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Hubble Scale Dark Energy Meets Nano Scale Casimir Energy and the Rational of Their T-Duality and Mirror Symmetry Equivalence*

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Abstract

We establish that ordinary energy, Casimir energy and dark energy are not only interlinked but are basically the same thing separated merely by scale and topology. Casimir energy is essentially a nano scale spacetime phenomenon produced by the boundary condition of the two Casimir plates constituting the Casimir experimental set up for measuring the Casimir force. By contrast dark energy is the result of the cosmic boundary condition, *i.e.* the boundary of the universe. This one sided Möbius-like boundary located at vast cosmic distance and was comparable only to the Hubble radius scales of the universe. All the Casimir energy spreads out until the majority of it reaches the vicinity of the edge of the cosmos. According to a famous theorem due to the Ukrainian-Israeli scientist I. Dvoretzky, almost 96% of the total energy will be concentrated at the boundary of the universe, too far away to be measured directly. The rest of the accumulated Casimir energy density is consequently the nearly 4% to 4.5%, the existence of which is confirmed by various sophisticated cosmic measurements and observations. When all is said and done, the work is essentially yet another confirmation of Witten's T-duality and mirror symmetry bringing nano scale and Hubble scale together in an unexpected magical yet mathematically rigorous way.

Keywords

Mirror Symmetry, Casimir Energy, Dark Energy, Zero Point Vacuum Energy, T-Duality, Nano Scale-Hubble Scale, Möbius Holographic Boundary, Dvoretzky's Theorem, Banach-Tarski Theorem

1. Introduction

The present paper is concerned with a wide range of problems connecting high energy quantum physics with

*Dedicated to the memory of Sir Herman Bondi and Prof. Thomas Barta—two very memorable teachers and fatherly friends.

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relativity, quantum gravity and cosmology [1]-[96].

One of the many memorable words of wisdom that Prof. Sir Herman Bondi was known for is to the effect that once a great truth about nature is uncovered, it becomes so obvious and self evident that we are bound to wonder how we did not notice it for such a long time and how it could have eluded us all despite its compelling simple logic [1]. Sir Herman used to cite Einstein' relativity as his favourite parade example [1]-[10].

Referring to the present work the author would like to think that the thesis that we are about to reveal belongs in the same category which Prof. Bondi would have liked [1]-[10]. We are pointing here at our discovery that we perceived a few months ago as tantalizing because it implies the realization that the Casimir energy involved in the famous Casimir effect as well as dark and ordinary energy density of the cosmos are nothing but different forms of the same physical reality by virtue of some mathematical theorem which goes by the name of mirror symmetry and T-duality [93]-[95]. With that we do not only mean that they are different manifestations of the zero point energy of vacuum fluctuation only [11]-[31]. It is far more than that. How much more is the subject of the present paper and we will, without much ado, turn our attention now to the present main task of explaining it in as simple a manor as possible with the only provision which Einstein would have said, "put not more".

2. An Exact Analytical-Topological Picture of Spacetime

Before reading a single word or equation of the present paper, our advise to the prospective reader is to first have a long contemplative look at **Figure 1** and **Figure 2** of the present work. The first figure is actually an artist impression of quantum or fractal-Cantorian spacetime (**Figure 1**). The second is an exact mathematical-topological picture in the sense of the scientific philosophy of people like Wittgenstein (**Figure 2**). Such a scientific picture is not the product of wild imagination but of stringent application of the dimensional function of von Neumann-Connes' continuous and noncommutative geometry, namely [71]

$$D = a + b\phi; \quad a, b \in Z \quad \text{and} \quad \phi = \left(\sqrt{5}-1\right)/2 .$$

