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Variation of Vacuum Energy if Scale Factor Becomes Infinitely Small, with Fixed Entropy Due to a Non Pathological Big Bang Singularity Accessible to Modified Einstein Equations

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ABSTRACT

When initial radius $R_{initial} \to 0$ if Stoica actually derived Einstein equations in a formalism which remove the big bang singularity pathology, then the reason for Planck length no longer holds. The implications of $R_{initial} \to 0$ are the first part of this manuscript. Then the resolution is alluded to by work from Muller and Lousto, as to implications of entanglement entropy. We present entanglement entropy in the early universe with a steadily shrinking scale factor, due to work from Muller and Lousto, and show that there are consequences due to initial entanged $S_{entropy} = 0.3 r_H^2 / a^2$ for a time dependent horizon radius r_H in cosmology, with for flat space conditions $r_H = \eta$ for conformal time. In the case of a curved, but not flat space version of entropy, we look at vacuum energy as proportional to the inverse of scale factor squared times the inverse of initial entropy, effectively when there is no initial time in line with $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$. The consequences for this initial entropy being entangled are elaborated in this manuscript. No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, t, re scaled does not go to zero. Even if the length goes to zero. This preserves a minimum non zero Λ vacuum energy, and in doing so keep the bits, for computational bits cosmological evolution even if $R_{initial} \to 0$.

Keywords: Fjortoft Theorem; Thermodynamic Potential; Matter Creation; Vacuum Energy Non Pathological Singularity Affecting Einstein Equations; Planck Length; Braneworlds

1. Introduction

This article is to investigate what happens physically if there is a non pathological singularity at the start of space-time, *i.e.* no reason to have a minimium nonzero length. The reasons for such a proposal come from [1] by Stoica who may have removed the reason for the development of Planck's length as a minimum safety net to remove what appears to be unadvoidable pathologies at the start of applying the Einstein equations at a space-time singularity, and are commented upon in this article. $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$ in particular is remarked upon. This is a counter part to *Fjortoft* theorem in **Appendix I** below.

Note a change in entropy formula given by Lee [3] about the inter relationship between energy, entropy and temperature as given by

$$m \cdot c^2 = \Delta E = T_U \cdot \Delta S = \frac{\hbar \cdot a}{2\pi \cdot c \cdot k_B} \cdot \Delta S \qquad (1)$$

Lee's formula is crucial for what we will bring up in the latter part of this document. Namely that changes in initial energy could effectively vanish if [1] is right, *i.e.* Stoica removing the non pathological nature of a big bang singularity.

If the mass m, *i.e.* for gravitons is set by acceleration (of the net universe) and a change in enthropy $\Delta S \sim 10^{38}$ between the electroweak regime and the final entropy value of, if $a \cong \frac{c^2}{\Delta x}$ for acceleration is used, so then we obtain

$$S_{Today} \sim 10^{88} \qquad (2)$$

Then we are really forced to look at (1) as a paring between gravitons (today) and gravitinos (electro weak) in the sense of preservation of information.

Having said this note by extention $\rho \sim H^2/G \Leftrightarrow H \approx a^{-1}$. As ρ changes due to $\rho \sim H^2/G$ and $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$, then a is also altered *i.e.* goes to zero.

What will determine the answer to this question is if $\Delta E_{initial}$ goes to zero if $R_{initial} \to 0$ which happens if there is no minimum distance mandated to avoid the pathology of singularity behavior at the heart of the Einstein equations. In doing this, we avoid using the $E \to 0^+$ situation, and instead refer to a nonzero energy, with $\Delta E_{initial}$ instead vanishing. In particular, the Entanglement entropy concept as presented by Muller and Lousto [4] is presented toward the end of this manuscript as a partial resolution of some of the pathologies brought up in this article before the entanglement entropy section. No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, *t*, re scaled goes to zero. This preserves a minimum non zero. Λ vacuum energy, and in doing so keep the bits, for computational bits cosmological evolution even if $R_{initial} \to 0$.

Before doing that, we review Ng [5-7] and his quantum foam hypothesis to give conceptual underpinnings as to why we later even review the implications of entanglement. Entropy, *i.e.* the concept of bits and computations is brought up because of applying energy uncertainty, as given by [3] and the Margolis theorem appears to indicate that the universe could not possibly evolve if [1] is applied, in a 4 dimensional closed universe. This bottle neck as indicated by Ng's [4] formalism is even more striking in its proof of the necessity of using entanglement entropy in lieu of the conclusion involving entanglement entropy, which can be non zero, even if $R_{initial} \to 0$.

