

The Study of Green Compression Strength of a Green Sand Mould Using Statistical Approach

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Received 10 July 2014; revised 30 August 2014; accepted 14 September 2014

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Abstract

The study of green compression strength of a green sand mould using statistical approach has been undertaken. Empirically generated data in National Metallurgical Development Centre, Jos Sand Testing Laboratory were used for the study. Coefficient of correlation, coefficients of determination and coefficient of multiple determinations were used to explain the relationship existing between the two independent variables of clay and moisture content and green compression strength, the dependent variable. The study showed that the coefficient of determination for $Y_s: X_1$ was 0.88 while the coefficient of correlation was 0.94, coefficient of determination for $Y_s: X_2$ was 0.90 while the coefficient of correlation was 0.95 and the coefficient of multiple determination was 0.72; these coefficients assisted tremendously in the study of green compression strength. A mathematical model was developed for the prediction of green compression strength; it was tested and proved to be a good estimation tool for estimating green compression strength values on the foundry shop floor. The study has clearly shown that statistical approach is a good tool for studying green compression strength of green sand moulds.

Keywords

Moulds, Statistical Approach, Study, Sands, Green Compression

1. Introduction

Jain [1] in his work said that “the green compression strength of a green sand mould depends on grain size, shape and distribution of sand grains, type and amount of clay or other binder and moisture content”. The researcher took time to explain how each of these affects green compression strength. Green compression strength

increases with moisture until a point is reached where further increase in moisture leads to decrease in compression strength. Green compression strength decreases with coarse grains but increases with increase in fineness of sand grains. Green compression strength increases with increase in clay content of the green sand mould. The findings of Jain agree with several authors who also confirm the above explanation [2]-[4]. Jain [1] tried to assess the combined effect of grain size variation and moisture content on green compression strength of green sand mould. He said that as moisture is increased, green compression strength also increases until a point is reached where further increase leads to decrease in green compression. He noted that as moisture increases coarse grained sand moulds have higher green compression strength than fine-grained sand moulds. Jain in his work also noted that as moisture is increased angular shaped grain sand moulds lose their green compression strength more than round grained sand moulds. Green compression strength is very important in the preparation of moulds. One reason is because it determines the mouldability of a sand mixture. For a sand mixture to be mouldable it must have a green compression strength value of 1.5 psi (10.34 kPa) and above [5]. Low green compression strength in green moulds may result in defective and unsound casting [6]. This is because there will be erosion, collapse, and damage of the mould wall and this will result in defects like rat tails, sand inclusions, incomplete molten metal flow, rough surfaces etc. This explains how important this property is in mould production and is always monitored in sand casting [4]-[7].

Statistical approach of the study of this important property will give in-depth information on how to produce good moulds. This fact has been upheld by several authors [8]-[12] who argued that statistical models will provide models for the quick estimation of green compression strength provided the clay content and moisture content is known. This will save the time used in setting up instrument for tests to determine green compression strength. They also agreed that statistical approaches will bring out clearly the nature of relationships that exists between green compression strength the dependent variable, clay content and moisture content the independent variables [8]-[12].

The objective of the work is to study green compression strength of a green sand mould using statistical approach.

2. Materials and Method

2.1. Materials

The materials used for this work were obtained from empirically generated data on various moulding sand systems at the sand testing laboratory of the National Development Centre, Jos. This particular data was generated from green compression strength tests on green moulds using Bacita foundry sand, clay binder and moisture (H₂O). Universal strength testing machines, manual green compression strength tester, sand rammer, mixer, measuring cylinders were used during the work as equipment. The main variables in the work were clay (binder) which was varied from 1.5% to 7% and moisture (water) which was varied from 2% to 8%. Other variables like mixing time, number of rams, sand (grain size and shape) were all kept constant. **Table 1** shows the data that was used for this study.

Table 1. Green compression strength data with variation in clay and moisture content.

