

# Fast Algorithm for DOA Estimation with Partial Covariance Matrix and without Eigendecomposition

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## ABSTRACT

A fast algorithm for DOA estimation without eigendecomposition is proposed. Unlike the available propagation method (PM), the proposed method need only use partial cross-correlation of array output data, and hence the computational complexity is further reduced. Moreover, the proposed method is suitable for the case of spatially nonuniform colored noise. Simulation results show the performance of the proposed method is comparable to those of the existing PM method and the standard MUSIC method.

**Keywords:** Fast Algorithm, DOA Estimation, Subspace-Based Method

## 1. Introduction

DOA estimation of spatial signal source with an array of sensors has been an active research problem in array signal processing due to its wide applications in radar, sonar and so on. Many classical algorithms have been developed in the past thirties years [1-3], in particular, a class of subspace-based methods such as MUSIC [1], Root-MUSIC [2], and ESPRIT [3] are drawn more attractive due to its higher resolution performance but without multiple-dimension search computation. However, most of the subspace-based methods are required to compute the eigendecomposition of covariance matrix of array output data in order to obtain the so-called signal subspace or noise subspace, which its application is limited in case of larger number of array sensors. To avoid the computational load of the eigendecomposition of covariance matrix, in recent years, some fast algorithms for DOA estimation have been proposed for certain condition in the literature [4-7]. In particular, the propagation method (PM) [6,7] without eigendecomposition has been discussed due to lower computational load. However, the available PM method need use the whole covariance of array output data to obtain the propagation operator, therefore, the PM-based algorithm is only suitable to the presence of white Gaussian noise, and its performance will be degraded in spatial nonuniform colored noise [8].

In this paper, we present a modified PM algorithm for DOA estimation with an ULA, a different computation method for the propagation operator is given, which is only obtained by the partially cross-correlation of array output data. As a result, the proposed algorithm is computationally simpler than the available PM method [6]. Moreover, the proposed algorithm is suitable for the case of spatially nonuniform colored noise due to using the off-diagonal elements of array covariance matrix.

## 2. Proposed Method

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Let an uniform linear array of  $N$  sensors receive  $P$  narrow band signals impinging from the sources with unknown spatial DOA's  $\{\theta_1, \dots, \theta_P\}$ . The sensor array outputs can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t), t = 1, 2, \dots, L \quad (1)$$

where  $\mathbf{x}(t) = [x_1(t), \dots, x_N(t)]^T$  and

$\mathbf{A}(t) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_p)]$  denote the received array data vector and the array manifold matrix, respectively.  $\mathbf{s}(t) = [s_1(t), \dots, s_p(t)]^T$  and

$\mathbf{n}(t) = [n_1(t), \dots, n_N(t)]^T$  stand for the source waveform vector and sensor noise vector.

$$\mathbf{a}(\theta_i) = [1, \exp(j2\pi d \sin \theta_i / \lambda), \dots, \exp(j2\pi(N-1)d \sin \theta_i / \lambda)]^T$$

is the steering vector and  $(\cdot)^T$  denotes the transposition of a matrix. The sensors noise is assumed to be a zero mean spatially and temporally white Gaussian process.

The following assumptions are made in the subsequent developments:

(A1) The number of sources  $P$  is known a priori and the number of sensors satisfies  $N > 2P$ .

(A2) The set of  $P$  steering vectors is linearly independent and the  $P$  signal sources are statistically independent of each other.

Under the assumption of  $N > 2P$ , the array manifold matrix  $\mathbf{A}$  can be partitioned as follows:

$$\mathbf{A} = [\mathbf{A}_1^T, \mathbf{A}_2^T, \mathbf{A}_3^T]^T \quad (2)$$

where  $\mathbf{A}_i$ ,  $i=1,2,3$  is a matrix with dimension  $P \times P$ ,  $P \times P$ ,  $(N-2P) \times P$ , respectively.

Based on the above partition of  $\mathbf{A}$  and using Equation (1), the following partially cross-correlation matrices of the array output are defined as:

$$\mathbf{R}_{12} = E[\mathbf{x}(t)(1:P,:) \mathbf{x}^H(t)((P+1):2P,:)] = \mathbf{A}_1 \mathbf{R}_{ss} \mathbf{A}_2^H \quad (3)$$

$$\mathbf{R}_{31} = E[\mathbf{x}(t)((2P+1):N,:) \mathbf{x}^H(t)(1:P,:)] = \mathbf{A}_3 \mathbf{R}_{ss} \mathbf{A}_1^H \quad (4)$$

$$\begin{aligned} \mathbf{R}_{32} &= E[\mathbf{x}(t)((2P+1):N,:) \mathbf{x}^H(t)((P+1):2P,:)] \\ &= \mathbf{A}_3 \mathbf{R}_{ss} \mathbf{A}_2^H \end{aligned} \quad (5)$$

where  $\mathbf{x}(t)(i:j,:)$  takes the  $i$ -th to  $j$ -th row of  $\mathbf{x}(t)$ ,  $(\cdot)^H$  denotes the Hermitian transpose and  $\mathbf{R}_{ss} = E[\mathbf{s}(t) \mathbf{s}^H(t)]$  is the source signal covariance matrix.

