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Labor Market Policies in Matching Models: Do Externalities Matter?

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Abstract

This paper analyzes the role played by five labor policy instruments (firing tax, hiring subsidies, taxation, unemployment benefits and tax structure) in a matching model with endogenous job destruction, when search externalities are not internalized and the market solution is inefficient. Since the theoretical model does not show univocal effects on equilibrium unemployment of some policy tools (such as hiring subsidies and firing tax), we propose a calibration and a numerical simulation of the model, in order to verify their real impact on unemployment and labor market structure. Results show that if, as is reasonable to assume, there are frictions on the labor market that generate search externalities, a labor market regulation becomes desirable and can be aimed at the internalization of externalities through an appropriate combination of labor policy instruments. In particular, our results have highlighted the crucial role of hiring subsidies and progressive taxation, not only for the achievement of the optimal solution, but also for supporting some forms of passive labor policies, mainly unemployment benefits and employment protection.

Keywords

Matching Models, Labor Market Policies, Search Externalities

1. Introduction

This paper proposes a matching model with endogenous job destruction [1] which evaluates the qualitative and quantitative effects of five labor policy tools: firing costs (employment protection), hiring subsidies, income tax, unemployment benefits and tax structure. Making use of this theoretical model, one can show that the effect on equilibrium unemployment of some of these tools, such as firing costs and hiring subsidies, is ambiguous because the variation in the average duration of unemployment is offset by the opposing variation in the flows into unemployment. This theoretical ambiguity needs to recourse to numerical simulations, via appropriate calibration of the model's equations, in order to establish which of the two effects prevails over the other. Most of the economic literature on this topic [2]-[8] assumes in the numerical simulation that search externalities are internalized by the market[1]; in this sense, the introduction of any labor policy tool determines a distortion which diminishes welfare.

The objective of this paper is to analyze the role of labor policies when search externalities are present and, in particular, to find the appropriate combination of the various policy tools available for internalizing them. The results obtained show that: 1) hiring subsidies can offset the distortions generated by employment protection with and without search externalities; 2) a progressive taxation combined with a hiring subsidy is able to correct taxation and unemployment benefit distortions and to internalize any search externalities, restoring job creation and job destruction rate to the levels required to guarantee the first best.

The organization of the paper is the following. Section 2 describes the model and introduces the labor policy tools. Section 3, by a comparative statics analysis, evaluates the qualitative effects of labor policies. Section 4 analyses compensating policy changes, namely policy tool combinations that leave the market equilibrium unaltered, while Section 5 analyses policy combinations that internalize search externalities. Section 6 provides the calibration of the theoretical model and discusses the results of the numerical simulations. Section 7 concludes.

2. The Model

The economy is made up of risk neutral workers and firms that consume their income entirely. Every worker can be employed or unemployed: when employed he earns a wage w, when unemployed he obtains an alternative income b. Every firm operating in the market has a job, which can be filled or vacant: when the job is filled, the resulting production activity generates a product y; the profit earned by the firm is thus $y - w$. When it is vacant, on the other hand, the firm incurs costs for its maintenance, c.

Workers and firms undertake a search process for finding a job and filling a vacancy, respectively. When the two meet, a worker decides whether to take the job at the current wage or to continue the job search and the firm decides whether to fill the vacancy with this worker or to continue its search. If the encounter generates a match, production activity starts and generates a product y.

Production activity continues until external events intervene to make its continuation no longer profitable. In this eventuality, the job is closed and the two parties undertake a new search process.

We assume that the productivity of a filled job is given by the product of two components: a general component y and an idiosyncratic component x ($0 \leq x \leq$

[1]Also, note some specific contributions such as that of [9] which considers a model with migration in developing countries calibrated to Korean data and that of [10] who incorporate the public sector in a matching context calibrating the model to Columbian data.

1). Firm's rational behavior implies that the production activity begins at maximum productivity level, *i.e.* with $x = 1$. When an idiosyncratic shock strikes the firm, it reduces productivity to a new level s. Corresponding to this new level, the firm decides whether to continue to produce or to close the job down. If the idiosyncratic shock impacts the individual firm at a constant rate λ and the idiosyncratic component x is distributed by a generic distribution function $G(x)$ (continuous and independent from the previous realization of x), the firm destroys the job when x goes down below a threshold value X, where continuing production is no longer profitable.

2.1. The Matching Function

The process that summarizes the number of matches is described by a matching function, assumed to be homogenous of degree-one, concave and increasing in the number of unemployed workers and in the number of vacancies.

For the constant returns to scale hypothesis, the value of the labor force can be assumed to be constant and normalized to one, for which the matching function is $m = m(u; v)$, where m is the number of matches, u is the unemployment rate and v is the ratio between vacancies and labor force. We denote by $\eta(\theta)$ the elasticity of the number of matches with regard to the unemployment rate.

Putting θ the ratio between vacancies and unemployment (v/u), this variable is the measure of the labor market tightness. The probability of moving out of unemployment and the probability of filling a vacancy depend on it.

