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Fitting of precipitation in 49 European capitals from 1901 to 1998 using random walk

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ABSTRACT

Mathematical modeling of precipitation is an important step to understand the precipitation patterns, and paves the way to possibly predict the precipitation. In this study, we attempt to use the random walk model to fit the annual precipitation in 49 European capitals from 1901 to 1998. At first, we used the simplest random walk model to fit the precipitation walk, which is the conversion of recorded precipitations into ±1 format, and then we used a more complex random walk model to fit the recorded precipitations. The results show that the random walk models can fit both precipitation walk and recorded precipitation. Thus this study provides a model to describe the precipitation patterns during this period in these cities.

Keywords: European Capital; Fitting; Precipitation; Random Walk

1. INTRODUCTION

Mathematical description of precipitation is important, not only because it can help up better understand the precipitation pattern, but also it can provide a tool to possibly predict the precipitation. However, a mathematical model is not so easy to build because the precipitation is related to enormous factors, which lead to the difficulty to use a deterministic model to describe the precipitation. Technologically, the deterministic model is generally based on the cause-consequence relationship, which can be modeled as differential equations. As many cause-consequence relationships coexist in precipitation process, one would have great difficulty to separate various cause-consequence relationships notably because we cannot conduct a control experiment to determine each cause-consequence relationship. Still we have many unknown factors, which are not possible to include in a deterministic model. Computationally, a deterministic model with many factors would have great difficulty to fit the recorded data.

In this context, we may consider alternative models, such as stochastic models, of which the random walk is an important model to describe natural phenomena, for examples, stock pattern [1] and temperature change [2, 3]. Thus an interesting question raised here is whether the random walk model can describe the precipitation along with other powerful deterministic models? Practically, we cannot find any clearly visible patterns in year-to-year precipitations if we scrutinize the recorded precipitations over year in certain geographic location. The non-patterned precipitations would provide us with the opportunity to use the random walk model to fit.

To answer this question, we use the random walk model to fit the annual precipitations in 49 European capitals from 1901 to m 1998 in this study.

2. MATERIALS AND METHODS

2.1. Data

The precipitations recorded in these 49 European capitals from 1901 to 1998 were obtained from the website of Oak Ridge National Laboratory [4], and their latitudes and longitudes were determined using Get Lat Lon [5] in order to define the precise precipitations according to the 0.5˚ by 0.5˚ latitude and longitude grid-box.

2.2. Precipitation Walk

We use the simplest random walk model, which starts at zero and moves by ±1 with equal probability at each step [6]. As the precipitation is in decimal format, thus we convert the precipitation into ±1 format as precipitation walk. Technically, when the precipitation at certain year is higher than its previous year, we classify it as 1, otherwise we classify it as –1, and then we add them together as the random walk does.

2.3. Generating Random Walk

We use the SigmaPlot [7] with different seeds to generate random sequences ranged from –1 to 1, and then we classify a random number as 1 if it is larger than its previous one and as –1 if it is smaller than its previous one. Thereafter we add the classified ±1 as random walk.

2.4. Searching for Seed

To the best of our knowledge, there is no algorithm available to find the right seed, which produces the best fit between random walk and observed data. However, this is not a problem with current computational technique, because we can simply search all the seeds in searching space and compare their outcomes.

2.5. Fitting Recorded Precipitation

Hereafter, we use a more complicated random walk model [8] to fit the recorded precipitation, which is in decimal format. In plain words, the simplest random walk comes from tossing of double-sided coin, while this random walk could be regarded as tossing of dice, which can be not only six-sided but as many as the decimal data. In this way, we generate random numbers, and add them to construct the random walk, and the fitting is again to search the best seed that generates best fit.

2.6. Comparison

For determining the best seed, we compare the least squared errors between precipitation walk and random walk, and between recorded precipitation and random precipitation generated from different seeds.

3. RESULTS AND DISCUSSION

Table 1 shows how we construct a precipitation walk and its corresponding random walk. For the precipitation walk, we have the follows: 1) the starting point is the annual precipitation in 1901, 847 mm (cell 2, column 2), and this starting point corresponds zero in sense of precipitation walk (cell 2, column 4); 2) the annual precipitation in 1902 is 737 mm (cell 3, column 2), which is smaller than the first one, 847 mm (cell 2, column 2), so we assign –1 as precipitation step (cell 3, column 3), 3) the precipitation walk is –1 (0 + (–1)) (cell 3, column 4), and 4) the similar computation is applied to all the data in columns 2, 3, and 4.

For the random walk, we have the follows: 1) a good seed we found is 6.98078, and this seed generates a series of random numbers (column 5), 2) the first random number, 0.36795 (cell 2, column 5), is considered as the starting point corresponding to 0 in random walk (cell 2, column 7), 3) the second random number, –0.74132 (cell 3, column 5), is smaller than the first random number, 0.36795 (cell 2, column 5), so we assign –1 (cell 3, column 6), 4) the random walk is –1 (0 + (–1)) (cell 3, column 7), and 5) the similar procedure is applied to all the data in columns 5, 6, and 7. In the same manner, we construct the precipitation walk and random walk.