2. Review of Ng [5-7] with Comments

First of all, Ng refers to the Margolus-Levitin theorem with the rate of operations $< E/\hbar \Rightarrow \#operations < E/\hbar \times time = \frac{Mc^2}{\hbar}\cdot\frac{l}{c}$. Ng wishes to avoid black-hole formation $\Rightarrow M \le \frac{lc^2}{G}$. This last step is not important to our view point, but we refer to it to keep fidelity to what Ng brought up in his presentation. Later on, Ng refers to the $\#operations \le (R_H/l_P)^2 \sim 10^{123}$ with R_H the Hubble radius. Next Ng refers to the $\#bits \propto [\#operations]^{3/4}$. Each bit energy is $1/R_H$ with $R_H \sim l_P \cdot 10^{123/2}$

The key point as seen by Ng [4] and the author is in

$$\#bits \sim \left[\frac{E}{\hbar}\cdot\frac{l}{c}\right]^{3/4} \approx \left[\frac{Mc^2}{\hbar}\cdot\frac{l}{c}\right]^{3/4} \quad (3)$$

Assuming that *E* of the universe is not set equal to zero, which the author views as impossible, the above equation says that the number of available bits goes down dramatically if one sets $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$? Also Ng writes entropy *S* as proportional to a particle count via *N*.

$$S \sim N \cong [R_H/l_P]^2 \quad (4)$$

We rescale R_H to be

$$R_H\big|_{rescale} \sim \frac{l_{Ng}}{\#}\cdot 10^{123/2} \quad (5)$$

The upshot is that the entropy, in terms of the number of available particles drops dramatically if # becomes larger.

So, as $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$ grows smaller, as # becomes larger.

a) The initial entropy drops;

b) The nunber of bits initially available also drops.

The limiting case of (4) and (5) in a closed universe, with no higher dimensional embedding is that both would vanish, *i.e.* appear to go to zero if # becomes very much larger.

3. Does It Make Sense to Talk of Vacuum Energy if $R_{initial} \neq 0$ Is Changed to $R_{initial} \to 0$? Only Answerable Straightforwardly if an Embedding Superstructure Is Assigned. Otherwise Difficult. Unless One Is Using Entanglement Entropy which Is Non Zero Even if $R_{initial} \to 0$

We summarize what may be the high lights of this inquiry leading to the present paper as follows.

a) One could have the situation if $R_{initial} \to 0$ of an infinite point mass, if there is an initial nonzero energy in the case of just four dimensions and no higher dimensional embedding even if [1] goes through verbatim. The author sees this as unlikely. But is prepared to be wrong. The infinite point mass construction is verbatim if one assumes a closed universe, with no embedding superstructure. Note this appears to nullify the parallel brane world construction author, in lieu of the manuscript sees no reason as to what would perturb this infinite point structure, so as to be able to enter in a big bang era. In such a situation, one would not have vacuum energy. That is unless one has a non zero entanglement entropy [4] present even if $R_{initial} \to 0$. See [8] for a smilar argument.

b) The most problematic scenario. $R_{initial} \to 0$ and no initial cosmological energy, *i.e.* this in a 4 dimensional closed universe. Then there would be no vacuum energy at all.initially. A literal completely empty initial state, which is not held to be viable by Volovik [9].

c) Finding that additional dimensions are involved, than just 4 dimensions may give credence to the authors speculation as to initial degrees of freedom reaching up to 1000, and the nature of a phase transition from essentially very low degrees of freedom, to over 1000 maybe in fact a chaotic mapping as speculated by the author in 2010 [10].

d) What the author would be particularly interested in knowing would be if actual semiclassical reasoning could be used to get to an initial prequantum cosmological state. This would be akin to using [11], but even more to the point, using [12] and [13], with both these last references relevant to forming Planck's constant from electromagnetic wave equations. The author points to the enormous Electromagnetic fields in the electroweak era as perhaps being part of the background necessary for such a semiclassical derivation, plus a possible Octonionic space-time regime, as before inflation flattens space-time, as forming a boundary condition for such constructions to occur [14].