The probability of filling a vacancy can be defined as $m/v = m(1/q;1) = q(\theta)$ (with $\partial q(\theta)/\partial\theta \le 0$), while the exit rate from unemployment is $m/u = m(1;\theta) = \theta q(\theta)$ (with $\partial \theta q(\theta)/\partial\theta \ge 0$). $1/q(\theta)$ and $1/\theta q(\theta)$ respectively represent average vacancy duration and average unemployment duration.

The dependence of these transition probabilities on the relative number of vacancies and unemployed workers generates a search externality in the model[2]: an increase in θ denotes an increase in the number of vacancies as compared to the number of unemployed workers: this reduces the firm's probability of filling a vacancy and increases the worker's probability of finding a job.

The equation of motion of the unemployment rate is given by the difference between the flows in and out of unemployment, with probabilities $\lambda G(X)$ and $\theta q(\theta)$ respectively:

$$\frac{\partial u}{\partial t} = \lambda G(X)(1-u) - \theta q(\theta)u \tag{1}$$

In steady state $\partial u/\partial \tau = 0$, so that the equilibrium unemployment rate is given by:

$$u = \frac{\lambda G(X)}{\lambda G(X) + \theta q(\theta)} \tag{2}$$

which is the Beveridge curve describing a decreasing relationship between un-

[2][11] is the pioneering article that illustrated the presence of trade externalities depending on the relative number of agents in the market.

employment and vacancies, since of the matching function has constant returns to scale.

Equation (2) is the first fundamental equation of the model and determines the unemployment rate for a given value of the reservation productivity X and the labor market tightness θ. Before determining the equilibrium values of X and θ, by analyzing worker and firm behavior, we introduce the labor market policy tools.

2.2. Labor Policy Tools

We consider the following labor policy tools: firing costs (employment protection), hiring subsidies, unemployment benefits and income tax.

Employment protection can vary in form and size, depending on labor market regulation. In general terms, employment protection can be thought of as: 1) a monetary transfer that the firm has to pay to a dismissed workers; 2) the whole of restrictions and bureaucratic procedures which firms have to obey, such as prior authorization before firing; 3) a warning period before the firing; 4) the employee right to undertake legal action against layoff. If employment protection is to be effective, for the class of models examined here, it must translate into a cost for the firms rather than a transfer, whose effects can be offset via the wage bargaining process[3]. A convenient way to consider employment protection is assuming it as a job destruction tax payable by the firm: when a filled job is closed, the firm pays a tax F to the policy maker.

Alongside this, labor policies can provide for a hiring or job creation subsidy H that the firm received when it hire a worker.

Unemployment benefits usually take the form of insurance against unemployment risk. It can be correlated with the unemployed worker's prior earnings and paid for a limited period of time, or not correlated with prior earnings and paid for the whole period of unemployment. In our model, we assume that the unemployment benefit is proportional to the average wage: if w_M is the average wage, subsidy z takes the form ρw_M where ρ is the replacement ratio.

Lastly, we consider taxation on wage, linear in the worker individual earnings. We assume that the worker receives a subsidy τ on his wage and then the wage inclusive of the subsidy is taxed with tax rate t. The net earning received by the worker is thus $(w+\tau)(1-t)$. If $\tau > 0$ taxation is progressive, if $\tau = 0$ taxation is proportional and if $\tau < 0$ it is regressive.

Summarizing, the policy tools are: the firing cost F, the hiring subsidy H, the replacement ratio ρ, the wage subsidy τ (reflecting taxation structure) and the marginal tax rate t.

2.3. Worker and Firm Behavior

Labor policies affect workers' and firms' optimization processes to the extent

[3]If a market is perfectly competitive and the firing cost takes the form of a transfer from firm to worker, an optimal contract between the two parties will result, which neutralizes the effect of employment protection. That is, the worker will pay the cost of firing when he is hired and get it back again when the contract is terminated. For a formal analysis of this, see [12].

that both parties take these into account when deciding whether to join a match and start the production activity. Moreover, we assume that the wage bargained between the two parties (in the form that will be described below) can be modify in a continuous way. This implies that a certain level of wage, namely w_0, is bargained in the initial phase of the match, corresponding to the highest level of productivity. Then, production continues until the job is hit by an idiosyncratic shock, so that the new level of the wage is $w(x)$, *i.e.* its value depends on the value assumed by the idiosyncratic component in productivity.

Indicating the value functions of the worker with V_E when employed and with V_U when unemployed, the following Bellman equations apply in the initial phase of the match:

$$rV_{E0} = (w_0 + \tau)(1-t) + \lambda \int_X^1 [V_E(s) - V_{E0}] \mathrm{d}G(s) + \lambda G(X)[V_U - V_{E0}] \tag{3}$$

$$rV_U = b + z + \theta q(\theta)[V_{E0} - V_U] \tag{4}$$

where V_{E0} denote the value of the employed worker with $x = 1$ and wage w_0.

After job creation, equation [3] becomes:

$$rV_E(x) = [w(x) + \tau](1-t) + \lambda \int_X^1 [V_E(s) - V_E(x)] \mathrm{d}G(s) + \lambda G(X)[V_U - V_E(x)] \tag{5}$$

where $V_E(x)$ shows the dependence of the employed worker value function on the value of idiosyncratic productivity component x.

