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A Fractal Rindler-Regge Triangulation in the Hyperbolic Plane and Cosmic de Sitter Accelerated Expansion

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Abstract

The well known finite elements Regge calculus is transformed to a triangulation in the hyperbolic plane using fractal Rindler wedges as tiling elements. The final result is an expanding de Sitter hyperbolic, *i.e.* Gauss-Bolyai-Lobachevsky universe with dark energy and ordinary energy densities in full agreement with cosmic observations and measurements. In the course of obtaining this vital result, the work addresses fundamental points connected to a host of subjects, namely Hardy's quantum entanglement, an extension of Turing's machine to a transfinite version, the phenomenon of measure concentration in the context of Banach-like spaces with high dimensionality as well as the pioneering work on the relation between quantum entanglement and computational efficiency.

Keywords

Component, Hyperbolic Regge Calculus, Finite Elements in Cosmology, de Sitter Universe, E-Infinity Theory, Transfinite Turing Golden Mean Computer, Rindler Triangulation, Endophysics, Anti-Bethes Poof, Topological Quantum Entanglement, Gauss-Bolyai-Lobachevsky Geometry

1. Introduction

Regge calculus is a powerful finite element-like method in four dimensions applied to solve Einstein's highly nonlinear equations [1]-[13]. The method was further developed, modified and extended to a quantum gravity theory by many researchers [3]. In recent times the work of J. Ambjorn and R. Loll [13] deserves special attention [1]-[3] [13]. It was against this background that the present work was planned and executed by transforming Regge calculus to a fractal hyperbolic theory [14] which is used to solve the mystery of accelerated cosmic ex-

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pansion and the associated dark energy believed to be an expression of a non-vanishing cosmological constant [14]-[24]. In short, the spacetime of our theory is a hyperbolic fractal where quantum entanglement is replaced by a Hardy type natural topological zero measure entanglement [22] following E-infinity theory [24]-[35].

The paper is subdivided for efficient reading as follows: After the Introduction we review in Section 2 some important dualities between inverse electroweak couplings and the various energy sections as well as the extension of Turing's machine to a transfinite version via a highly structured golden mean ring as a number system [34]-[48]. Section 3 is a condensed account of the Regge "finite element" method and its adaption to the present work [35]. In Section 4 we discuss the ideas and hunches leading to a hyperbolic expanding de Sitter universe [35]. Subsequently in Section 5 the fundamental results are wrapped up with the analogy presented earlier in Section 2 [49]-[60]. Finally in Section 6 we present a short recapitulation of the paper and the relevance of quantum entanglement enhanced computation [61]-[63].

The present work is by no means self contained but we rely on an adequate list of references to fill in the inevitable gaps in a condensed presentation [20]-[63].

2. E-Infinity Dualities between Various Cosmic Energy Sections and Electroweak Couplings—The Transfinite Turing Machine

Let us consider here a most instructive and somewhat surprising duality between the coupling constant of the electroweak interaction (*i.e.* $\bar{\alpha}_1 = 60$, $\bar{\alpha}_2 = 30$, $\bar{\alpha}_3 = 9$ and $\bar{\alpha}_4 = 1$) [25]-[40] and the four fundamental sections of cosmic energy (*i.e.* pure dark energy 74%, dark matter 22%, ordinary energy 3% and ordinary matter 1%) [14]-[24]. The values mentioned above for $\bar{\alpha}_i$ are the exact E-infinity theoretical value which are very close to the experimental one [25]-[32] and also satisfies the reconstruction equation of $\bar{\alpha}_o = 137 + k_o = 137.082039325$ where $k_o = \phi^5\left(1-\phi^5\right)$, $\phi^5 = P$ (Hardy's quantum entanglement) and $\phi = \left(\sqrt{5}-1\right)/2$ [22]. Inserting in the said equation one finds [28] [33]

$$\begin{aligned}\bar{\alpha}_o &= \left(\bar{\alpha}_1\right)\left(1/\phi\right) + \bar{\alpha}_2 + \bar{\alpha}_3 + \bar{\alpha}_4 = \left(60\right)\left(1/\phi\right) + \left[\left(60/2\right) = 30\right] + 9 + 1 \\ &= \left(97 + k_o\right) + 30 + 10 = 137 + k_o = 137.082039325\end{aligned} \tag{1}$$