The relevant template for examinging such questions is given in the following **Table 1**.

e) The meaning of Octonionic geometry prior to the introduction of quantum physics presupposes a form of embedding geometry and in many ways is similar to Penrose's cyclic conformal cosmology speculation. Note the following argument, as:

f) We are stuck with how a semiclassical argument can be used to construct **Table 1** below. In particular, we look at how Planck's constant is derived, as in the electroweak regime of space-time, for a total derivative [12, 13]

$$E_y = \frac{\partial A_y}{\partial t} = \omega \cdot A'_y\left(\omega \cdot (t-x)\right) \quad (6)$$

Similarly [12,13]

$$B_z = -\frac{\partial A_y}{\partial x} = \omega \cdot A'_y\left(\omega \cdot (t-x)\right) \quad (7)$$

The *A* field so given would be part of the Maxwell's equations given by [11] as, when $[\ ]$ represents a D'Albertain operator, that in a vacuum, one would have for an A field [12,13]

$$[\ ]A = 0 \quad (8)$$

And for a scalar field ϕ

$$[\ ]\phi = 0 \quad (9)$$

Following this line of thought we then would have an

Table 1. Time interval dynamical consequences does qm/wdw apply?

Just before Electroweak Era	Form $\hbar$ from early E & M fields, and use Maxwell's Equations with necessary to implement boundary conditions created from change from Octonionic geometry to flat space	No
Electro-Weak Era	$\hbar$ kept constant due to Machian relations	Yes
Post Electro-Weak Era to Today	$\hbar$ kept constant due to Machian relations	Yes Wave function of Universe

energy density given by, if ε_0 is the early universe permeability [12]

$$\eta = \frac{\varepsilon_0}{2}\cdot\left(E_y^2 + B_z^2\right) = \omega^2 \cdot \varepsilon_0 \cdot A'^2_y\left(\omega\cdot(t-x)\right) \quad (10)$$

We integrate (10) over a specified E and M boundary, so that, then we can write the following condition namely [12,13].

$$\begin{aligned}&\iiint \eta \mathrm{d}(t-x)\mathrm{d}y\mathrm{d}z \\ &= \omega\varepsilon_0 \iiint A'^2_y\left(\omega\cdot(t-x)\right)\mathrm{d}(t-x)\mathrm{d}y\mathrm{d}z\end{aligned} \quad (11)$$

(11) would be integrated over the boundary regime from the transition from the Octonionic regime of space time, to the non Octonionic regime, assuming an abrupt transition occurs, and we can write, the volume integral as representing [12,13]

$$E_{gravitational-energy} = \hbar \cdot \omega \quad (12)$$

Our contention for the rest of this paper, is that Mach's principle will be necessary as an information storage container so as to keep the following, *i.e.* having no variation in the Planck's parameter after its formation from electrodynamics considerations as in (11) and (12). Then by applying [12,13] we get $\hbar$ formed by semiclassical reasons and need to have Machs principle (1) to have the same value up to the present era. In semi classical reasoning similar to [11]

$$\hbar(t) \xrightarrow[Apply-Machs-Relations]{} \hbar \quad \text{(Constant value)} \quad (13)$$

The question we can ask, is that can we have a pre-quantum regime commencing for (11) and (12) for $\hbar$ if $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck}$? And a closed 4 dimensional universe? If so, then what is the necessary geometrial regime of space-time so that the integration performed in (11) can commence properly? Also, what can we say about the formation of (12) above, as a number, $\#$ gets larger and larger, effectively leading to. Also,with an Octonionic geometry regime which is a pre quantum state [14].

In so many words, the formation period for $\hbar$ is our pre-quantum regime. This **Table 1** could even hold if $R_{initial} \to 0$ but that the 4 dimensional space-time exhibiting such behavior is embedded in a higher dimensional template. That due to $R_{initial} \to 0$ not removing entanglement entropy as is discussed near the end of this article.

4. If $R_{initial} \to 0$, Then if There Is an Isolated, Closed Universe, There Is a Disaster Unless One Uses Entanglement Entropy

One does not have initial entropy, and the number of bits initially disappears.

Abandoning the idea of a completely empty universe, this unperturbed point of matter-energy appears to be a recipede for a static point with no perturbation, as may be the end result of applying Fjortoft theorem [15] to the thermodynamic potential as given in [16], *i.e.* the non definitive anwer for fufillment of criteria of instability by applying Fjortoft's theorem [15] to the potential [16] leading to no instability as given by the potential given in [16] may lead to a point of space-time with no change, *i.e.* a singular point with "infinite" mass which does not change at all.

5. Can an Alternative to a Minimum Length Be Put in? Consider the Example of Planck Time as the Minimal Component, Not Planck Length

From J. Dickau, the following was given to the author, as a counter part as to how to view threshholds as to how a Mandelbrot set may pre select for critical behavior different from what is being pre supposed in this manuscript. [17].