Under the assumption of (A2), both  $\mathbf{R}_{ss}$  and  $\mathbf{A}_i$ ,  $i = 1,2$  are invertible matrices, therefore, the following equation holds:

$$\mathbf{R}_{32} \mathbf{R}_{12}^{-1} \mathbf{A}_1 = \mathbf{A}_3 \mathbf{R}_{ss} \mathbf{A}_2^H (\mathbf{A}_2^H)^{-1} \mathbf{R}_{ss}^{-1} \mathbf{A}_1^{-1} \mathbf{A}_1 = \mathbf{A}_3 \quad (6)$$

Similarly, the following equation can also be obtained:

$$\mathbf{R}_{31} \mathbf{R}_{21}^{-1} \mathbf{A}_2 = \mathbf{A}_3 \quad (7)$$

Combines Equation (6) and Equation (7) and yields:

$$\mathbf{R}_{32} \mathbf{R}_{12}^{-1} \mathbf{A}_1 + \mathbf{R}_{31} \mathbf{R}_{21}^{-1} \mathbf{A}_2 = 2\mathbf{A}_3 \quad (8)$$

Equivalently, the above equation can be written as:

$$[\mathbf{R}_{32} \mathbf{R}_{12}^{-1} \quad \mathbf{R}_{31} \mathbf{R}_{21}^{-1} \quad -2\mathbf{I}_{N-2P}] \mathbf{A} = \mathbf{0} \quad (9)$$

where  $\mathbf{I}_{N-2P}$  is an identical matrix with dimension  $N-2P$  and  $\mathbf{0}_{N-2P}$  a zero matrix, respectively.

Let  $\mathbf{Q}^H = [\mathbf{R}_{32} \mathbf{R}_{12}^{-1} \quad \mathbf{R}_{31} \mathbf{R}_{21}^{-1} \quad -2\mathbf{I}_{N-2P}]$  and we have

$$\mathbf{Q}^H \mathbf{A} = \mathbf{0} \quad (10)$$

which implies that the columns of  $\mathbf{Q}^H$  form a basis of the null space of  $\mathbf{A}$ , *i.e.*,

$$\mathbf{Q}^H \mathbf{a}(\theta_k) = \mathbf{0} \quad (k = 1, 2, \dots, P)$$

Using the estimated matrix  $\hat{\mathbf{Q}}$  from the finite array output data in real application, similar to the MUSIC-based method, we may form the following spatial spectrum function and obtain the estimates of  $\theta_k$  ( $k = 1, 2, \dots, P$ ) from  $P$  spectrum peaks:

$$f(\theta) = \frac{1}{\|\hat{\mathbf{Q}}^H \mathbf{a}(\theta)\|^2} \quad (11)$$

where

$$\mathbf{a}(\theta_i) = [1, \exp(j2\pi d \sin \theta_i / \lambda), \dots, \exp(j2\pi(N-1)d \sin \theta_i / \lambda)]^T$$

Alternately, using the Root-MUSIC-based method [2] to get the estimation value of  $\theta_k$  directly.

It is worthy to note that the estimation of  $\hat{\mathbf{Q}}$  need not any eigendecomposition, and the noise information is not involved in  $\hat{\mathbf{Q}}$ , therefore, the proposed method can be used in the case of spatial non-uniform noise or spatial band limited noise [8].

Regarding major computational complexity, the number of multiplications for calculating  $\mathbf{Q}$  includes  $P(N-P)L$  in the cross-correlation computation in Equations (3)-(4) and  $O(P^3)$  for the inversion of  $\mathbf{R}_{12}$  in Equations (6)-(7), respectively, while the MUSIC method [1] involves  $N^2L$  for covariance matrix computation and  $O(N^3)$  for the eigenvalue decomposition of the resultant covariance matrix. On the other hand, the number of multiplications for computing the propagation operator  $\mathbf{Q}$  in the available PM method [6] is  $NPL + O(P^3)$ . Apparently, the computational complexity of the proposed algorithm is lower than those of the MUSIC method and the propagation method.