where $\left(1/\phi\right)$ is the E-infinity Clebsch factor [25]-[27] [39] and $k_o = \phi^5\left(1-\phi^5\right)$, where ϕ^5 is Hardy's probability of quantum-topological zero measure entanglement [22], could be expressed in terms of 'tHooft's order parameter of dimensional regularization, namely $k = \phi^3\left(1-\phi^3\right)$ to give $k_o = \left(k/2\right) - \left(k^2/4\right)$. On the other hand, the integer value of $\bar{\alpha}_o$ namely $\bar{\alpha}_o - k_o = 137$ could be found using the same reconstruction formula but after replacing $\bar{\alpha}_1$ and $\left(1/\phi\right)$ by the following values [14]-[24]

$$\begin{aligned}&\bar{\alpha}_1 = 60 && \leftrightarrow \ \gamma_E\left(D\right) = 74\% \text{ pure dark energy} \\ &\bar{\alpha}_2 = 30 && \leftrightarrow \ \gamma_m\left(D\right) = 22\% \text{ dark matter} \\ &\bar{\alpha}_3 + \bar{\alpha}_4 = 10 && \leftrightarrow \ \gamma_{E,m}\left(O\right) = 4\% \text{ ordinary energy and ordinary matter} \\ &\sum_1^4 \bar{\alpha}_i = 100 && \leftrightarrow \ \sum_1^4 \gamma_i = 100\%.\end{aligned} \tag{2}$$

In addition the E-infinity transfinite value the Clebsch factor [39] must be made rational, *i.e.*

$$1 + \phi = \left(1/\phi\right) \to 1 + \left(\frac{1}{2}\right) = 3/2 \tag{3}$$

We note on passing that the Clebsch factor is needed because the electroweak is based on three different Lie symmetry groups U(1), SU(2) and SU(3) [39] [40] which are made compatible by this very same factor. Inserting in our renormalization equation one finds

$$\bar{\alpha}_o\left(\text{integer}\right) = \left(3/2\right)74 + 22 + 3 + 1 = 111 + 22 + 4 = 111 + 26 = 137 \tag{4}$$

where 22 + 4 = 26 is the dimensionality of the Veneziano-Nambu bosonic string space [39]. It is also noteworthy that [11] [12] [19] [20]

$$\left(\bar{\alpha}_1 = 60\right) + \left(\bar{\alpha}_2 = 30\right) + \left(\bar{\alpha}_3 + \bar{\alpha}_4 = 9 + 1 = 10\right) = 100 \tag{5}$$

and similarly

$$\left(\gamma\left(\text{pure D}\right)=74\right)+\left(\gamma\left(\text{dark matter}\right)=22\right)+\left(3+1=4\right)=100. \tag{6}$$