S/NO	%Clay Content Variation X_1	% Moisture content Variation X_2	Green Compression Strength, kN/m ² (Y_s)
1	1.5	2	11.72
2	2	3	17.24
3	3	4	21.38
4	4	5	22.75
5	5	6	24.13
6	6	7	24.82
7	7	8	33.74

2.2. Method

2.2.1. Multiple Regression Model Development

The basic two variable models (one dependent and one independent variable) is

$$Y = a + bx \quad (1)$$

Which can be solved using the normal equations thus:

$$\sum Y = an + b \sum x \quad (2)$$

$$\sum XY = a \sum X + b \sum X^2 \quad (3)$$

From this can be developed models with more than two variables and this is illustrated below using a 3 variable model (one dependent and two independent variables, Y , X_1 , and X_2)

$$Y = a + b_{1X_1} + b_{2X_2} \quad (4)$$

Which can be solved by the normal equations for a three variable model, as follows:

$$\sum Y = an + b_1 \sum X_1 + b_2 \sum X_2 \quad (5)$$

$$\sum X_1 Y = a \sum X_1 + b_1 \sum X_1^2 + b_2 \sum X_1 X_2 \quad (6)$$

$$\sum X_2 Y = a \sum X_2 + b_1 \sum X_1 X_2 + b_2 \sum X_2^2 \quad (7)$$

The line of best fit gives way to a plane of best fit, b_1 is the slope of the plane along the X_1 axis, b_2 is the slope along the X_2 axis and the plane cuts the Y axis at “ a ”. The aim of adding to the simple two variable models is to improve the fit of the data. The closeness of fit is measured by the coefficient of multiple determination R^2 for which the general formula and a useful computational formula is given below

$$R^2 = \frac{a \sum Y + b_1 \sum X_1 Y + b_2 \sum X_2 Y - \frac{(\sum Y)^2}{n}}{\sum Y^2 - \frac{(\sum Y)^2}{n}} \quad (8)$$

Table 2 gives the calculation of separate regressions with green compression strength as Y , clay content as X_1 and moisture content as X_2 .

2.2.2. For Regression Y , on X_1 (Clay Content %)

$$b_{X_1} = \frac{n \sum X_1 Y_p - \sum X_1 \sum Y_p}{n \sum X_1^2 - (\sum X_1)^2} = \frac{7 \times 713.07 - 28.5 \times 155.81}{7 \times 141.25 - (28.5)^2} = 3.12$$

$$a_{X_1} = \frac{\sum Y_p}{n} - \frac{b_{X_1} \sum X_1}{n} = \frac{155.81}{7} - \frac{3.12 \times 28.5}{7} = 9.56$$

The regression equation for the relationship of clay content and green compression strength value of the green mould is

$$Y_{X_1} = a_{X_1} + b_{X_1} X_1 = 9.56 + 3.12 X_1$$

$$Y_{X_1} = 9.56 + 3.12 X_1 \quad (9)$$

The coefficient of correlation for this relationship is

$$r_{X_1} = \frac{n \sum X_1 Y_p - \sum X_1 \sum Y_p}{\sqrt{n \sum X_1^2 - (\sum X_1)^2} \times \sqrt{n \sum Y_p^2 - (\sum Y_p)^2}} \quad (10)$$

Substituting the values in **Table 2** in Equation (10) we have

Table 2. Calculation of separate regressions with green compression strength as Y_s .

n	Y_s	Y_s^2	X_1	X_1^2	X_2	X_2^2	X_1Y_s	X_2Y_s	X_1X_2
1	11.72	137.36	1.5	2.25	2	4	17.58	23.44	3.0
2	17.24	297.22	2	4	3	9	34.48	51.72	6.0
3	21.38	457.10	3	9	4	16	64.14	85.52	12.0
4	22.78	518.93	4	16	5	25	91.12	113.9	20.0
5	24.13	582.26	5	25	6	36	120.65	144.78	30.0
6	24.82	616.03	6	36	7	49	148.92	173.74	42.0
7	33.74	1138.39	7	49	8	64	236.18	269.92	56.0
Σ	155.81	3747.29	28.5	141.25	35	203	713.07	863.02	169.0

$$r_{X_1} = 0.94$$

$r_{X_1}^2 = 0.88$ i.e. coefficient of determination for Y_s : X_1 .