The figures in the left side of **Figure 1** show the fittings of precipitation walk in 7 European capitals using random walk model. As can be seen, the curve generated by random walk generally passes through the precipitation walk. Theoretically, the chance for a completely perfect fitting of precipitation walk is an extremely rare event. In our case, there are 98 annual precipitations, thus the completely perfect fitting has the chance of $(1/2)^{98}$ theoretically, which is extremely small. Clearly this probability is very difficult to achieve in limited time because the space of our search is limited to one million of seed. So the fitting results in the left side of **Figure 1** suggest that a good seed can be relatively easily found, thus we consider that the random walk can describe the precipitation walk, although we cannot compare our results with other results because the other models do not set an equal-sized step.

Actually we can view the precipitation walk, which is the conversion from its annual precipitation, as the trend of recorded precipitation. This is so because this trend answers the very basic question of whether the precipitation at certain year is larger (1) or smaller (–1) than its previous year.

Yet, the cities in **Figure 1** cross the whole Europe, thus there would be uncountable factors affecting the precipitations, but the random walk still can fit them. This furthermore suggests that the random walk can describe the precipitation patterns in terms of precipitation walk.

Table 2 shows how we use a random walk model to fit the recorded annual precipitation, here we only need to construct the random precipitation: 1) the starting point is the first recorded annual precipitation, which is 847 mm (cell 2 in column 2 and column 4), 2) the seed for **Table 2** is 1.31923, 3) the first random number generated by the seed is 64.17878 (cell 3, column 3), 4) we add this value to the previous precipitation datum (847) resulting in 911.17878 mm (cell 3, column 4), and 5) along this procedure, we get the random precipitation in column 4.

The figures in the right side of **Figure 1** display the fittings of recorded precipitation with random precipitation in 7 European capitals. In these figures, the precipitation demonstrates very remarkable fluctuations along the time course, which do not show any clear sign of visible pattern. This is the basis for conducting random-