From the preceding elementary analysis, it is obvious even to non-mathematicians that the mathematical scheme behind this calculation is trying to tell us something which transcends mathematics in the common naïve understanding as a mere tool and elevate it to something indistinguishable from physics, albeit the physics of what G. 'tHooft calls the elementary building block of space, time and matter [41]. Let us attempt in good faith to translate what is superficially numbers to what is equally superficially means physics. Since 75% corresponds to $\bar{\alpha}_1=60$ we feel strongly inclined to view pure dark energy as being of electromagnetic origin at the energy scale of the electroweak unification. Next $\bar{\alpha}_2=30$ corresponds to the weak force and again at the electroweak unification. Proceeding with our educated guesswork, it would seem quite evident that ordinary energy and matter must be clearly connected to the strong interaction $\bar{\alpha}_3=8+1$ and the quantum gravity coupling $\bar{\alpha}_4=1$ [25]-[30]. Considering that the topology required by high energy physics is not the classical "tame" topology but the Cantor set ramified "wild topology" [28]-[33], the preceding guesswork was not "wild" at all. However, the next guesswork may be closer to scientific speculation than to scientific-mathematical deduction. The point concerns the division of ordinary energy into non-luminous matter and luminous matter. Following the statistical analysis of cosmic energy measurement, it seems that 74% should be nearer to 73% while 22% is reported to be 23% [42]. This is not grave because we could discuss these things away once we take the next set of data seriously. These are 3.6% for non-luminous matter and 0.4% for luminous matter [42]. We can for instance see 0.4% as the difference between the core Hausdorff dimension $4+\phi^3=4.23606$ and the scaling dimension of 'tHooft fractal dimensional regularization spacetime $4-k=4.018033989=(10)\left(\phi^2\right)$ [17]. That means $\left(4+\phi^3\right)-\left(4-k\right)=\phi^3+k\cong 0.41\%$. Concentrating now on the larger picture we could say from reading the preceding dualities road map that the ordinary energy and matter inhabits Einstein's four dimensional spacetime, *i.e.* 3 + 1 = 4 dimensions with a corresponding about 4% energy density [14]-[24]. Dark matter on the other hand is uniquely connected to the compactified 26 − 4 = 22 dimensions of bosonic string spacetime. This leaves from our canonical 100 dimensions exactly 100 − (22 + 4) = 74 dimensions for pure dark energy to roam in [19]-[22]. The word is too complex to have been at the beginning and we say that with the outmost due respect to all religions who believe to the contrary. The number must have been there first because one, two, three seems infinitely simpler than a word although we admit that this depends crucially upon what we mean by the word 'word'. However, this is not entirely true because the integers are a very special kind of numbers and they may be infinite but there seems to be much larger infinity than this simple infinite number. It is here where our golden mean based number system constituting a transfinite version of A. Turing's machine [34] [43], a golden mean computer if you want [43], is infinitely more efficient to handle quantum problems in high energy physics and cosmology. In fact, this number theoretical wonder weapon is subtly related to our choice of units and it can be shown on deep reflection that Hans Bethe's famous spoof [44] is not really a spoof. We actually mean that the most convenient choice of units is a natural requirement of accurate quantum and micro physics expressed properly in the language of the most irrational number ϕ in all its combinations. In this sense, a part of naïve conception of number coincidence or numerology of the customary loose sense in which this word is misused although if we encounter such "numerology" [44]-[52] then it is just the pattern of a not yet understood fundamental physical or cosmological phenomena. That is also where the circle closes because fundamental physics and cosmology are fundamental mathematics [49]-[60]. In this sense, Sir A.S. Eddington's rational intuition was far ahead of his and our mastery of pure mathematical reasoning at the time [52] [56]-[60].

3. From Transfinite Triangulation in the Hyperbolic Plane to Dark Energy

In what follows, we give a concise outline for a general theory of quantum relativity and dark energy based on an exact transfinite extension of Regge triangulation calculus in the Poincare-Beltrami hyperbolic plane [39]. In our present theory, not only the basic idea is very easy to grasp but also the actual computation is surreal in its simplicity [14]. We suggest starting by recalling the geometrical shape of a circular region covered by triangular tiles which are a crossbreed between those used in Klein's modular curve and those utilized in Penrose tiling [19] [20] [39]. Compactifying these shapes would mean that we are approaching a fractal Penrose universe or in more stringent mathematical terminology, our circular region becomes an exact realization of a fiber bundle ma-

nifold [39] or in the language used by A. Connes, a noncommutative x quotient space [36]. It is thus neither difficult to imagine nor mathematical to reason that the following applies to the topology and geometry we are dealing with:

1) In the center of our Regge "finite elements" tessellation [1]-[5], the triangular tiles are rather ordinary classical shapes with straight sides. However, as we move from the center of the Beltrami-Poincaré plane outwardly the triangular straight lines start to deform and shrink hyperbolically [35] [39].

2) The circular boundary lies naturally at infinity and the area of the hyperbolic triangle tends towards a classical zero.

3) Adding uniform randomness to the so obtained fractal Regge tiling covering of the hyperbolic plane will result in a topology endowed with the golden mean inherited from the two legitimate parents, namely the Mauldin-Williams random triadic Cantor sets [28]-[34] and the golden mean proportioned kite and dart of Penrose's universe [28]-[34] [39].

4) Having went as far as we did, our basic spacetime blocks will be looked upon not as mere hyperbolic triangles but as a Rindler triangle, *i.e*. Rindler wedge with the familiar spear form containing the correlated part of spacetime, *i.e*. the entangled part while the moon section rear part contains the uncorrelated part of space which amounts to about 95.5% of the total "area" of the Rindler wedge [14] [35].

