

Existence of Nonoscillatory Solutions of a Class of Nonlinear Dynamic Equations with a Forced Term

Shanliang Zhu, Xinli Zhang

College of Mathematics and Physics, Qingdao University of Science and Technology, Qingdao, China

Email: zhushanliang77@163.com

Received June 26, 2012; revised July 19, 2012; accepted July 30, 2012

ABSTRACT

In this paper, we consider the following forced higher-order nonlinear neutral dynamic equation

$$\left[x(t) + p(t)x(\tau(t)) \right]^{\Delta^m} + f\left(t, x(\tau_1(t)), x(\tau_2(t)), \dots, x(\tau_k(t))\right) = q(t), t \in [t_0, \infty)_{\mathbb{T}}$$

on time scales. By using Banach contraction principle, we obtain sufficient conditions for the existence of nonoscillatory solutions for general $p(t)$ and $q(t)$ which means that we allow oscillatory $p(t)$ and $q(t)$. We give some examples to illustrate the obtained results.

Keywords: Dynamic Equation; Higher Order; Non-Oscillation; Time Scale; Neutral

1. Introduction

The study of dynamic equations on time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his Ph.D. thesis in 1988 in order to unify continuous and discrete analysis [1]. Dynamic equations on time scales have an enormous potential for modelling a variety of applications such as in population dynamics. Several authors have expounded on various aspects of this new theory, see the survey paper by Agarwal, Bohner, O'Regan and Peterson [2] and references cited therein. A book on the subject of time scales, by Bohner and Peterson [3], summarizes and organizes much of the time scale calculus. We refer also to the last book by Bohner and Peterson [4] for advances in dynamic equations on time scales.

Recently, much attention is concerned with oscillation and nonoscillatory solutions for dynamic equations on time scales [5-12].

In Li and Zhang [6] studied the existence of nonoscillatory solutions to neutral dynamic equation

$$\left[x(t) + p(t)x(\tau(t)) \right]^{\Delta^n} + f_1\left(t, x(\tau_1(t))\right) - f_2\left(t, x(\tau_2(t))\right) = 0.$$

Li, Han, Sun and Yang [10] established the existence of nonoscillatory solutions to the following second order neutral delay dynamic equation

$$\left[x(t) + p(t)x(\tau_0(t)) \right]^{\Delta\Delta} + q_1(t)x(\tau_1(t)) - q_2(t)x(\tau_2(t)) = e(t).$$

Zhang and Sun [13] studied the existence of nonoscillatory solutions of the forced nonlinear difference equation

$$\Delta\left(x_n - p_n x_{\tau(n)}\right) + f\left(n, x_{\sigma(n)}\right) = q_n.$$

Zhou and Zhang [14] obtained some sufficient conditions of nonoscillatory solutions for the higher order delay difference equation with positive and negative coefficients

$$\Delta^m\left(x_n + cx_{n-k}\right) + p_n x_{n-r} - q_n x_{n-l} = 0.$$

Lu [15] obtained some necessary and sufficient conditions for the existence of nonoscillatory solutions for the following first order neutral equation

$$\left(x(t) - \sum_{i=1}^m p_i(t)x(h_i(t)) \right)' + \sum_{j=1}^n f_j\left(t, x(g_j(t))\right) = Q(t).$$

Motivated by these works, in this paper, we consider the higher-order nonlinear neutral dynamic equation

$$\left[x(t) + p(t)x(\tau(t)) \right]^{\Delta^m} + f\left(t, x(\tau_1(t)), x(\tau_2(t)), \dots, x(\tau_k(t))\right) = q(t), \quad (1)$$

where $t \in [t_0, \infty)_{\mathbb{T}}$, $m \in \mathbb{N}$, $\sup \mathbb{T} = \infty$. We assume $p, q \in C_{rd}\left([t_0, \infty)_{\mathbb{T}}, \mathbb{R}\right)$ and allow $p(t)$ and $q(t)$ to be oscillatory. $\tau, \tau_i \in C_{rd}\left([t_0, \infty)_{\mathbb{T}}, \mathbb{T}\right)$ satisfy $\lim_{t \rightarrow \infty} \tau(t) = \lim_{t \rightarrow \infty} \tau_i(t) = +\infty, i = 1, 2, \dots, k$, $f(t, u_1, u_2, \dots, u_k) \in C\left(\mathbb{T} \times \mathbb{R}^k, \mathbb{R}\right)$ is nondecreasing for

u_j and $u_1 f(t, u_1, u_2, \dots, u_k) \geq 0$ for
 $u_1 u_j \geq 0, j = 1, 2, \dots, k$.