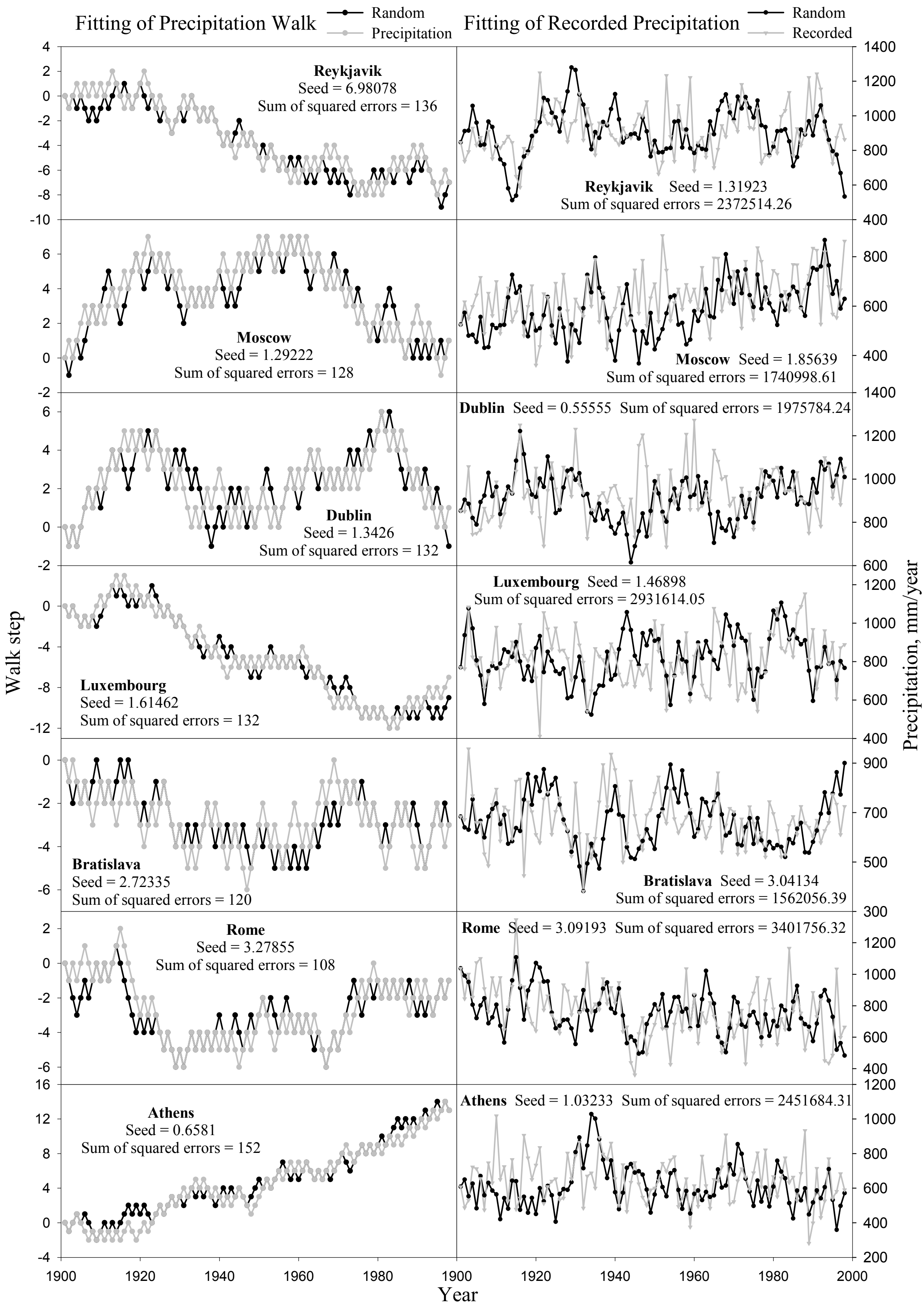

Figure 1. Comparison of precipitation walk with random walk and of recorded precipitation with random precipitation in 7 European capitals from 1901 to 1998.

Table 1. Conversion of recorded precipitation into precipitation walk and generation of random walk for precipitation in Reykjavík from 1901 to 1998.

Year	Precipitation mm/year	Precipitation Step	Precipitation Walk	Generated Random Number	Random Step	Random Walk
1901	847		0	0.36795		0
1902	737	–1	–1	–0.74132	–1	–1
1903	793	1	0	0.03941	1	0
1904	929	1	1	–0.05685	–1	–1
1905	830	–1	0	0.27137	1	0
1906	876	1	1	–0.54282	–1	–1
1907	807	–1	0	–0.85436	–1	–2
1908	911	1	1	0.02900	1	–1
1909	752	–1	0	–0.31867	–1	–2
1910	833	1	1	0.67880	1	–1
…	…	…	…	…	…	…
1991	1243	1	–4	–0.03583	–1	–6
1992	1155	–1	–5	0.53701	1	–5
1993	918	–1	–6	–0.31605	–1	–6
1994	841	–1	–7	–0.44354	–1	–7
1995	723	–1	–8	–0.63220	–1	–8
1996	875	1	–7	–0.87762	–1	–9
1997	947	1	–6	–0.80406	1	–8
1998	863	–1	–7	–0.10839	1	–7

The seed for generation of random numbers is 6.98078 using SigmaPlot.

Table 2. Generation of recorded precipitation into random precipitation in Reykjavík from 1901 to 1998.

Year	Recorded Precipitation mm/year	Generated Random Number	Random Precipitation mm/year
1901	847		847.00000
1902	737	64.17878	911.17878
1903	793	2.96367	914.14245
1904	929	143.23057	1057.37302
1905	830	–97.43795	959.93507
1906	876	–130.95922	828.97585
1907	807	4.73200	833.70785
1908	911	132.97379	966.68164
1909	752	–32.00623	934.67541
1910	833	–110.27817	824.39724
…	…	…	…
1991	1243	112.18813	998.44105
1992	1155	60.58394	1059.02499
1993	918	–92.92398	966.10101
1994	841	–105.85999	860.24102
1995	723	–64.91220	795.32882
1996	875	–20.30670	775.02212
1997	947	–106.11975	668.90237
1998	863	–136.10377	532.79860

The seed for generation of random numbers is 1.31923 using SigmaPlot.

walk to fit the precipitation data. As can be seen, the random model did generate the curves similar to the recorded annual precipitations. This is particularly important, because the recorded annual precipitation presents a very difficult pattern to be fit by any other models.

Due to the limitation of space, we did not present our fittings for all 49 European capitals, not only because seven European capitals in **Figure 1** come from almost each corner and the central of Europe, but also because one can make graphic observation with the seeds listed in **Table 3** using SigmaPlot to generate fitted curves to compare with recorded ones.

The data used in our study spanned for almost a century. With 98 annual precipitations, we would find a good estimate because the seed is the only model parameters for random walk. However, the uncertainty would increase if we use a more complicated model, which contains more model parameters.

Actually, the literature search does not show many studies using random walk to fit the historical data. There could be several reasons for the lack of use of random walk. 1) Our fitting technique mainly concen trated in deterministic models in the past while the fitting using random walk is largely ignored. 2) Although computational technique advanced significantly, to fully try each possibility of tossing of coin is still very diffi-

Table 3. Model parameters (seeds) and fitted results for fitting precipitation change in 49 European capitals from 1901 to 1998 using random walk model.