Dickau writes:

"*If we examine the Mandelbrot Set along the Real axis, it informs us about behaviors that also pertain in the Quaternion and Octonic case-because the real axis is invariant over the number types. If numbers larger than* 0.25 *are squared and summed recursively* (*i.e.* –z = z^2 + c) *the result will blow up, but numbers below this threshold never get to infinity, no matter how many times they are iterated. But once space-like dimensions are added, i.e. an imaginary compoent—the equation blows up exponentially, faser than when iterated*".

Dickau concludes:

"*Anyhow there may be a minimum* (*space-time length*) *involved but it is probably in the time direction*".

This is a counter pose to the idea of minimum length, *i.e.* the idea being a replacement for what the author put in here: looking at a beginning situation with a crucial parameter $R_{initial}$ even if the initial time step is "put in by hand". First of all, look at [4], if *E* is *M*, due to setting c = 1, then

$$\Delta E_{initial} \approx 4\pi\rho\left(R_{initial}\right)^2 \Delta R_{initial} \tag{14}$$

Everything depends upon the parameter $R_{initial}$ which can go to zero. The choice as to $R_{initial}$ going to zero, or not going to zero will be conclusion of our article.

We have to look at what (14) tells us, even if we have an initial time step for which time is initially indeterminate, as given by a redoing of Mitra's g_{00} formula [8] which we put in to establish the indeterminacy of the initial time step if quantum processes hold.

$$\left(g_{00} = \exp\left[\frac{-2}{1+\left(\rho(t)/p(t)\right)}\right]\right) \xrightarrow{\rho+p=0} 0 \tag{15}$$

What Dickau is promoting is, that the Mandelbrot set, if applicable to early universe geometry, that what the author wrote, with $R_{initial} \sim \frac{1}{\#}\ell_{Ng} < l_{Planck} \xrightarrow{\#\neq\infty} small-value$ potentially going to zero, is less important than a minimum time length. To which the author states, if Dickau is correct as to applicability of the Mandelbrot set, that he, the author is happily corrected, but he also thinks that the Mandelbrot set is a beautiful example of the fungability of space-time metrics used, *i.e.* how one sets the initial space-time potential is to determine the correctness of the Mandelbrot set, *i.e.* the [16] reference, as given, by Thanu Padmanabhan appears not to have a Mandelbrot set, in its thermodynamic potential. The instability issue is reviewed in **Appendix II**. For those who are interested in the author's views as to lack proof of instability. It uses [16] which the author views as THE reference as far as thermodynamic potentials and the early universe.

6. Muller and Lousto Early Universe Entanglement Entropy, and Its Implications. Solving the Spatial Length Issue, Provided a Minimum Time Step Is Preserved in the Cosmos, in Line with Dickau's Suggestion

We look at [4]

$$S_{entropy} = 0.3 r_H^2 / a^2 \quad \text{for a time dependent horizon radius } r_H \text{ in cosmology} \tag{16}$$

Equation (16) above was shown by the author to be fully equivalent to

$$S_{entropy} = 0.3 r_H^2 / a^2 \sim \frac{0.3}{a^2}\exp\left[-t\cdot\sqrt{\frac{\Lambda}{3}}\right] \tag{17}$$

i.e.

$$\left[-t\sqrt{\frac{\Lambda}{3}}\right] \sim \ln_e\left(\frac{a^2}{0.3}\cdot S_{entropy}\right) \tag{18}$$

So, then one has

$$\Lambda \approx \frac{3}{t^2} \cdot \left[\ln_e \left(\frac{a^2}{0.3} \cdot S_{entropy} \right) \right]^2 \qquad (19)$$

No matter how small the length gets, $S_{entropy}$ if it is entanglement entropy, will not go to zero. The requirement is that the smallest length of time, *t*, re scaled does not go to zero. *This preserves a minimum non zero* Λ *vacuum energy*, *and in doing so keep the bits, for computational bits even if* $R_{initial} \to 0$.

7. Conclusions

a) The universe if $R_{initial} \to 0$ [1] and if it is an isolated system, i.e. not as embedded in higher dimensions as referred to in [18] may have no bits, or computations as thought of by Ng [5-7]. This would be in tandem with the authors conclusion that one would have an initial infinite point mass and no evolution. And no generation of entropy. The only way about this, as indicated in section 6 would be to use entanglement entropy, [4] and to keep the minimum time step from going to zero.

b) If $R_{initial} \to 0$ [1] but the universe is embedded in a higher dimensional system, as given by [19], then there is no reason to say there are no bits, or computations, and the universe will continue to evolve with entropy as a by product of that evolution.