### 3. Simulation Results

In the first simulation, the experiments are performed with an uniform linear array (ULA) with  $N = 10$  sensors

and half-wavelength inter-element spacing. Two equally powered narrow-band sources with DOA's  $\theta_1 = -7^\circ$  and  $\theta_2 = 8^\circ$  impinge on the array and the two sources are statistically independent of each other. Let  $L = 100$  and the average results for 200 independent runs are used to evaluate the estimation performance of different methods.

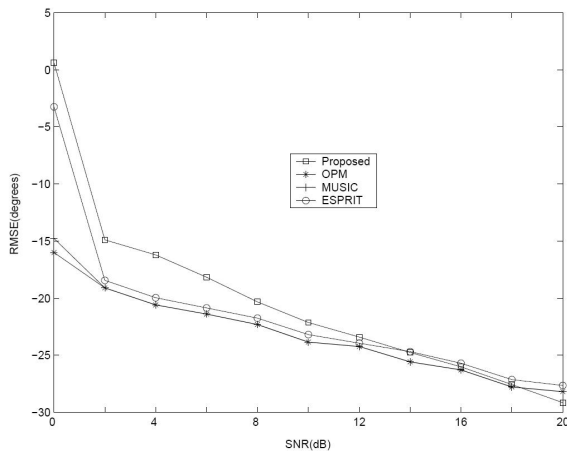
**Figures 1-2** show the RMSEs (root-mean-square error) for DOA estimation versus different SNR conditions. The results using the conventional MUSIC algorithm [1], the Standard ESPRIT method [3] and the available orthonormalisation PM (OPM) [7] method are also included to contrast the performance of the proposed algorithm. It is seen that the estimation accuracy of the proposed method is comparable to those of the subspace-based MUSIC method, ESPRIT and OPM method at all the SNRs.

In the second experiment, we assume  $[\theta_1, \theta_2] = [-5^\circ, 6^\circ]$  and other parameters is same as the above experiment, however, the spatially nonuniform independent sensor noise has the following covariance matrix:  $\mathbf{Q} = \sigma_n^2 \text{diag}[1, 1.2, 4, 12, 11, 3, 0.4, 10, 2]$ .

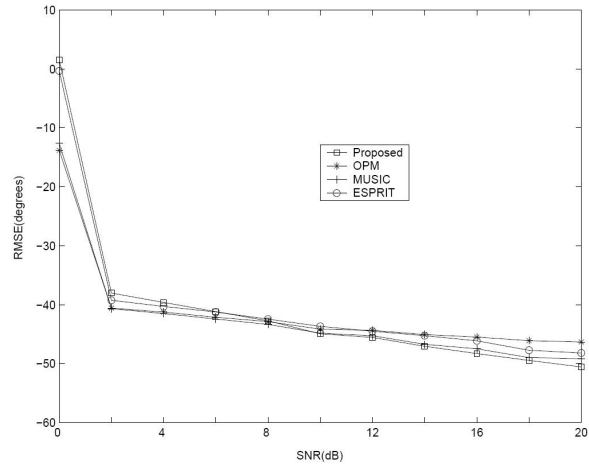
The definition of signal-noise-ratio (SNR) is the same as that in [8]. The estimation results of the proposed method for 10 independent runs in the case of SNR = 5dB are plotted in **Figure 3**. It is seen that the proposed method can resolve accurately two closely spatial sources in the presence of spatially nonuniform noise.

**4. Conclusions**

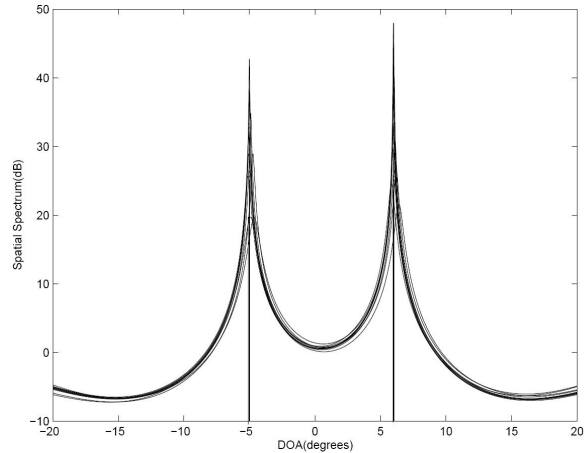
A computationally efficient algorithm for DOA estimation with an uniform linear array has been presented. The partial cross-correlation of array outputs is utilized to compute the propagation operator, and hence the proposed method is suitable to the case of spatially non-uniform noise. Finally, it is shown that the estimation performance of the



**Figure 1. RMSEs of  $\theta_1$  versus SNR.**



**Figure 2. RMSEs of  $\theta_2$  versus SNR.**



**Figure 3. Spectrum of proposed method in spatially non-uniform colored noise.**

proposed algorithm is comparable to those of the available PM method as well the conventional MUSIC method at sufficiently higher SNR conditions.

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