We recall x is a solution of Equation (1) provided that $x(t) + p(t)x(\tau(t))$ is m times differentiable, and x satisfies Equation (1). A solution x of Equation (1) is called nonoscillatory if x is of one sign when eventually.

2. Existence Results for Nonoscillatory Solutions

In this section, we establish sufficient conditions of the existence of nonoscillatory solutions for Equation (1). First we define a sequence of functions $g_k(s, t), k \in \mathbb{N}_0$ as follows:

$$g_0(s, t) \equiv 1, g_{k+1}(s, t) = \int_t^s g_k(\sigma(\tau), t) \Delta \tau.$$

For $g_k(s, t)$, we have the following Lemma.

Lemma 2.1. (Li and Zhang [6]) Assume s is fixed, and let $g_k^\Delta(s, t)$ be the derivative $g_k(s, t)$ with respect to t . Then

$$g_k^\Delta(s, t) = -g_{k-1}(s, t), k \in \mathbb{N}, t \in \mathbb{T}^k.$$

Let BC denote the Banach space of all bounded functions $x(t), t \geq t_0$, with the norm $\|x\| = \sup_{t \geq t_0} |x(t)| < \infty$. We will use the following assumptions:

(i) there exists $\alpha > 0$ such that

$$\begin{aligned} &|f(t, u_1, u_2, \dots, u_k) - f(t, v_1, v_2, \dots, v_k)| \\ &\leq L(t) \max_{1 \leq i \leq k} |u_i - v_i| \end{aligned}$$

for $t \geq t_0$ and $0 \leq u_i, v_i \leq \alpha, j = 1, 2, \dots, k$, where $L(t) \in C_{rd}(\mathbb{T}, \mathbb{T})$;

(ii) $\int_{t_0}^\infty g_{m-1}(\sigma(s), 0)L(s)\Delta s < \infty$;

(iii) $\int_{t_0}^\infty g_{m-1}(\sigma(s), 0)|q(s)|\Delta s < \infty$;

(iv) there exists $p \in \left(\frac{1}{2}, 1\right)$ such that

$$|p(t)| \leq 1 - p, t \geq t_0;$$

(v) there exists $p \in (-1, 0]$ such that

$$p \leq p(t) \leq 0, t \geq t_0;$$

(vi) there exist $p_1, p_2 \in (-\infty, -1)$ such that

$$p_1 \leq p(t) \leq p_2, t \geq t_0;$$

(vii) there exists $p \in (0, 1)$ such that

$$0 < p(t) \leq p, t \geq t_0;$$

(viii) there exist $p_1, p_2 \in (1, +\infty)$ such that

$$p_1 \leq p(t) \leq p_2, t \geq t_0.$$

Theorem 2.1. Assume that (i), (ii), (iii) and (iv) hold, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. Choose d_1, c_1 such that $0 < d_1 < (2p-1)\alpha$ and $d_1 + (1-p)\alpha < c_1 < p\alpha$. Let

$c = \min\left\{\frac{p}{2}\alpha, p\alpha - c_1, c_1 - d_1 - (1-p)\alpha\right\}$. There exists a

$t_1 \geq t_0$ large enough such that when $t \geq t_1$, we have $\tau(t), \tau_i(t) \geq t_0, i = 1, 2, \dots, k$ and

$$\int_{t_1}^\infty g_{m-1}(\sigma(s), 0)[\alpha L(s) + |q(s)|]\Delta s \leq c. \quad (2)$$

