbf{k}^T &= [k_{1,1}, \cdots, k_{N-1,1}, k_{1,2}, \cdots, k_{N-1,M-1}]_{1\times(N-1)(M-1)} \end{aligned}$$

where

$$\begin{aligned} \mathbf{s}_i^T &= \begin{bmatrix} u_{i1}^2 & u_{i2}^2 & u_{i3}^2 & 2u_{i1}u_{i2} & 2u_{i1}u_{i3} & 2u_{i2}u_{i3} \end{bmatrix} \\ k_{i,j} &= -(u_{i1} v_{11} w_{1j} + u_{i2} v_{22} w_{2j} + u_{i3} v_{33} w_{3j}) + d_{i,j}^2 - d_{1,j}^2 \end{aligned}$$

As a result, we may rewrite the cost function in (9) as $\mathbf{f} = \arg\min (\mathbf{pf} - \mathbf{k})^T (\mathbf{pf} - \mathbf{k})$. It is an LSE, whose solution is

$$\mathbf{f} = [f_1, f_2, \ldots, f_6]^T = (\mathbf{p}^T \mathbf{p})^{-1} \mathbf{p}^T \mathbf{k}. \tag{11}$$

Note that $\mathbf{f}$ contains six entries only, which comes from the fact that under the constraints in (3), the number of unknowns in R reduces to six (also see (10)). Combining the definition of $\mathbf{f}$ and (11) yields

$$\begin{aligned} &r_1^2 + r_2^2 + r_3^2 = f_1, r_4^2 + r_5^2 + r_6^2 = f_2, \\ &r_7^2 + r_8^2 + r_9^2 = f_3, \ r_1 r_4 + r_2 r_5 + r_3 r_6 = f_4, \\ &r_1 r_7 + r_2 r_8 + r_3 r_9 = f_5, r_4 r_7 + r_5 r_8 + r_6 r_9 = f_6 \end{aligned} \tag{12}$$

Putting (10) into (12) yields a system of six nonlinear equations with respect to r_1, r_2, …, r_6. Solving it via numerical techniques such as the MATLAB routine f solve() gives the estimate of **R**. Substituting the result back into (6) yields the desired estimates of the sensor and the source positions, which completes the WSN self-localization task.

4. Simulations

The simulation scenario has 20 sensors and 20 sources deployed in a unit cube. A Monte Carlo simulation of 100 ensemble runs is conducted. **Figure 1** plots the sensor node localization results from the proposed technique and the algorithm from [11] at a particular ensemble run when the standard deviation of the TOF noise is $\sigma=0.01$. **Figure 2** compares the localization mean square errors (MSEs) of the two algorithms in consideration as a function of the TOF noise variance. An amount of 2 - 4 dB reduction in the self-localization MSE can be obtained with the newly proposed algorithm, as compared to the method in [11].

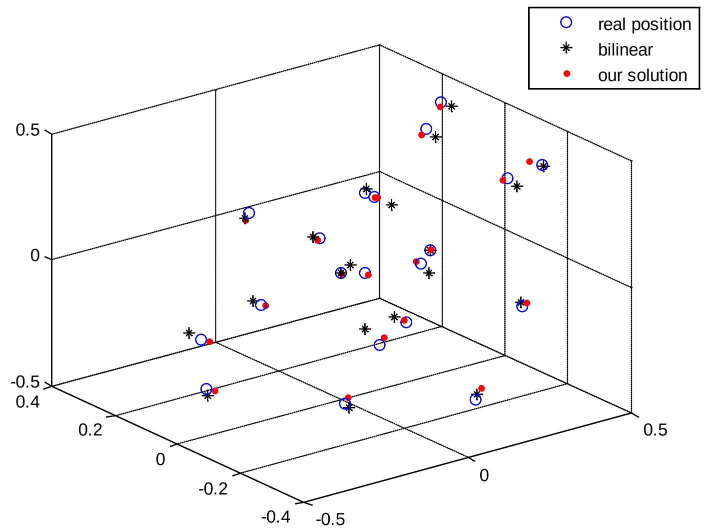

Figure 1. Sensor node self-localization accuracy of the proposed method and the algorithm from [11] when $\sigma=0.01$.

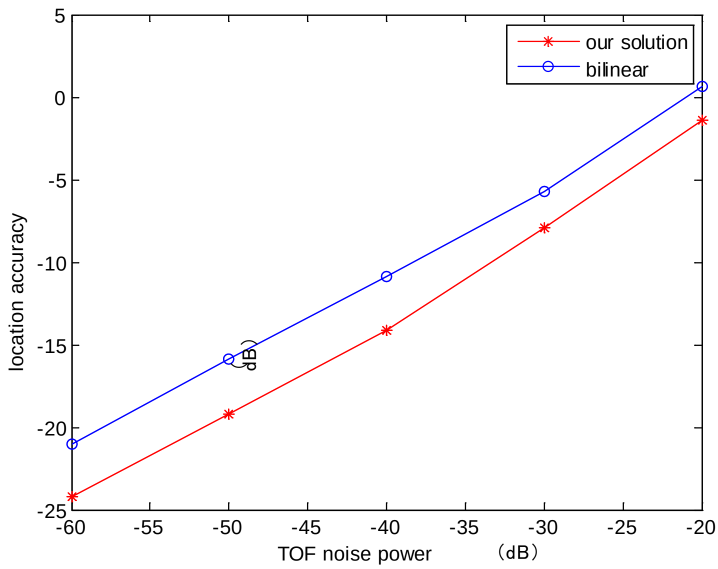

Figure 2. Sensor node self-localization MSEs of the proposed method and the algorithm from [11] as function of the TOF noise variance.

5. Conclusion

This paper developed an enhanced bilinear approach for the WSN self-localization problem. The presence of emitting sources is assumed so that source signal TOFs can be obtained to deduce source-sensor distances for the self-localization task. The new method first utilizes the bilinear technique in [11] to attain initial estimates of the sensor and source positions. Then, it refines these estimates via an LSE that takes into account the geometric constraints imposed by the desired global coordinate system. Simulations verified that due to the use of the geometric constraints, the newly developed algorithm offers better self-localization accuracy over the existing bilinear method in [11].

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