The future endeavor to investigate, is if entanglement entropy can be set up so as to have Vacuum energy no matter what in terms of (19). Satisfying this will make $R_{initial} \to 0$ a tractable cosmological problem, and [1] very useful [4].

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Appendix I. Fjortoft Theorem

A **necessary condition for instability** is that if z_* is a point in spacetime for which $\frac{\mathrm{d}^2U}{\mathrm{d}z^2}=0$ for any given potential U, then there must be some value z_0 in the range $z_1 < z_0 < z_2$ such that

$$\left.\frac{\mathrm{d}^2U}{\mathrm{d}z^2}\right|_{z_0}\cdot\left[U\left(z_0\right)-U\left(z_*\right)\right]<0 \quad (1)$$

For the proof, see [13] and also consider that the main discussion is to find instability in a physical system which will be described by a given potential U. Next, we will construct in the boundary of the EW era, a way to come up with an optimal description for U.

Appendix II. Constructing an Appropriate Potential for Using Fjortoft Theorem in Cosmology for the Early Universe Cannot Be Done. We Show Why

To do this, we will look at Padamanabhan [16] and his construction of (in Dice 2010) of thermodynamic potentials he used to have another construction of the Einstein GR equations. To start, Padamanabhan [16] wrote

If P^{ab}_{cd} is a so called Lovelock entropy tensor, and T_{ab} a stress energy tensor

$$\begin{aligned}U\left(\eta^a\right) &= -4\cdot P^{cd}_{ab}\nabla_c\eta^a\nabla_d\eta^b + T_{ab}\eta^a\eta^b \\ &\quad +\lambda(x)g_{ab}\eta^a\eta^b \\ &= U_{gravity}\left(\eta^a\right)+U_{matter}\left(\eta^a\right)+\lambda(x)g_{ab}\eta^a\eta^b \quad (1)\\ &\Leftrightarrow U_{matter}\left(\eta^a\right)=T_{ab}\eta^a\eta^b; U_{gravity}\left(\eta^a\right) \\ &= -4\cdot P^{cd}_{ab}\nabla_c\eta^a\nabla_d\eta^b\end{aligned}$$

We now will look at

$$\begin{aligned}U_{matter}\left(\eta^a\right) &= T_{ab}\eta^a\eta^b \quad (2)\\ U_{gravity}\left(\eta^a\right) &= -4\cdot P^{cd}_{ab}\nabla_c\eta^a\nabla_d\eta^b\end{aligned}$$

So happens that in terms of looking at the partial derivative of the top (1) equation, we are looking at

$$\frac{\partial^2U}{\partial\left(\eta^a\right)^2}=T_{aa}+\lambda(x)g_{aa} \quad (3)$$

Thus, we then will be looking at if there is a specified η^a_* for which the following holds.

$$\begin{aligned}&\left[\frac{\partial^2U}{\partial\left(\eta^a\right)^2}=T_{aa}+\lambda(x)g_{aa}\right]_{\eta^a_0} \\ &*\Big[-4\cdot P^{cd}_{ab}\left(\nabla_c\eta^a_0\nabla_d\eta^b_0-\nabla_c\eta^a_*\nabla_d\eta^b_*\right) \quad (4)\\ &\quad +T_{ab}\cdot\left[\eta^a_0\eta^b_0-\eta^a_*\eta^b_*\right]+\lambda(x)g_{ab}\cdot\left[\eta^a_0\eta^b_0-\eta^a_*\eta^b_*\right]\Big] \\ &<0\end{aligned}$$

What this is saying is that there is no unique point, using this η^a_* for which (4) holds. Therefore, we say there is no official point of instability of η^a_* due to (3). The Lagrangian structure of what can be built up by the potentials given in (3) with respect to η^a_* mean that we cannot expect an inflection point with respect to a 2nd derivative of a potential system. Such an inflection point designating a speed up of acceleration due to DE exists a billion years ago [21]. Also note that the reason for the failure for (4) to be congruent to Fjoroft's theorem is due to

$$\left[\frac{\partial^2U}{\partial\left(\eta^a\right)^2}=T_{aa}+\lambda(x)g_{aa}\right]\neq 0, \text{for } \forall\eta^a_* \text{ choices} \quad (5)$$