By condition (i) and the hypotheses on $f(t, u_1, \dots, u_k)$, for any $t \geq t_0, 0 \leq u_i \leq \alpha, i = 1, 2, \dots, k$, we have

$$f(t, u_1, \dots, u_k) \leq \alpha L(t). \quad (3)$$

We define a set $\Omega \subset BC$ as follows:

$$\Omega = \{x \in BC : d_1 \leq x(t) \leq \alpha, t \geq t_0\}. \quad (4)$$

Then Ω is a closed, bounded and convex subset of BC . Define a map Γ on Ω as follows:

$$\begin{aligned} (\Gamma x)(t) &= \begin{cases} c_1 - p(t)x(\tau(t)) + (-1)^{m-1} \int_t^\infty g_{m-1}(\sigma(s), t) \\ \cdot [f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) - q(s)] \Delta s, \\ t \geq t_1, \\ (\Gamma x)(t_1), t_0 \leq t \leq t_1. \end{cases} \end{aligned}$$

First, we shall show that for any $x \in \Omega$ and $t \geq t_0$, $(\Gamma x)(t) \in \Omega$. For any $x \in \Omega$ and $t \geq t_1$, by (2), (3) and (4), we get

$$\begin{aligned} (\Gamma x)(t) &\geq c_1 - p(t)x(\tau(t)) - \int_t^\infty g_{m-1}(\sigma(s), t) \\ &[f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) + |q(s)|]\Delta s \\ &\geq c_1 - |p(t)|x(\tau(t)) \\ &\quad - \int_t^\infty g_{m-1}(\sigma(s), 0)[\alpha L(s) + |q(s)|]\Delta s \\ &\geq c_1 - (1-p)\alpha - [c_1 - d_1 - (1-p)\alpha] = d_1. \end{aligned}$$

Furthermore, we have

$$\begin{aligned} (\Gamma x)(t) &\leq c_1 - |p(t)|x(\tau(t)) - \int_t^\infty g_{m-1}(\sigma(s), t) \\ &[f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) + |q(s)|]\Delta s \\ &\leq c_1 + (1-p)\alpha \\ &\quad + \int_t^\infty g_{m-1}(\sigma(s), 0)[\alpha L(s) + |q(s)|]\Delta s \\ &\leq c_1 + (1-p)\alpha + p\alpha - c_1 = \alpha. \end{aligned}$$

Hence when $t \geq t_0$, we obtain $d_1 \leq (\Gamma x)(t) \leq \alpha$, so $(\Gamma x)(t) \in \Omega$ for any $x \in \Omega$.

Next, we show that Γ is a contraction mapping on Ω . In fact for any $x, y \in \Omega$ and $t \geq t_1$, we have

$$\begin{aligned} & (\Gamma x)(t) \\ & \leq |p(t)| |x(\tau(t)) - y(\tau(t))| \\ & + \int_t^\infty g_{m-1}(\sigma(s), t) \left| f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) \right. \\ & \left. - f(s, y(\tau_1(s)), y(\tau_2(s)), \dots, y(\tau_k(s))) \right| \Delta s \\ & \leq (1-p) \|x-y\| + \int_t^\infty g_{m-1}(\sigma(s), 0) L(s) \|x-y\| \Delta s \\ & \leq \left(1-p + \frac{p}{2}\right) \|x-y\| = \left(1 - \frac{p}{2}\right) \|x-y\|. \end{aligned}$$

Since $0 < 1 - \frac{p}{2} < 1$, we conclude that Γ is a contraction mapping on Ω . By the Banach fixed point theorem, Γ has a fixed point $x^* \in \Omega$. By Lemma 2.1, it is easy to see that $x^*(t)$ is a bounded nonoscillatory solution of the Equation (1). This completes the proof of Theorem 2.1.

Theorem 2.2. Assume that (i), (ii), (iii) and (v) hold, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. Choose $\beta > 0$, such that $\beta \leq \frac{2(1+p)\alpha}{3}$. Obviously

$(1+p)\alpha - \beta \geq \frac{\beta}{2}$. There exists a $t_1 \geq t_0$ sufficiently large such that when $t \geq t_1$, we have $\tau(t), \tau_i(t) \geq t_0, i = 1, 2, \dots, k$ and

$$\int_{t_1}^\infty g_{m-1}(\sigma(s), 0) [\alpha L(s) + |q(s)|] \Delta s \leq \frac{\beta}{2}.$$

We define a closed, bounded and convex subset Ω of BC as follows:

$$\Omega = \left\{ x \in BC : \frac{\beta}{2} \leq x(t) \leq \alpha, t \geq t_0 \right\}.$$