2.2.3. For Regression of Y_s on X_2 (% Moisture Content)

$$b_{X_2} = \frac{n \sum X_2 Y_p - \sum X_2 \sum Y_p}{n \sum X_2^2 - (\sum X_2)^2} = \frac{6041.14 - 5453.35}{7 \times 203 - (35)^2} = 3$$

$$a_{X_2} = \frac{\sum Y_p}{n} - \frac{b_{X_2} \sum X_2}{n} = \frac{155.81}{7} - \frac{3 \times 35}{7} = 7.26$$

The regression equation for the relationship of moisture content with green compression strength value of the green mould is

$$Y_{sX_2} = a_{X_2} + b_{X_2} X_2 = 7.26 + 3X_2 \tag{11}$$

The coefficient of correlation for this relationship is

$$r_{X_2} = \frac{n \sum X_2 Y_p - \sum X_2 \sum Y_p}{\sqrt{n \sum X_2^2 - (\sum X_2)^2} \times \sqrt{n \sum Y_p^2 - (\sum Y_p)^2}} = \frac{587.79}{618.90} = 0.95$$

$r_{X_2}^2 = 0.90$ i.e. coefficient of determination for Y_s : X_2 .

2.2.4. The Multiple Regression (Y_s : X_1 and X_2)

The multiple regression calculations are carried out using the three variable normal Equations (4)-(7) with substitution with values from **Table 2** new set of equations are derived as follows

$$155.81 = 7a + 28.5b_1 + 35b_2 \tag{12}$$

$$713.07 = 28.5a + 141.25b_1 + 169b_2 \tag{13}$$

$$863.02 = 35a + 169b_1 + 203b_2 \tag{14}$$

Solving these three equations simultaneously gave

$$a = 9.34$$

$$b_1 = 3.07$$

$$b_2 = 0.03$$

Now substituting the model equation for three variables in Equation (4), this new model equation is obtained

$$Y_s = 9.34 + 3.07X_1 + 0.03X_2 \tag{15}$$

This mathematical model is derived with the combined influence of clay content and moisture content all taken into account and therefore can be used to predict the green compression strength of the green sand mould. Table below shows empirical values with calculated values using the mathematical model.

2.2.5. Coefficient of Multiple Determination, R^2

From Equation (9)

$$R^2 = \frac{202.17}{279.18} = 0.72$$

3. Results and Discussion

3.1. Results

The various coefficients of determination can now be summarized

$$r_{x_1}^2 = 0.88$$

$$r_{x_2}^2 = 0.90$$

$$R^2 = 0.72$$

Equations (9), (11) and (15) are the developed regression models for simple and multiple linear regression relationships existing between green compression strength and the two variables of clay content and moisture content in the green mould.

Table 2 is the result of separate calculation of the regression with green compression strength as the dependent variable

Table 3 gives the empirical values of green compression strength against mathematical model values and predicted values of green compression strength.

3.2. Discussion

The various coefficients are interpreted as follows:

$r_{x_1}^2$: this indicates that about 88% of the variation in green compression is caused by variation in clay content this is a major influence and the influence is positive which means that an increase in clay content % will bring about an increase in green compression strength.

$r_{x_2}^2$: this indicates that 90% of the variation in green compression strength is caused by variation in moisture content %; this is a major positive influence that moisture content % has on green compression strength which means as moisture content increases green compression strength also increases.

Table 3. Empirical values and mathematical model values.

n	X_1 (%)	X_2 (%)	Y_s (Empirical values of Green compression strength) kN/m ²	Y_c (Mathematical model values of Green compression strength) kN/m ²
1	1.5	2	11.72	14.00
2	2.0	3	17.24	15.57
3	3.0	4	21.38	18.67
4	4.0	5	22.75	21.77
5	5.0	6	24.13	24.87
6	6.0	7	24.82	27.97
7	7.0	8	33.74	31.07
8	8.0	9	-	34.17
9	9.0	10	-	37.27

R^2 : this shows the combined influence of the two variables of clay content and moisture content %. The influence is a major influence and it is positive. 72% of the changes in green compression strength values in green sand mould are brought about by the combined influence of clay content % and moisture content %.