Equations (3) and (5) state that the expected income flow of an employed worker (corresponding to the two phases of the match) must be equal to the wage, inclusive of the subsidy and net of taxation, plus the variation in value when the idiosyncratic shock maintains productivity above the threshold value and net of variations in value deriving from the shift from employed to unemployed status, if the idiosyncratic component of productivity falls below the threshold value.

In the same way, Equation (4) states that the expected income flow of an unemployed worker must be equal to the sum of the alternative income and the unemployment benefit plus the variation in value from unemployed to employed status, which happens with probability $\theta q(\theta)$.

For the firm, the flow of expected profits of a filled job rV_F and a vacancy rV_V in the initial phase of the match satisfies the following equations:

$$rV_{F0} = y - w_0 + \lambda \int_X^1 [V_F(s) - V_{F0}] \mathrm{d}G(s) + \lambda [V_V - F - V_{F0}] \tag{6}$$

$$rV_V = -c + q(\theta)[V_{F0} + H - V_V] \tag{7}$$

where V_{F0} is the value of a filled job for $x = 1$ and wage w_0.

In the continuous phase of the match, Equation (6) becomes:

$$rV_F(x) = yx - w(x) + \lambda \int_X^1 [V_F(s) - V_F(x)] \mathrm{d}G(s) + \lambda G(X)[V_V - F - V_F(x)] \tag{8}$$

Equations (6) and (8) state that, in the initial (continuous) phase of the match, a filled job yields a flow of profit equal to $y - w_0$ ($yx - w(x)$). If the shock is in the range $1 \le x \le X$, the value of the filled job changes from V_{F0} ($V_F(x)$) to $V_F(s)$; otherwise, the job is destroyed, the firm has to pay the firing cost F, and the asset value of the job changes to that of a vacancy V_V.

In Equation (7) the flow of expected profits of a vacancy must be equal to its maintenance costs plus the variation in value resulting from its change in status, inclusive of the hiring subsidy, that happens with probability $q(\theta)$.

The optimization process by workers and firms described above generates economic rents resulting from the implementation of the production activity. Wage bargaining determines the distribution of this rents in the way described in the next section.

2.4. Wage Bargaining

The production activity generates a surplus which is divided up via wage bargaining. We assume that this surplus is shared out by a Nash bargaining.

The assumption that the wage can be re-negotiated in a continuous way means that the initial wage differs from the continuation wage because of the presence of hiring subsidies and firing costs.

The wage level derives from the maximization of the geometric mean of the surplus generated by the two parties weighted by their relative bargaining power. In the first phase of the bargaining, when the firm fills the vacancy, it obtains the hiring subsidy, so that the value of the filled job is $V_{F0} + H$. This implies that the maximization program is:

$$w_0 = \arg\max \left[V_{E0} - V_U\right]^{\beta} \left[V_{F0} + H - V_V\right]^{1-\beta} \tag{9}$$

where β represents the worker's bargaining power.

In the continuous phase of the bargaining, if the firm decides to close the job down, it incurs the firing cost and thus the value of a vacancy is $V_V - F$. The continuous wage is thus given by the outcome of the following maximization process:

$$w(x) = \arg\max \left[V_E(x) - V_U\right]^{\beta} \left[V_F(x) + F - V_V\right]^{1-\beta} \tag{10}$$

The two maximization programs (9) and (10) generate two different surplus sharing rules in the two bargaining phases.

For the initial wage, we obtain:

$$V_{E0} - V_U = \frac{\beta(1-t)}{1-\beta}\left[V_{F0} + H - V_V\right] \tag{11}$$

while for the continuous wage:

$$V_E(x) - V_U = \frac{\beta(1-t)}{1-\beta}\left[V_F(x) + F - V_V\right] \tag{12}$$

As can be noted, tax rate modifies the quota of surplus due to the worker while the firing cost and the hiring subsidy affect the total amount of the surplus [13] [14].

The two sharing rules (11) and (12) generate two different wage equations: an entry or outsider wage in the initial phase of the match and a continuation or insider wage in the second phase [15]. Using Equations ((3), (4), (6) and (7)) and substituting them in Equation (11), after bit of algebra, the outsider wage is given by:

$$w_0 = \frac{(1-\beta)(b+z)}{1-t} - (1-\beta)\tau + \beta\left[y + c\theta - \lambda F + (r+\lambda)H\right] \quad (13)$$

Equation (13) describes an increasing relationship between wage w and labor market tightness θ; the presence of firing costs F reduces wage because the firm considers them when they become binding in the continuous phase of the match.

Similarly, by substituting Equations ((3), (5), (6) and (8)) in Equation (12) the insider wage equation is given by:

$$w(x) = \frac{(1-\beta)(b+z)}{1-t} - (1-\beta)\tau + \beta\left[yx + c\theta + rF\right] \quad (14)$$

In Equation (14), firing costs increase wage levels because workers are now protected to a greater extent: this increases their bargaining power pushing the wage upwards.

2.5. Job Creation and Job Destruction

Exploiting all the market profit opportunities, resulting from rational behavior by economic agents, makes the free entry condition applicable $V_V = 0$. If this condition apply, Equation (7) becomes:

$$V_{F0} + H = \frac{c}{q(\theta)} \quad (15)$$

which states that the value of a filled job at the maximum level of productivity and with the initial wage level, inclusive of the subsidy, is equal to the cost of maintaining a vacancy for the whole period in which it remains so.