Define a map Γ on Ω as follows:

$$\begin{aligned} & (\Gamma x)(t) \\ & = \begin{cases} \beta - p(t)x(\tau(t)) + (-1)^{m-1} \int_t^\infty g_{m-1}(\sigma(s), t) \\ \cdot [f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) - q(s)] \Delta s, \\ t \geq t_1, \\ (\Gamma x)(t_1), t_0 \leq t \leq t_1. \end{cases} \end{aligned}$$

The rest of the proof is similar to that of Theorem 2.1 and hence omitted. The proof is complete.

Theorem 2.3. Assume that (i), (ii), (iii) and (vi) hold. τ has the inverse $\tau^{-1} \in C(\mathbb{T}, \mathbb{T})$, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. We choose positive constants M_1, M_2, β , such that $M_2 \leq \alpha, -p_1 M_1 < \beta < (-p_2 - 1)M_2$. Let

$$c = \min \left\{ \frac{\beta + p_1 M_1}{p_1} p_2, (-p_2 - 1)M_2 - \beta, -\frac{1 + p_2}{2} \alpha \right\}.$$

There exists a $t_1 \geq t_0$ large enough such that when $t \geq t_1$, we have $\tau^{-1}(\tau_i(t)) \geq t_0, i = 1, 2, \dots, k$, and

$$\int_{\tau^{-1}(t)}^\infty g_{m-1}(\sigma(s), 0) [\alpha L(s) + |q(s)|] \Delta s \leq c.$$

We define a closed, bounded and convex subset Ω of BC as follows:

$$\Omega = \{x \in BC : M_1 \leq x(t) \leq M_2, t \geq t_0\}.$$

Define a map $\Gamma : \Omega \rightarrow BC$ as follows:

$$\begin{aligned} & (\Gamma x)(t) = \\ & \begin{cases} -\frac{\beta}{p(\tau^{-1}(t))} \frac{x(\tau^{-1}(t))}{p(\tau^{-1}(t))} + \frac{(-1)^{m-1}}{p(\tau^{-1}(t))} \int_{\tau^{-1}(t)}^\infty g_{m-1}(\sigma(s), t) \\ \cdot [f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) - q(s)] \Delta s, \\ t \geq t_1, \\ (\Gamma x)(t), t_0 \leq t \leq t_1. \end{cases} \end{aligned}$$

First, we shall show that $\Gamma\Omega \subset \Omega$. For any $x \in \Omega$ and $t \geq t_1$, note that

$$\begin{aligned} & (\Gamma x)(t) \\ & \geq -\frac{\beta}{p(\tau^{-1}(t))} \\ & + \frac{1}{p(\tau^{-1}(t))} \int_{\tau^{-1}(t)}^\infty g_{m-1}(\sigma(s), t) [\alpha L(s) + |q(s)|] \Delta s \\ & \geq -\frac{\beta}{p_1} + \frac{1}{p_2} \int_{\tau^{-1}(t)}^\infty g_{m-1}(\sigma(s), 0) [\alpha L(s) + |q(s)|] \Delta s \\ & \geq -\frac{\beta}{p_1} + \frac{(\beta + p_1 M_1) p_2}{p_1 p_2} = M_1 \end{aligned}$$

and

$$(\Gamma x)(t) \leq -\frac{\beta}{p_2} - \frac{M_2}{p_2} - \frac{(-p_2 - 1)M_2 - \beta}{p_2} = M_2.$$

Thus $(\Gamma x)(t) \in \Omega$ for $x \in \Omega$, this is $\Gamma\Omega \subset \Omega$.