In particular it can be shown that $D(O)$ is the dimensionality of the zero set which models the pre-quantum particle while $D(-1)$ is the dimensionality of the empty set which models the quantum wave. More specifically the pre-quantum particle is accurately represented by the bidimension [64] [71]

$$D(O) = (D_T; D_H) = (O, \phi)$$

where $D = O$ is the Menger-Urysohn deductive topological dimension and $D_H = \phi$ is the corresponding Hausdorff dimension which happens to be the same dimension as that of a triadic Cantor set that is constructed randomly using a uniform distribution. In other words, we could use the follow shorthand notation [64] [71]

$$D(QP) \equiv (O, \phi) \equiv D(O) \equiv \text{zero set}$$

for the pre-quantum particle and

$$D(QW) \equiv \left(-1, \phi^2\right) \equiv D(-1) \equiv \text{empty set}$$

for the pre-quantum wave. Now using Fibonacci's growth law and starting with $D(O)$ using $(a, b) = (0, 1)$ and $D(1)$ $(a, b) = (1, 0)$ as said, one obtains for positive dimension the following series

$$\begin{aligned}
&D(O) = 0 + \phi = \phi \\
&D(1) = 1 + 0 = 1 \\
&D(2) = 1 + \phi = 1 + \phi = (1/\phi) \\
&D(3) = 2 + \phi = 2 + \phi = (1/\phi)^2 \\
&D(4) = 3 + 2\phi = 4 + \phi^3 = (1/\phi)^3 \\
&D(5) = 5 + 3\phi = 6.854101 = (1/\phi)^4 \\
&\qquad \vdots \\
&D(n) = \cdots = \cdots = (1/\phi)^{n-1} .
\end{aligned}$$