If $V_F(x)$ represents the value of a filled job with productivity x and if the job is destroyed when productivity x falls below threshold value X, when the job is destroyed the firm gives up $V_F(x)$ and pays the firing cost F. Thus a job with an idiosyncratic productivity component x is kept active when $V_F(x) > -F$ and destroyed when $V_F(x) < -F$. This implies that, at the reservation productivity value X, we have:

$$V_F(X) + F = 0 \quad (16)$$

Making use of Equations (15) and (16) and of the two wage Equations (13) and (14), we obtain the following the job creation condition:

$$\frac{c}{q(\theta)} = (1-\beta)\left[\frac{y(1-X)}{r+\lambda} + H - F\right] \quad (17)$$

The job creation condition (17) states a decreasing relationship between θ and X, by the equality between the costs of keeping a vacancy (the left hand side) and the expected profits of the firm (the right hand side). The job creation is decreasing because an increase in the productivity threshold reduces the job's life expectancy (jobs are destroyed according to $\lambda G(X)$ probabilities by time interval), so that firms create fewer vacancies, getting worse labor market conditions, *i.e.* reducing θ.

Making use of the insider wage Equation (14) and the arbitrage Equations (8) and (16), the job destruction condition is given by:

$$yX + rF - \frac{\beta c\theta}{1-\beta} - \frac{b+z}{1-t} + \tau + \frac{\lambda y}{r+\lambda}\int_X^1 (s-X)\,\mathrm{d}G(s) = 0 \tag{18}$$

Equation (18) states an increasing relationship between θ and X because higher labor market tightness increase workers' external opportunities: this generates higher wages, lower profits and higher job destruction.

Market equilibrium is described by the job creation condition (17) and the job destruction condition (18), which determine the equilibrium values of X and θ, as shown in **Figure 1**. Then, given the equilibrium values of the reservation productivity and the labor market tightness, the equilibrium unemployment rate can be obtained by the Beveridge curve (2).

3. Comparative Statics: The Qualitative Effects of Labor Policies

The hiring subsidies H leads to an upwards right shift in the job creation conditions. In **Figure 2** this effect is described by the shift of the equilibrium from point A to point C. The X and θ equilibrium values both increase and this has two effects on the equilibrium unemployment, which work in opposing directions: higher θ reduces the average duration of unemployment while higher X increases flows into unemployment. As a result, the overall effect on the unemployment rate is ambiguous.

The firing cost F leads to a shift in the job destruction condition below right and in the job creation condition below left. Equilibrium point moves from B to A, with a lower level of X and θ[4]. The equilibrium unemployment increases only if the effects of θ prevail over those of X.

An interesting result of the model is when the hiring subsidy and the firing cost are equal, *i.e.* $H = F$. In this case, the job creation curve does not shift because

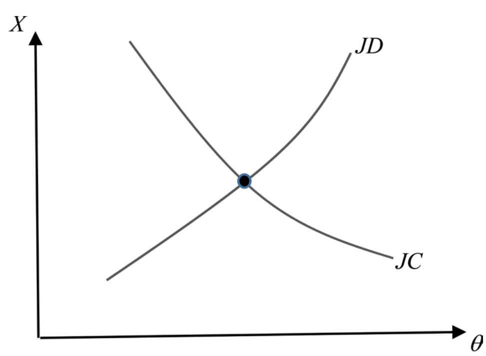

Figure 1. The labor market equilibrium.

[4]The figure seems to show that the effect on θ is ambiguous. However, differentiating Equations (17) and (18) with respect to F shows that θs equilibrium value diminishes [4] [6].

the net subsidy received by the firm is nil. The firing cost, on the other hand, shifts the job destruction curve down right. Equilibrium moves from point D to point A in **Figure 2** with a reduction in X (with lower flows into unemployment) and an increase in θ (with a decrease in the average duration of unemployment). The equilibrium unemployment decreases unambiguously. The reason is that whilst the firing cost causes lower job destruction, the hiring subsidy boosts firms to fill vacancies. The combination of this two policy tools is able to keep active workers with low productivity, offsetting this productivity gap by the hiring subsidy.

With regard to the unemployment benefit z and the marginal tax rate t, they do not affect the job creation condition but only the job destruction because of the increase of the reservation productivity caused by the reduction in the opportunity cost of being unemployed. In terms of **Figure 2**, a shift in the job destruction curve up leftwards occurs and the equilibrium passes from A to D, with an increase in X and a reduction in θ. In this case, both the average duration of unemployment and the flows into unemployment move in the same direction: the equilibrium unemployment increases unambiguously.

Lastly, a progressive taxation system (in the form of a wage subsidy τ) shifts the job destruction curve down rightwards moving the equilibrium from D to A in **Figure 2**. This leads to an increase in θ's equilibrium value and a reduction in X's equilibrium value. An increase in θ reduces the average duration of unemployment while a reduction in X reduces the flows into unemployment. The equilibrium unemployment decreases. The impact of progressive taxation is to induce worker towards wage moderation, reducing labor costs for firms and lowering job destruction.