Next, we show that Γ is a contraction mapping on Ω . In fact for any $x, y \in \Omega$ and $t \geq t_1$, we have

$$\begin{aligned}
 (\Gamma x)(t) &\leq -\frac{1}{p(\tau^{-1}(t))} \left| x(\tau^{-1}(t)) - y(\tau^{-1}(t)) \right| \\
 &\quad - \frac{1}{p(\tau^{-1}(t))} \int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s), t) \left| f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) \right. \\
 &\quad \left. - f(s, y(\tau_1(s)), y(\tau_2(s)), \dots, y(\tau_k(s))) \right| \Delta s \\
 &\leq -\frac{1}{p_2} \|x - y\| - \frac{1}{p_2} \int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s), 0) L(s) \|x - y\| \Delta s \\
 &\leq \frac{1}{p_2} \left(-1 + \frac{1 + p_2}{2} \right) \|x - y\| = \frac{p_2 - 1}{2p_2} \|x - y\|.
 \end{aligned}$$

Since $0 < \frac{p_2 - 1}{2p_2} < 1$, we conclude that Γ is a contraction mapping on Ω .

By the Banach fixed point theorem, Γ has a fixed point $x^* \in \Omega$. By Lemma 2.1, it is easy to see that $x^*(t)$ is a bounded nonoscillatory solution of the Equation (1). This completes the proof of Theorem 2.3.

Theorem 2.4. Assume that (i), (ii), (iii) and (vii) hold, then equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. Choose $\beta > 0$, such that $p\alpha < \beta < \alpha$. Let

$c = \min \left\{ \alpha - \beta, \frac{\beta - p\alpha}{2} \right\}$. There exists a $t_1 \geq t_0$ large enough such that when $t \geq t_1$, we have

$$\tau(t), \tau_i(t) \geq t_0, i = 1, 2, \dots, k,$$

and

$$\int_{t_1}^{\infty} g_{m-1}(\sigma(s), 0) [\alpha L(s) + |q(s)|] \Delta s \leq c.$$

Easily to know

$$\Omega = \left\{ x \in BC : \frac{\beta - p\alpha}{2} \leq x(t) \leq \alpha, t \geq t_0 \right\}$$

is a closed, bounded and convex subset of BC . Define a map $\Gamma : \Omega \rightarrow BC$ as follows:

$$\begin{aligned}
 (\Gamma x)(t) &= \begin{cases} \beta - p(t)x(\tau(t)) + (-1)^{m-1} \int_t^{\infty} g_{m-1}(\sigma(s), t) \\ \cdot [f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) - q(s)] \Delta s, \\ t \geq t_1, \\ (\Gamma x)(t_1), t_0 \leq t \leq t_1. \end{cases}
 \end{aligned}$$

The rest of the proof is similar to that of Theorem 2.1 and hence omitted. The proof is complete.

Theorem 2.5. Assume that $\alpha \geq 1$, (i), (ii), (iii) and (viii) hold. τ has the inverse $\tau^{-1} \in C(\mathbb{T}, \mathbb{T})$, then Equation (1) has a bounded nonoscillatory solution which is bounded away from zero.

Proof. We choose β , such that $1 < \beta < p_1$. Let

$c = \min \left\{ \beta - 1, \frac{p_1 - \beta}{2}, \frac{\beta - 1}{2} \alpha \right\}$. There exists a $t_1 \geq t_0$

large enough such that when $t \geq t_1$, we have $\tau^{-1}(\tau_i(t)) \geq t_0, i = 1, 2, \dots, k$, and

$$\int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s), 0) [\alpha L(s) + |q(s)|] \Delta s \leq c.$$

We define a closed, bounded and convex subset Ω of BC as follows:

$$\Omega = \left\{ x \in BC : \frac{p_1 - \beta}{2p_2} \leq x(t) \leq \frac{p_1 + \beta}{2p_1}, t \geq t_0 \right\}.$$

Define a map $\Gamma : \Omega \rightarrow BC$ as follows:

$$\begin{aligned}
 (\Gamma x)(t) &= \begin{cases} \frac{\beta}{p(\tau^{-1}(t))} - \frac{x(\tau^{-1}(t))}{p(\tau^{-1}(t))} + \frac{(-1)^{m-1}}{p(\tau^{-1}(t))} \int_{\tau^{-1}(t)}^{\infty} g_{m-1}(\sigma(s), t) \\ \cdot [f(s, x(\tau_1(s)), x(\tau_2(s)), \dots, x(\tau_k(s))) - q(s)] \Delta s, \\ t \geq t_1, \\ (\Gamma x)(t_1), t_0 \leq t \leq t_1. \end{cases}
 \end{aligned}$$

The rest of the proof is similar to that of Theorem 2.3 and hence omitted. The proof is complete.