5) The genesis from classical triangular geometry to that of hyperbolic geometry may be likened to a very far extent with that of the logistic map which starts by period doubling classical bifurcation [48] [50] [52] but at approximately $\lambda = 3.8$ we have triple bifurcation leading according to Zarkovski's number theory to a quasi chaotic region [50] following the title of the famous J. Yorke paper "Period three implies chaos" and from $\lambda = 4.23606$ onwards we enter into a hyperbolic region [48] [50] [52]. It is incredible how generic this behaviour is when we consider that the Hausdorff dimensions of 'tHooft's fractal regularization space is $4 - k = 3.81966011$ while the core of E-infinity Cantorian spacetime is characterized by a Hausdorff dimension expectation of exactly $D = 4.23606$, *i.e*. $D = \lambda = 4.23606$ of the logistic map [57]. Needless to say that in all of our physical analysis quantum entanglement is de facto replaced by the notion of a zero measure topological Hardy type entanglement [22].

6) Also as we move away from the center with our normal triangle, we leave behind us slowly but surely an exo-physical universe and approach an endo-physical one [53] [54]. At infinity we cannot reconcile experiment with theory except when we recognize the exo-endo duality as uncovered in the pioneering work of D. Finkelstein, O.E. Roessler, H. Primas and their schools [53] [54].

7) Most importantly, we must realize that as our random space grows more "hyperbolic" it does not acquire a negative curvature but against naïve expectation, the curvature corresponding to the entire hyperbolic projection is not negative but positive.

8) Based on the above we have to conclude that our space corresponds to a de Sitter universe [39].

9) The preceding scenario of a de Sitter space entails not only an expanding but an accelerated expansion of the spacetime described above.

10) Taking points one to nine, we conclude that the cosmological constant is non-zero and that our universe is self similar hyperbolic fractal and that with a 95.5% volume concentration at the surface of the spacetime manifold. This is actually a mathematical theorem on volume measure concentration of convex highly dimensional Banach-like spaces.

4. Discussion and Our Initial Hunch That E-Infinity Leads to an Expanding Hyperbolic de Sitter University

It is quite instructive and rather helpful to a proper understanding of the hyperbolic fractal nature of our alleged de Sitter universe to explain how we arrived at what must have been just a hunch and foggy idea, at least initially. Any idea, no matter how simple is produced by a highly complex and as yet not really understood perceptive, combinatorial and imaginative brain process, but in the present case it may well have started by the author noticing the similarity between the Rindler wedge and the Regge triangulation used in recent times intensively in a computer oriented theory by Ambjorn and Loll [13]. In addition, we know that a normal triangle is fat in the spherical case but thin in the hyperbolic case. Therefore in the limit the area tends to zero. Furthermore as we could describe a flat space as space with a trivial axiom for parallel lines and a spherical space as zero parallel lines space, a hyperbolic space has infinitely many parallel lines just as a Cantorian space has infinitely many

dimensions. The only point which may not be straight forwards in our largely hand waving but accurate arguments is that we are claiming "Rindler" tiling space will be globally a de Sitter space although on account of its hyperbolicity, it should have a negative curvature, an anti de Sitter space [39]. However, noting the intrinsic randomness coupled with the Poincare-Beltrami projection and the basic averaging used, our space is not simply hyperbolic space but actually a very high dimensional Banach-like Cantorian fractal [14] [35] which is projected on the hyperbolic plane to give us this unique structure. In the case of any doubt, the reader needs to ponder the simplicity of the exact expressions of the Rindler areas involved. These are the arrow like area which is equal to the density of ordinary energy of the exo-physical universe [53] [54] and is given by [18]-[20]

$$\gamma(O) = \phi^5/2 = 1/22 \cong 4.5\% \tag{7}$$

and the circular section area behind the arrow which is equal to the total ark energy density of an endo-physical universe [53] [54], namely [18]-[20]

$$\gamma(D) = 5\phi^2/2 = 21/22 \simeq 95.5\% \tag{8}$$

If the proof of the pudding is in the eating, as a common English proverb says, then the above result and the associated energy-mass relationship [18]-[20]

$$E = E(O) + E(D) = \left(\phi^5/2\right) + \left(5\phi^2/2\right) = mc^2 = E(\text{Einstein}) \tag{9}$$

are sufficient proof for us that Einstein's formula is confirmed rather than refuted by the absence of measurable dark energy and the observation of accelerated cosmic expansion as well as our present reinterpretation of $E = mc^2$ as the sum of the measurable energy of the quantum particle $E(O)$ and the energy of the quantum wave $E(D)$ which we cannot measure because of the Hartle-Hawking collapse of Wheeler-Dewitt wave function of the cosmos [12] [39]. For the avoidance of any possible misunderstanding, we stress again that our dissection not only presupposes Einstein's beautiful formula but in conjunction with the cosmic endo-physical measurement, it confirms $E = mc^2$ theoretically and experimentally beyond a trace of a doubt.