4. Compensating Policy Changes

By compensating policy changes, we mean a specific combination of policy tools that maintain the market equilibrium unchanged [14]. Making this possible is the variety of tools available to policy makers. Studying these compensating policy changes can be useful because of the number of objectives of labor policies: think, in particular, of redistributive goals designed to support the earnings of

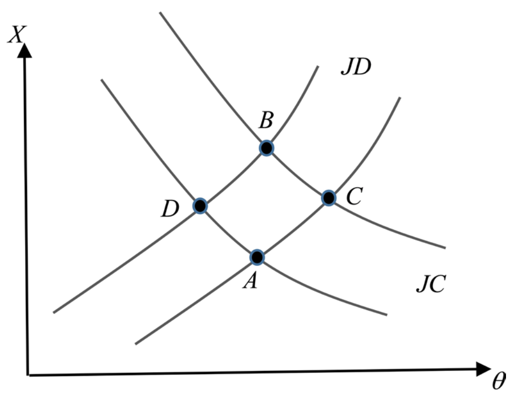

Figure 2. The qualitative effects of labor market policies.

unemployed or low paid workers [6] [16]. In this case, the most efficient way of structuring policy intervention is to do so without altering the equilibrium generated by the market[5].

To do this, we have to consider the job creation and the job destruction condition with and without policies. Comparing Equations (17) and (18) with those obtainable in the absence of policies, a compensating policy changes has to satisfy the following conditions:

$$H = F \tag{19}$$

$$\tau = \frac{tb+z}{1-t} - rF \tag{20}$$

When Equations (19) and (20) are satisfied, the X and θ equilibrium values, and thus the job creation rate, the job destruction rate and the unemployment rate are the same with and without policies.

Condition (19) states that the effects generated by firing costs need to be offset by hiring subsidies of equal amount. Note that, in the absence of any other policy tools, the unique values of hiring subsidy and firing tax which does not alter the equilibrium are $F = 0$ and $H = F$, *i.e.* the absence of policies. In fact, condition (20) states that unemployment benefits and marginal tax rate can be offset with progressive taxation. The presence of firing costs reduces the degree of progressivity of taxation to maintain the market equilibrium unchanged. In other words, Equation (20) justifies a progressive taxation and a job destruction tax for financing unemployment benefits.

Consider now some characteristics of the choice of tools available to the policy maker.

To simplify the analysis, we assume no hiring subsidies ($H = 0$) and firing costs ($F = 0$) and a nil value of leisure ($b = 0$). If there are not unemployment benefits, Equation (20) becomes $\tau = 0$, which implies that marginal taxation, without any form of progressivity, can be used by policy makers to increase the public revenues without any distortion in the market equilibrium. Moreover, if the value of leisure is nil, any distortion would be generated only by profit variations, so that absence of distortions means that taxation is entirely paid by workers.

We now assume that policy makers decide to introduce an unemployment benefit financed by taxation revenues. If $b = 0$, Equation (20) becomes $\tau = z/(1-t)$, that is the taxation that finance the unemployment benefit must necessarily be progressive in order to maintain the equilibrium unchanged.

If instead the value of leisure is positive, for compensating policy changes the following condition must be satisfy:

$$\tau = \frac{tb+z}{1-t} \tag{21}$$

With positive value of leisure, taxation must be more progressive to finance unemployment benefits. Furthermore, the progressive taxation must be higher

[5]Strictly speaking, this also applies when the market produces an efficient result.

the higher the level of tax rate. This result derives from the fact that τ is equivalent to an employment subsidy for the worker: the value of leisure increases because of the unemployment benefit while the labor income increases because of the wage subsidy, with consequent non-variation in the relative value of the two incomes.

5. The Role of Search Externalities

Compensating policy changes can be useful if the equilibrium is a first best, that arises if the well-known Hosios condition is satisfied [17]. This condition states that search externalities are internalized if the worker's surplus share β is equal to the elasticity of the matching function with regard to the unemployment rate $\eta(\theta)$, and thus the job destruction rate is not too low, as in the case in which search externalities are present. Unfortunately, the Hosios condition is difficult to fulfil because the β and $\eta(\theta)$ values are determined in two different behavioral contexts. We thus need to ask if labor policies can be structured in such a way as to internalize search externalities [14].

Examining this requires comparing job creation and destruction conditions in the private solution with the efficient one, a comparison that brings out the following:

$$\frac{c}{q(\theta)(1-\beta)} - H + F = \frac{c}{q(\theta)[1-\eta(\theta)]} \tag{22}$$

$$\frac{\beta}{1-\beta}c\theta + \frac{b+z}{1-t} - \tau - rF = b + \frac{\eta(\theta)}{1-\eta(\theta)}c\theta \tag{23}$$

From Equation (22), it emerges that the optimal hiring subsidy is:

$$H = F + \left[\frac{1}{1-\beta} - \frac{1}{1-\eta(\theta)}\right]\frac{c}{q(\theta)} \tag{24}$$

This latter equation affirms that, when $\beta > \eta(\theta)$, hiring subsidies must be higher than firing costs and lower in the opposite case. The reason is that when $\beta > \eta(\theta)$, the labor costs are higher than those necessary to ensure efficiency. Consequently, firms open a too low number of vacancies. Hiring subsidies can boost firms to open more vacancies and thus correct this type of inefficiency.