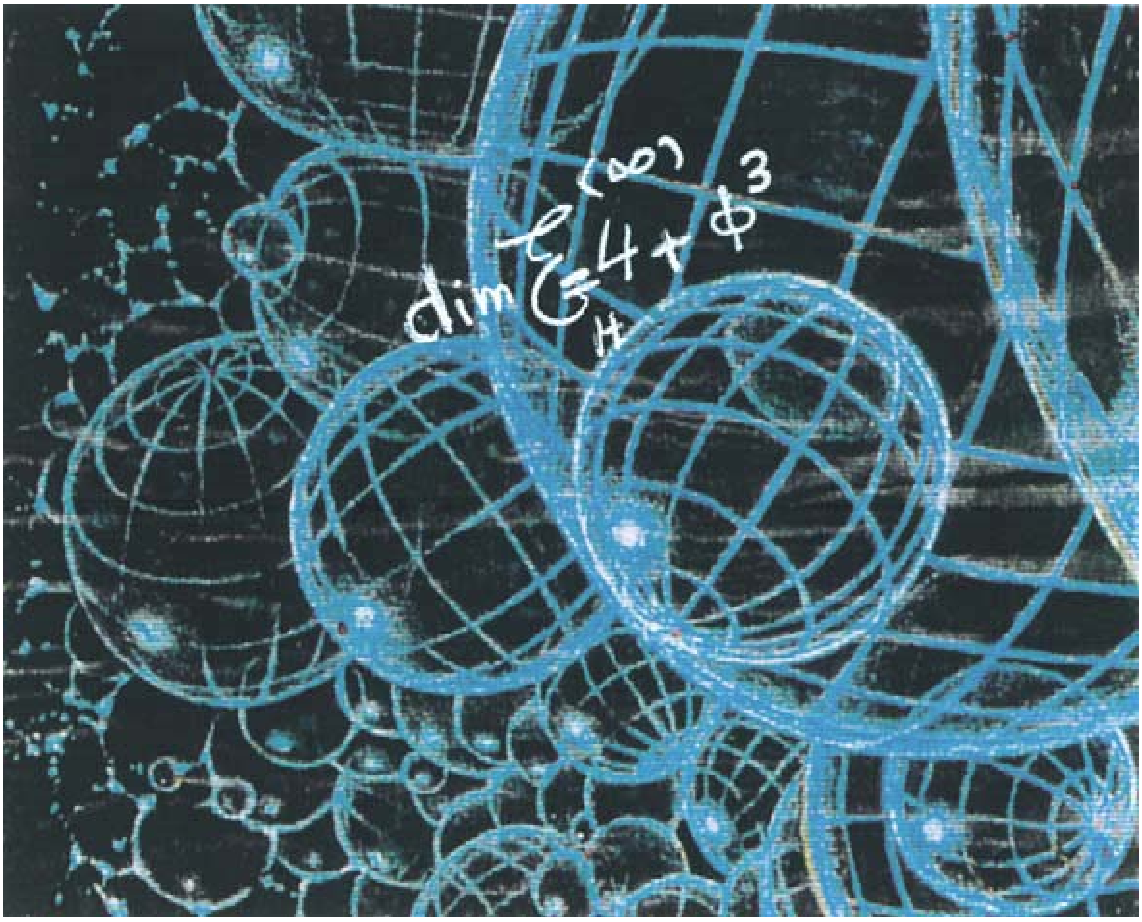

Figure 1. T-duality and Banach-Tarski sphere decomposition Cantorian spacetime of E-infinity theory that is considered here to model our actual spacetime may be envisaged advantageously as in this artist impression. This is basically a two dimensional projection in which each of the larger balls (circles) are a zero set $(0;\phi)$ representing the quantum particle while the surface (circumference) represents the empty set $(-1,\phi^2)$ which in turn represents the quantum wave [1] [17]. This wave is then surrounded by an infinite hierarchy of smaller (fractal) spheres (surfaces), which may be seen as the emptier set $(-2;\phi^3)$, *i.e.* the surface of the empty set quantum wave. Remarkably the average set of all zero and empty sets is an expectation value equal $\langle -2;\phi^3 \rangle$. In other words $\langle -2;\phi^3 \rangle$ is our quantum spacetime, which is the cobordism of the quantum wave, which in turn is the cobordism of the quantum particle, floating and propagating with the help of its wave in our Cantorian E-infinity spacetime [1] [2] [10] [11]. It is likewise remarkable that ϕ^3 is simultaneously equal to the topological Casimir force as well as the topological mass of the ordinary energy of spacetime [96]. Thus all matter and energy manifestations in our cosmos are essentially a manifestation of the zero point energy of the vacuum of spacetime. To obtain Einstein maximal energy density we just need to find first the topological energy density by adding Kaluza-Klein D = 5 to ϕ^3 of the spacetime vacuum and find the fractal Kaluza-Klein dimension $5+\phi^3$ then multiply this with the average Cantorian interval speed of light c = ϕ squared. The result is $(5+\phi^3)\phi^2 = 2$. Inserting in Newton's kinetic energy one finds $E(Einstein) = \frac{1}{2}m(v \to c)^2 (2) = mc^2$ exactly as should be. The preceding explanation amounts to a paradigm shift in physics where the totally empty vacuum of spacetime is taken as fundamental and everything else is derivable from it. To prove this point was a dream of Serbian American inventor *N*. Tesla who died in 1943 as well as Soviet physicist A. Zakharov. In fact in his later years Nobel Laureate J. Schwinger was a champion of cold fusion [12] which comes very near to our present concept of a Casimir-nano energy reactor [10] [11]. We also stress that we are making tacit use of the Banach-Tarski decomposition theorem as a Schwinger-like source [18] [21] [34]. The main conclusion that jumps out of this picture is that scale in physics is not a trivial idea and can only be deeply comprehended via mathematical tools such as P-Adic quantum mechanics and Witten's T-duality [93]-[95].

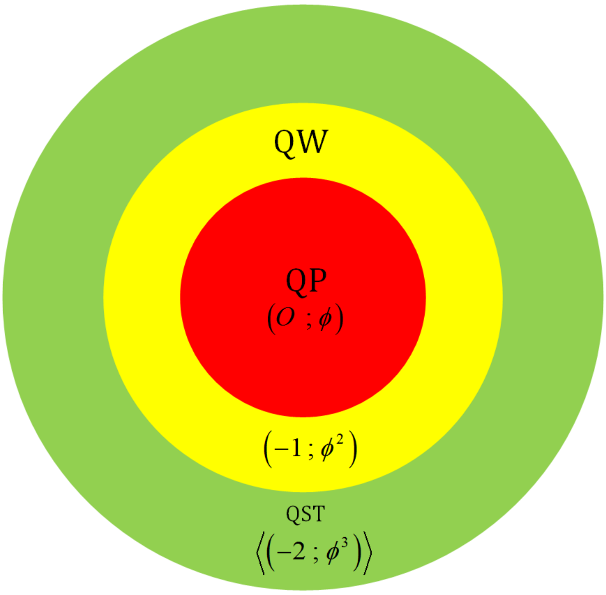

Figure 2. The quantum spacetime E-infinity hierarchy [12] [13].