State	Capital	Fitting of Precipitation Walk		Fitting of Recorded Precipitation	
		Seed	Sum of Squared errors	Seed	Sum of Squared errors
Albania	Tirana	5.89746	160	3.09458	4480260.88
Andorra	Andorra la Vella	0.76994	120	2.04962	3762379.49
Armenia	Yerevan	5.11706	140	1.03292	1654018.73
Austria	Vienna	5.29984	120	6.26795	1854605.22
Azerbaijan	Baku	1.83503	144	1.17243	580310.28
Belarus	Minsk	3.91196	124	6.87272	1136251.86
Belgium	Brussels	1.32931	128	3.79933	2793049.95
Bosnia and Herzegovina	Sarajevo	2.34269	120	3.89254	4488970.73
Bulgaria	Sofia	1.47405	136	3.50057	1574839.07
Croatia	Zagreb	4.75272	132	1.62778	4530028.69
Cyprus	Nicosia	0.96966	124	1.68592	1959226.37
Czech Republic	Prague	2.41744	156	1.75624	1180350.59
Denmark	Copenhagen	2.67161	136	1.75624	979500.29
Estonia	Tallinn	3.68127	112	0.75372	1719360.48
Finland	Helsinki	1.17561	119	3.88438	1601835.51
France	Paris	0.71257	137	0.01458	1671903.49
Georgia	Tbilisi	4.04021	143	1.03292	1876296.69
Germany	Berlin	1.53962	100	8.04385	1267406.51
Greece	Athens	0.65810	152	1.03233	2451684.31
Hungary	Budapest	2.24208	124	2.26215	3274954.55
Iceland	Reykjavík	6.98078	136	1.31923	2372514.26
Ireland	Dublin	1.34260	132	0.55555	1975784.24
Italy	Rome	3.27855	108	3.09193	3401756.32
Latvia	Riga	1.63481	120	2.23707	982200.71
Liechtenstein	Vaduz	5.32749	140	7.48196	6544143.92
Lithuania	Vilnius	2.64178	128	3.80609	1817826.21
Luxembourg	Luxembourg	1.61462	132	1.46898	2931614.05
Republic of Macedonia	Skopje	2.61614	132	2.21326	2413733.26
Malta	Valletta	2.29461	124	1.03233	1683481.23
Moldova	Chişinău	1.02884	116	0.53868	1976370.09
Monaco	Monaco	0.07616	128	3.50057	5805776.86
Montenegro	Podgorica	0.64299	184	3.89254	7373847.85
Netherlands	Amsterdam	1.11756	96	1.8384	2103720.16
Norway	Oslo	5.15239	128	0.49518	2762908.96
Poland	Warsaw	6.38046	136	3.73365	1099078.36
Portugal	Lisbon	5.62135	128	1.94803	3988957.69
Romania	Bucharest	2.96749	124	2.82657	1634074.60
Russia	Moscow	1.29222	128	1.85639	1740998.61
San Marino	San Marino	2.86281	160	1.65197	2562453.91
Serbia	Belgrade	5.30232	145	2.82657	2285551.27
Slovakia	Bratislava	2.72335	120	3.04134	1562056.39
Slovenia	Ljubljana	1.07884	136	3.89254	5247413.75
Spain	Madrid	1.81411	132	0.56663	1181419.10
Sweden	Stockholm	9.22779	120	3.24016	834669.37
Switzerland	Bern	6.31144	152	1.03233	7547487.27
Turkey	Ankara	2.57067	147	1.6555	633833.41
Ukraine	Kiev	1.24473	124	6.08046	1628394.92
United Kingdom	London	9.43653	136	3.79933	1801453.26
Vatican City	Vatican City	3.28152	108	3.09193	3401756.32

cult. For example, the complete fitting of 98 annual precipitations require 316, 912, 650, 057, 057, 350, 374, 175, 801,344 trials $(1/2)^{98}$, which is a very difficult task because not only the time for computation is very considerable but also it is doubt whether the current Monte-Carlo algorithm could generate so many different seeds. We found the trend within 1,000,000 trials, so the probability is extremely small $(1/2)^{91}$, which does suggest the precipitation trend.

On the other hand, it is not clear whether we can satisfyingly use the seed, which fits the first 49-year precipitations, to predict the second 49-year precipitations, not only because this research area is far less studied but also it is difficult for any deterministic model to use the parameters obtained from fitting of first half data to predict the second half data. Moreover, the focus in this study is to see whether a random walk can fit the recorded precipitation, which should be the first step for the predictions that need many studies in the future.

Currently, no results are available from other models on fitting the precipitation of these cities for comparison. However, our results are encouraging because the random walk model provides a way to describe the precipitation pattern.

In conclusion, the results show that the random walk model can describe the annual precipitation pattern.

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