If Equation (24) applies, Equation (23) implies that the other labor policies tools must fulfil:

$$\tau = \left[\frac{\beta}{1-\beta} - \frac{\eta(\theta)}{1-\eta(\theta)}\right]c\theta + \frac{tb+z}{1-t} - rF \tag{25}$$

In this case, when $\beta > \eta(\theta)$ the level of τ is greater than in the efficient solution: higher τ can correct the inefficiently low levels of job creation, which in turn affects the job destruction rate.

6. Numerical Simulation: The Quantitative Effects of Labor Policies

In Section 3, we have shown the ambiguous effects of some labor policies on equilibrium unemployment, because the variation in the flows into unemployment and in the average duration of unemployment move in the opposite directions. In order to establish which of the two effects prevails to the other requires using a numerical simulation of the model. To do this, we need the following: 1) specific functional forms for the matching function and for the probability distribution of the idiosyncratic shock; 2) a calibrating of the parameters of the theoretical model. The purposes is to obtain quantitative answers on the role played by labor policy tools introduced previously.

6.1. Functional Forms and Calibration

The criterion is that of simplicity and parsimony. We assume Cobb-Douglas constant returns to scale matching function:

$$m = u^{\eta} v^{1-\eta}$$

where η is the elasticity of the number of matches with regard to the unemployment rate. Idiosyncratic shock distribution is assumed uniform in the $[\gamma;\ 1]$ support; the cumulative distribution function is thus:

$$G(x) = \frac{x-\gamma}{1-\gamma}$$

The values of the baseline parameters are reported in Table 1.

These parameters conform to those used in the literature and reflect the characteristics of the US economy[6].

We assume initially that the system does not generate search externalities, *i.e.* that the elasticity of the matching function with regard to unemployment η is equal to the worker's surplus share β as Hosios condition established. The value assigned to the parameters is quarterly. The productivity of a new job y is normalized to one, the interest rate r is fixed at 2%, the frequency of idiosyncratic shock λ is 0.1 and the hiring cost c is 0.4 per worker. Furthermore, we assume a moderate level of marginal tax rate t and replacement ratio ρ at 20%. Lastly, the

Table 1. Baseline parameters assumed in the model simulation.

Worker's surplus share	β	0.5
Productivity of a new job	y	1
Risk free interest rate	r	0.02
Idiosyncratic shock arrival rate	λ	0.1
Vacant job maintenance costs	c	0.4
Value of leisure	b	0.14
Elasticity of the matching function	η	0.5
Replacement ratio	ρ	0.2
Marginal taxation	t	0.2
Lower idiosyncratic shock probability distribution support	γ	0.53

[6]On this point see [4] [5] [6] and [8].

value of leisure b and the lower bound of the idiosyncratic shock probability distribution γ were chosen in such a way as to obtain an initial equilibrium unemployment value of 6.5%, with corresponds to an average duration of unemployment of about three months. With no tax rate and replacement ratio the equilibrium unemployment rate is 5%, with an average duration of unemployment of about two months.

6.2. Results

The effects of marginal taxation and replacement ratio are shown in panel A of Table 2. Both policy tools lead to an increase in the equilibrium unemployment with more marked effects generated by the replacement ratio. We can also note that moderate levels of ρ (=0.15) do not determine substantial losses in welfare (measured by the aggregate income). Such welfare losses, on the other hand, increase significantly when both policy tools begin to assume higher values. This result suggests that measures involving limited reductions in both tools can have greater effects than considerable reductions of just one [18].

The effects of hiring subsidies and firing costs are shown in panel B of Table 2. 0.33 increases correspond to around 1.5 times the monthly average product per worker. The results obtained by the simulation show that hiring subsidies increase the equilibrium unemployment while firing costs reduce it. In both cases,

Table 2. Effects of labor policies when search externalities are internalized. (aggregate income percentage variations in brackets).

A) Marginal taxation and replacement ratio								
	t = 0		**t = 0.15**		**t = 0.3**		**t = 0.4**	
ρ = 0	5.0		5.1	(0.0)	5.3	(0.0)	5.4	(0.0)
ρ = 0.15	5.7	(0.0)	6.0	(−0.1)	6.4	(−0.2)	7.0	(−0.3)
ρ = 0.3	6.6	(−0.2)	7.2	(−0.4)	8.5	(−1.0)	10.3	(−2.0)
ρ = 0.4	7.4	(−0.5)	8.6	(−1.0)	11.2	(−2.6)	17.3	(−7.3)
B) Hiring subsidies and firing costs (r = 0.2; t = 0.2)								
	F = 0		**F = 0.33**		**F = 0.67**		**F = 1**	
H = 0	6.5	(−0.2)	6.0	(−0.3)	5.4	(−0.8)	4.7	(−1.6)
H = 0.33	7.0	(−0.3)	6.5	(−0.2)	6.0	(−0.3)	5.4	(−0.8)
H = 0.67	7.4	(−0.8)	7.0	(−0.3)	6.5	(−0.2)	6.0	(−0.3)
H = 1	7.8	(−1.6)	7.4	(−0.8)	7.0	(−0.3)	6.5	(−0.2)
C) Progressive taxation (ρ = 0.2; t = 0.2)								
τ = 0	6.5	(−0.2)						
τ = 0.33	4.8	(0.0)						
τ = 0.67	3.8	(−0.4)						
τ = 1	3.1	(−0.9)						

Source: Author's own calculations.

this means that the effects on average unemployment duration are proportionally lower than those generated on incoming flows[7].