Remark 2.1. Theorem 1 - 5 not only unify the known results for differential and difference equations corresponding to Equation (1), but also generalize and improve essentially the existing results of [13-15] using the time scale theory.

We will give the following examples to illustrate our main results.

Example 2.1. Consider the forth-order dynamic equation on the time scale $\mathbb{T} = \{q^n : n \in \mathbb{N}_0, q > 1\}$

$$\begin{aligned}
 &\left(x(t) - \frac{1}{\sqrt{q}} x\left(\frac{t}{q}\right) \right)^{\Delta^4} + \frac{(1 - \sqrt{q})(q+1)^2(q^2+1)}{q^{10}} \\
 &\quad \times \frac{(q^2+q+1)}{t^2(t+q^3)^3} x^3\left(\frac{t}{q^3}\right) \\
 &= 2 \frac{(1 - \sqrt{q})(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^5}
 \end{aligned} \tag{5}$$

Here $m = 4, p(t) = -\frac{1}{\sqrt{q}}, \tau(t) = \frac{t}{q}$,

$$q(t) = 2 \frac{(1 - \sqrt{q})(q+1)^2(q^2+1)(q^2+q+1)}{q^{10}t^5},$$

$$L(t) = 3\alpha^2 \frac{\left[(1 - \sqrt{q})(q+1)^2(q^2+1)(q^2+q+1) \right]}{q^{10}t^2(t+q^3)^3}.$$

By the definition of $g_k(s, t)$, we have

$$\begin{aligned} & g_{4-1}(\sigma(s), 0)L(s) \\ & \leq s^3 \frac{3\alpha^2(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10} s^2 (s+q^3)^3} \\ & \leq \frac{3\alpha^2(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10} s^2}, \\ & g_{4-1}(\sigma(s), 0)|q(s)| \\ & \leq s^3 2 \frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10} s^5} \\ & = 2 \frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10} s^2}. \end{aligned}$$

Then

$$\begin{aligned} & \int_{t_0}^{\infty} \frac{3\alpha^2(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10} s^2} \Delta s < \infty, \\ & \int_{t_0}^{\infty} 2 \frac{(\sqrt{q}-1)(q+1)^2(q^2+1)(q^2+q+1)}{q^{10} s^2} \Delta s < \infty. \end{aligned}$$

It is obvious that Equation (5) satisfies all conditions of Theorem 2.2. Hence Equation (5) has a bounded nonoscillatory solution which is bounded away from zero. In fact $x(t) = 1 + \frac{1}{t}$ is a solution of Equation (5). However, to the best of our knowledge, there are no results dealing with the existence of nonoscillatory solutions for Equation (5).

Example 2.2. Consider the third-order dynamic equation on the time scale $\mathbb{T} = \mathbb{N}$

$$(x(t) - 2x(t-1))^{\Delta^3} + \frac{1}{2^t} x(t-1) = \frac{11(2^t) + 16}{8(2^{2t})}, \quad (6)$$

$$t \geq 2.$$

Here $m = 3, p(t) = -2, \tau(t) = t - 1,$

$$f(t, x(\tau_1(t))) = \frac{1}{2^t} x(t-1)$$

and $q(t) = \frac{11(2^t) + 16}{8(2^{2t})}$. It is easy to see that all condi-

tions of Theorem 2.3 are satisfied and hence Equation (6) has a bounded nonoscillatory solution which is bounded away from zero. In fact $x(t) = 1 + \frac{1}{2^t}$ is a solution of Equation (6).

3. Acknowledgements

The authors sincerely thank the reviewers for their valuable suggestions and useful comments that have lead to the present improved version of the original manuscript.