The individual correlation between clay content and green compression strength is given by the coefficient of correlation r_{x_1} and the value is 0.94 and is a positive relationship. This means that 94% of changes in green compression strength are caused by clay content. The individual influence on green compression strength is therefore significant. Equation (9) which is a model for predicting the green compression strength of green sand mould based on the influence of clay content only can be used assuming the only influence on green compression is coming from clay content. The model will not be correct if there are other variables involved in determining green compression strength. The correlation agrees completely with existing theories as clay content increases green compression strength also increases. This explains why clay that does not have sufficient binding ability in small quantity in moulding mixtures becomes mouldable when the quantity of clay is increased. Literatures reviewed for this work also support the above strong correlation between clay content and green compression strength [13] [14].

Moisture content has a very strong positive correlation with green compression strength of green sand moulds. The correlation is 0.95 which means that 95% of the increase in green compression strength will be caused by increase in moisture content of the moulding mixture if the moisture content is the only variable. The importance of moisture is also observed practically while preparing moulds. Moulds with moisture content below optimum do tend to be lacking in green compression strength. Literatures that have been reviewed for this work have also shown that moisture content increase do lead to increase in green compression strength until an optimum point is reached where further increase leads to a decrease in green compression strength [6] [13] [14].

The combined effect of clay content and moisture content as well as the influence on green compression strength has been given by the coefficient of multiple determination and interpreted above. The established relationship from the coefficient of multiple determination agrees with what is obtained practically and empirically generated results both clay content and moisture content increase green compression strength of moulding mixture making it mouldable [6] [13] [14]. Jain has made this observation in his work [1]. Equation (15) is developed based on the influence of both clay content and moisture content variation on green compression strength of green sand moulds as indicated by the coefficient of multiple determination R^2 . This mathematical model is derived with the combined influence of clay content and moisture content all taken into account and therefore can be used to predict the green compression strength of the green sand mould. On the strength of this **Table 3** was produced.

Table 3 gives the empirical values of green compression strength against green compression strength values calculated using the developed mathematical model. A look at **Table 3** shows that the difference between the empirical values of green compression strength and the calculated green compression strength values is not so much. The table showed that green compression strength has been predicted for 8% clay content and 9% moisture content, and 9% clay content and 10% moisture content and the result of the prediction is within a reasonable range as can be seen in the table. The results in **Table 3** have shown that the developed model can be a quick method or tool for the prediction of green compression strength on the shop floor. Given the importance of this parameter the green compression strength of a moulding mixture can be estimated using the model and the information can be used in predicting the mouldability of the moulding mixture. As stated earlier green compression strength of 10.34 kPa and above is required for mouldability of green sand moulds [4]-[6] [14]. Statistical approach have indeed proven to be a good method for the study of green compression strength as can be seen above it had provided a step by step analysis of the relationship between green compression strength and the two variables of clay content and moisture content. Their interactive influence on green compression strength has also been studied through the use of multiple coefficient of determination R^2 . No wonder it had been supported by many researchers as a useful research tool [8]-[14].

4. Conclusions

The study of green compression strength of a green sand mould using statistical approach has been successfully carried out and the following conclusions are from the study:

1) Statistical approach has been found to be a good method of studying green compression strength of green sand mould.

2) The coefficients of correlation, coefficients of determination and coefficient of multiple determinations are great tools for studying the relationship between the variables (independent and dependent variables).

3) The study has shown the positive strong relationship that exists between green compression strength of the dependent variable and the two independent variables of clay content and moisture content. The study showed a coefficient of multiple determination of 0.72 which translates to 72% of the variation in green compression strength coming from the combined effect of clay content and moisture content.

4) The study developed a mathematical model which can be used for the prediction and estimation of green compression strength in green sand moulds. This clearly confirms the crucial nature of statistical approach in the study of green sand moulds.

Acknowledgements

The authors hereby extend their sincere appreciation to the staff of the Sand Testing Laboratory of NMDC Jos who assisted with the generation of this data. We also acknowledge your effort for the enormous amount of data bank that now exists in that Laboratory on Nigerian foundry sands.

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