Whilst hiring subsidies determine an increase in the equilibrium unemployment, these are in any case capable of offsetting the negative effects generated by high employment protection on aggregate income. As is shown by the principal diagonal in **Table 2**, panel B, a combination of hiring subsidies and firing costs of equal amount can maintain the unemployment rate at the initial level of 6.5% without generating significant losses in aggregate income.

The effects generated by progressive taxation are shown in panel C of **Table 2**. Positive τ values lead to reductions in the equilibrium unemployment caused by a lower in average duration and lower inflows into unemployment. However, despite the decrease of the unemployment rate, progressive taxation is not capable of generating a higher aggregate income[8]. This is due to the reduction in the job destruction rate that, with no search externalities, falls below efficient levels.

The results obtained so far accord with those to be found in the standard economic literature on matching models. In general, the objective is to explain high unemployment levels in terms of different institutional structures, in particular in high levels of taxation, unemployment benefits and employment protection. High tax pressure and high unemployment benefits would explain the higher equilibrium unemployment, while rigid forms of employment protection would be responsible for the higher average duration of unemployment. In terms of our simulation, for example, a 35% level of replacement ratio and tax rate with a firing cost around three times the average monthly product per worker (F = 0.67) would take the equilibrium unemployment to 11%, with double average duration of unemployment than the initial situation.

Together with the problem of choosing which simulation parameters to use, maintained identical in the definition of the structures of different economies, an important hypothesis is considering the matching process efficient. In this sense, every policy action generates distortions, determining more or less significant welfare losses. We saw above that, in this latter case, labor policy tools can be modelled in such a way as not to alter initial equilibrium. In particular, a progressive taxation accompanied by hiring subsidies is able to correct the distortions generated by the presence of other policy actions and internalizing search externalities, restoring job creation and destruction rates to the levels necessary to guarantee the first best.

Table 3 shows the results of the simulation relating to a combination of hiring subsidies and tax structure when search externalities are internalized by the market.

Panel A starts from an initial first best situation corresponding to a 5% unemployment rate and in the absence of further policy action. As can be observed,

[7]This result accords with simulations proposed by [4] [5] and [6].

[8]The positive aggregate income variation observable in the table corresponding to τ = 0.33 is due to correction of the distortions prompted by the presence of positive values in marginal taxation t and replacement ratio ρ.

Table 3. Compensating policy changes. (percentage variations in aggregate income in brackets).

A) $\beta=\eta$; $\rho=0$; t = 0; F = 0								
	$\tau=0$		$\tau=0.33$		$\tau=0.67$		$\tau=1$	
H = 0	5.0		3.9	(−0.3)	3.2	(−0.8)	2.7	(−1.4)
H = 0.33	5.4	(0.0)	4.3	(−0.1)	3.6	(−0.4)	3.1	(−0.9)
H = 0.67	5.8	(−0.3)	4.7	(−0.2)	4.0	(−0.4)	3.4	(−0.7)
H = 1	6.2	(−0.9)	5.1	(−0.7)	4.3	(−0.7)	3.8	(−0.9)
B) $\beta=\eta$; $\rho=0.3$; t = 0.4; F = 0								
	$\tau=0$		$\tau=0.33$		$\tau=0.67$		$\tau=1$	
H = 0	10.3	(−2.0)	6.2	(−0.1)	4.6	(−0.1)	3.7	(−0.5)
H = 0.33	10.8	(−2.4)	6.8	(−0.3)	5.0	(0.0)	4.1	(−0.2)
H = 0.67	11.3	(−3.1)	7.3	(−0.7)	5.5	(−0.2)	4.5	(−0.3)
H = 1	11.7	(−4.0)	7.8	(−1.5)	6.0	(−0.8)	4.9	(−0.7)
C) $\beta=\eta$; $\rho=0.3$; t = 0.4; F = 0.67								
	$\tau=0$		$\tau=0.33$		$\tau=0.67$		$\tau=1$	
H = 0	8.9	(−1.9)	5.0	(−0.8)	3.6	(−1.3)	2.8	(−2.1)
H = 0.33	9.5	(−1.8)	5.6	(−0.3)	4.1	(−0.5)	3.2	(−1.1)
H = 0.67	10.1	(−1.9)	6.2	(−0.1)	4.6	(−0.1)	3.6	(−0.5)
H = 1	10.7	(−2.3)	6.7	(−0.2)	5.0	(0.0)	4.1	(−0.2)

Source: Author's own calculations.

whilst leading in some cases to unemployment rates below 5%, positive H and τ values determine negative variations in aggregate income.

Furthermore, when taxation is highly progressive, initial hiring subsidy increases succeed in partially offsetting the negative effect on job creation but not to an extent sufficient to permit a welfare level on a par with the initial situation.