5. Analogies between Analogies and into Deep 5 Dimensional de Sitter Waters

From the preceding analysis, we see that a most intriguing duality or maybe what we could call analogies among analogies as befitting a self affine universe exists, which we could represent as earlier in Section 2:

$$\begin{aligned}
&\bar{\alpha}_1 = 60 && \leftrightarrow \ \gamma_E(D) = 74\% \text{ pure dark energy, i.e. electromagnetic} \\
&\bar{\alpha}_2 = 30 && \leftrightarrow \ \gamma_m(D) = 22\% \text{ dark matter, i.e. weak force} \\
&\bar{\alpha}_3 + \bar{\alpha}_4 = 10 && \leftrightarrow \ \gamma_{E,m}(O) = 4\% \text{ ordinary energy and ordinary matter, i.e.} \\
& && \qquad \text{strong force plus quantum gravity} \\
&\sum_1^4 \bar{\alpha}_i = 100 && \leftrightarrow \ \sum_1^4 \gamma_i, \quad \text{i.e. the total sum is the same.}
\end{aligned} \tag{10}$$

However, this is not yet where things end for there are some even deeper interrelationships not only between time and matter but also between mater expressed as mass and space as well as spacetime dimensionality. It turns out that a unit of dimensionless energy corresponds to a unit of Cantorian spacetime and that tangible and directly detectable mass (m) enters into our equations as a three dimensional, *i.e.* space-like Cantor set given by the treble intersection ϕ^3. This means "measurable" mass is the inverse of the core Hausdorff dimension $D_H = 4 + \phi^3$ of our Cantorian de Sitter spacetime. By contrast dark matter enters into our equation as a five dimensional topological mass $m = 5$. Recalling that the interval topological speed of light is given by $v = \phi$, we see that we must have two kinds of energy densities, namely

$$\gamma_1 = \frac{1}{2}mv^2 = \frac{1}{2}\phi^3\phi^2 = \phi^5/2 \tag{11}$$

and

$$\gamma_2 = \frac{1}{2}5\phi^2 = 5\phi^2/2 \tag{12}$$

which then leads to the two expressions for ordinary energy $E(O) = \left(\phi^5/2\right)mc^2$ and dark energy

$E(D)=(5\phi^2/2)mc^2$. Naturally and provided we remember that ϕ is the Hausdorff dimension of the zero set while ϕ^2 is the Hausdorff dimension of the empty set, both expressions, *i.e.* γ_1 and γ_2 could be viewed as having been grown out of a five dimensional Kaluza-Klein theory with the only difference that γ_1 is a multiplicative, *i.e.* ϕ^5, while γ_2 is additive, *i.e.* $5\phi^2$ as explained in great detail in previous publications [17]-[22] [43].

6. Conclusion

The classical Regge method of relativity is transformed using Gauss-Bolyai-Lobachevsky hyperbolic geometry to the context of quantum relativity and cosmology to give information about the accelerated cosmic expansion and the associated dark energy density. The task was achieved by transforming the method to the hyperbolic plane and using Rindler-like triangles to form a random self similar fractal tiling akin to that of Klein-Penrose. The final conclusion is that our model effectively represents a fractal de Sitter universe with ordinary measurable exo-physical energy density of $E(O)=(\phi^5/2)mc^2=mc^2/22$ and a complimentary endo-physical dark energy [48] [49] density which cannot be measured directly and amounts to $E(D)=(5\phi^2/2)mc^2\simeq mc^2(21/22)$. We stress that all the preceding results could not be arrived at without the experimentally verified magnificent result of L. Hardy that a two quantum particle entanglement is given by ϕ^5 where $\phi=(\sqrt{5}-1)/2$ and the interpretation that this is due to zero measure topological entanglement [22]. In fact, we are tempted to speculate on the imaginative but quite realistic idea that computational efficiency may be related to entanglement and consequently to a hidden golden mean number system [61] [62].

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