REFERENCES

- [1] S. Hilger, "Analysis on Measure Chains—A Unified Approach to Continuous and Discrete Calculus," *Results in Mathematics*, Vol. 18, No. 1-2, 1990, pp. 18-56.
- [2] R. Agarwal, M. Bohner, D. O'Regan and A. Peterson, "Dynamic Equations on Time Scales: A Survey," *Journal of Computational and Applied Mathematics*, Vol. 141, No. 1-2, 2002, pp. 1-26. [doi:10.1016/S0377-0427\(01\)00432-0](https://doi.org/10.1016/S0377-0427(01)00432-0)
- [3] M. Bohner and A. Peterson, "Dynamic Equations on Time Scales: An Introduction with Applications," Birkhäuser, Boston, 2001.
- [4] M. Bohner and A. Peterson, "Advances in Dynamic Equations on Time Scales," Birkhäuser, Boston, 2003. [doi:10.1007/978-0-8176-8230-9](https://doi.org/10.1007/978-0-8176-8230-9)
- [5] B. G. Zhang and S. L. Zhu, "Oscillation of Second-Order Nonlinear Delay Dynamic Equations on Time Scales," *Computers & Mathematics with Applications*, Vol. 49, No. 4, 2005, pp. 599-609.
- [6] Q. L. Li and Z. Zhang, "Existence of Solutions to Nth Order Neutral Dynamic Equations on Time Scale," *Electronic Journal of Differential Equations*, Vol. 2010, No. 151, 2010, pp. 1-8. <http://ejde.math.txstate.edu> or <http://ejde.math.unt.edu>, [ftp ejde.math.txstate.edu](http://ftp.ejde.math.txstate.edu)
- [7] D. X. Chen, "Oscillation and Asymptotic Behavior for Nth-Order Nonlinear Neutral Delay Dynamic Equations on Time Scales," *Acta Applicandae Mathematicae*, Vol. 109, No. 3, 2010, pp. 703-719. [doi:10.1007/s10440-008-9341-0](https://doi.org/10.1007/s10440-008-9341-0)
- [8] T. S. Hassan, "Oscillation of Third Order Nonlinear Delay Dynamic Equations on Time Scales," *Mathematical and Computer Modelling*, Vol. 49, No. 7-8, 2009, pp. 1573-1586. [doi:10.1016/j.mcm.2008.12.011](https://doi.org/10.1016/j.mcm.2008.12.011)
- [9] Z. Q. Zhu and Q. R. Wang, "Existence of Nonoscillatory Solutions to Neutral Dynamic Equations on Time Scales," *Journal of Mathematical Analysis and Applications*, Vol. 335, No. 2, 2007, pp. 751-762. [doi:10.1016/j.jmaa.2007.02.008](https://doi.org/10.1016/j.jmaa.2007.02.008)
- [10] T. Li, Z. Han, S. Sun and D. Yang, "Existence of Nonoscillatory Solutions to Second-Order Neutral Delay Dynamic Equations on Time Scales," *Advances in Difference Equations*, Vol. 2009, 2009, pp. 1-10. [doi:10.1155/2009/562329](https://doi.org/10.1155/2009/562329)
- [11] T. X. Sun, H. Xi, X. Peng and W. Yu, "Nonoscillatory Solutions for Higher-Order Neutral Dynamic Equations on Time Scales," *Abstract and Applied Analysis*, Vol. 2010, 2010, pp. 1-16. [doi:10.1155/2010/428963](https://doi.org/10.1155/2010/428963)
- [12] B. G. Zhang and X. H. Deng, "Oscillation of Delay Differential Equations on Time Scales," *Mathematical and Computer Modelling*, Vol. 36, No. 11-13, 2002, pp. 1307-1318. [doi:10.1016/S0895-7177\(02\)00278-9](https://doi.org/10.1016/S0895-7177(02)00278-9)
- [13] B. G. Zhang and Y. J. Sun, "Existence of Nonoscillatory

Solutions of a Class of Nonlinear Difference Equations with a Forced Term," *Mathematica Bohemica*, Vol. 126, No. 3, 2001, pp. 639-647.

- [14] Y. Zhou and B. G. Zhang, "Existence of Nonoscillatory Solutions of Higher-Order Neutral Delay Difference Equations," *Computers & Mathematics with Applications*,

Vol. 45, No. 6-9, 2003, pp. 991-1000.
[doi:10.1016/S0898-1221\(03\)00074-9](https://doi.org/10.1016/S0898-1221(03)00074-9)

- [15] W. D. Lu, "Existence of Nonoscillatory Solutions of First Order Nonlinear Neutral Equations," *Journal of the Australian Mathematical Society Series B*, Vol. 32, No. 2, 1990, pp. 180-192.