On the other hand, when we introduce marginal taxation and unemployment benefits, an opportune H and τ combination can reduce the equilibrium unemployment with no losses in welfare. As we can see from panel B of Table 3, with $\tau=0.67$ and $H=0.33$ the unemployment rate is at 5% and the aggregate income is equal to the first best.

Lastly, in panel C of Table 3 a positive firing cost value has been introduced ($F=0.67$). In this case, compensating policy changes requires a progressive taxation equal to the earlier case ($\tau=0.67$), and a higher level of hiring subsidies ($H=1$) to offset the welfare loss prompted by firing costs.

Consider now Table 4, which report the quantitative effects of hiring subsidies and progressive taxation when search externalities are not internalized by the market. We assume a worker's surplus share higher than the elasticity of the matching function with regard to the unemployment rate ($\beta=0.6$ and $\eta=0.5$), so that the Hosios condition does not apply. This implies that the equilibrium

Table 4. Labor policies and research externalities (percentage variations in aggregate income in brackets).

A) $\beta = \eta$; $\rho = 0$; t = 0; F = 0								
	$\tau = 0$		$\tau = 0.33$		$\tau = 0.67$		$\tau = 1$	
H = 0	6.0	(−0.2)	4.7	(−0.2)	3.9	(−0.6)	3.3	(−1.0)
H = 0.33	6.5	(−0.2)	5.2	(0.0)	4.3	(−0.1)	3.7	(−0.4)
H = 0.67	7.0	(−0.4)	5.7	(0.0)	4.8	(0.0)	4.1	(−0.2)
H = 1	7.5	(−0.9)	6.1	(−0.4)	5.2	(−0.2)	4.5	(−0.2)
B) $\beta = \eta$; $\rho = 0.3$; t = 0.4; F = 0								
	$\tau = 0$		$\tau = 0.33$		$\tau = 0.67$		$\tau = 1$	
H = 0	12.4	(−3.5)	7.5	(−0.6)	5.6	(−0.2)	4.4	(−0.3)
H = 0.33	13.0	(−3.9)	8.2	(−0.8)	6.1	(−0.1)	5.0	(0.0)
H = 0.67	13.6	(−4.6)	8.8	(−1.3)	6.7	(−0.3)	5.5	(−0.1)
H = 1	14.1	(−5.5)	9.4	(−2.0)	7.2	(−0.8)	6.0	(−0.4)
C) $\beta = \eta$; $\rho = 0.3$; t = 0.4; F = 0.67								
	$\tau = 0$		$\tau = 0.33$		$\tau = 0.67$		$\tau = 1$	
H = 0	10.7	(−3.2)	6.1	(−1.2)	4.3	(−1.4)	3.3	(−2.1)
H = 0.33	11.2	(−3.2)	6.8	(−0.8)	4.9	(−0.7)	3.9	(−1.0)
H = 0.67	12.2	(−3.3)	7.5	(−0.6)	5.5	(−0.2)	4.4	(−0.3)
H = 1	12.8	(−3.8)	8.1	(−0.8)	6.1	(−0.1)	5.0	(0.0)

Source: author's own calculations.

unemployment rate is higher than in the efficient solution.

Panel A relates to the hypothesis that no further policy action occurs. In this case, moderate hiring subsidy levels and progressive taxation (both between 0.33 and 0.67, not shown in the table) enable to obtain an unemployment rate and an aggregate income corresponding to the efficient situation.

In panel B, the introduction of an unemployment benefit ($\rho = 0.3$) and marginal tax rate ($t = 0.4$) requires, for reaching the first best, a moderate level of hiring subsidies ($H = 0.33$) and a more progressive taxation ($\tau = 1$) than in the case of no externalities.

Lastly, when in panel C we introduce the firing cost ($F = 0.67$), the corrective action required by hiring subsidies ($H = 1$) is greater than that relating to the cases examined previously.

7. Conclusions

In this paper, we have focused on both the qualitative and quantitative effects of five labor policy instruments (firing tax, hiring subsidies, taxation, unemployment benefits and tax structure) in a search and matching model with endogenous job destruction.

The comparative statics on the theoretical model shows how some policy instruments, such as firing tax and hiring subsidies, produce an ambiguous effect on equilibrium unemployment, given that variations in the average duration of unemployment and inflows into unemployment work in opposite directions. This implies that, in order to establish which of the two effects will prevail, a quantitative analysis based on numerical simulation of the model is needed.

For numerical simulation purposes, most of the economic literature assumes that the search externalities arising in the theoretical model are fully internalized by the market and thus play no role in the computational solution. Therefore, the introduction of any labor policy tools generates a distortion that unavoidably reduces welfare.

By contrast, we have evaluated the role of policy when search externalities are not internalized and the market solution is inefficient. Following this approach, the set of policy tools available to policy makers allows them to be combined in order to internalize search externalities and reach the first best solution.

The results obtained show that a labor market regulation becomes opportune and necessary and can be aimed at the internalization of externalities through an appropriate combination of labor policy instruments. In particular, our results have highlighted the crucial role of hiring subsidies and progressive taxation, not only for the achievement of the optimal solution, but also for supporting some forms of passive labor policies, mainly unemployment benefits and employment protection.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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