Remarkably our Fibonacci-like dimension series could be extended into the negative side using the same logic as follows [64] [71]

$$\begin{aligned}
&D(1)=1+0\\
&D(O)=0+\phi\\
&D(-1)=(1-0)+(0-\phi)=1-\phi=\phi^2\\
&D(-2)=(0-1)+\phi-(-\phi)=-1+2\phi=\phi^3\\
&D(-3)=1-(-1)+(\phi)-2\phi=(1+1)-3\phi=\phi^4\\
&D(-4)=\cdots=\phi^5\\
&\qquad\vdots\\
&D(-n)=\cdots=\phi^{n+1}.
\end{aligned}$$

Although the preceding computation is mechanistic and essentially elementary, it is quite tedious because of the heavy and slightly confusing notation for negative dimensions. It is at this point that one discovers the advantage of the equivalent notation used in E-infinity theory [69]. The equivalent formula to that of the dimensional function is simply the bijection formula [33] [65] [69]

$$d_c^{(n)}=(1/\phi)^{n-1}.$$

Here n is the Menger-Urysohn topological dimension while $d_c^{(n)}$ denotes the Hausdorff dimension corresponding to n. Consequently for the zero set and the empty set we have [69] [70]

$$d_c^{(0)}=(1/\phi)^{0-1}=\phi$$

and

$$d_c^{(-1)}=(1/\phi)^{-1-1}=\phi^2$$

correspondingly. For the core Hausdorff dimension of E-infinity we just need to set $n = 4$ and find [69]-[71]

$$d_c^{(4)}=(1/\phi)^{4-1}=(1/\phi)^3=4+\phi^3$$

We note for later use that for $n = 2$, which means the surface of the pre-quantum wave we find

$$d_c^{(-2)} = (1/\phi)^{-2-1} = (1/\phi)^{-3} = \phi^3.$$

Now we ask an important question, namely what happens when $n \to \pm\infty$? In the positive domain we find of course that

$$d_c^{(n)} = (1/\phi)^{\infty-1} = (1/\phi)^{\infty} = \infty.$$

By contrast for $n \to -\infty$ one finds [69]-[71]

$$d_c^{(n)} = (1/\phi)^{-\infty-1} = (1/\phi)^{-\infty} = \phi^{\infty} = \text{zero}.$$

This is clearly the end of our and any universe. Said in a mundane way, we basically have a Cantorian onion in the two dimensional projection as shown in **Figure 2**. First we have the zero set quantum pre-particle $D(O) \equiv (O,\phi)$. This is surrounded by its cobordism, *i.e.* its surface, namely the empty set quantum pre-wave $D(1) \equiv (-1,\phi^2)$. Next the surface of the quantum wave is also an empty set but not only simply empty. It is an emptier set than the empty set. The highly interesting point is that the inversion of this ϕ^3 empty set given by $D(-2) = \phi^3$ is simply the Hausdorff dimension of the core of our E-infinity Cantorian spacetime. This is so because [69]-[71]

$$1/(D(-2)) = 1/\phi = 4 + \phi^3$$

while the average dimension of the entire spacetime is given by

$$\sum_{n=1}^{\infty}\phi^n + \phi^0 + \phi^{-1} = 1/\phi + 1 + 1/\phi = 1 + \phi + 1 + 1 + \phi$$
$$= 3 + 2\phi = 3 + 1 + \phi^3 = 4 + \phi^3$$

which is precisely the same value as that of the inversion of the cobordism of the empty set quantum wave. Thus we conclude that the picture of **Figure 2** is accurate in the most stringent mathematical and transfinite topological sense and we just have to be put $D(-2,\phi^3)$ between averaging brackets to read $D(-2,\phi^3)$. That way the picture is complete and will play a pivotal role in understanding the mirror symmetry and T-duality of Casimir and dark energy [93]-[95].

3. The Geometry and Topology of the Casimir Effect [89] [91]

Consider the classical Casimir set up and the hydromechanical analogy that goes with it. Bringing the two Casimir plates as near as a few nano units, we can consider the space between the plates practically totally empty of any “waves” or “particles” pushing them apart. Outside the plates spacetime resembles a stormy sea, which pushes the plates towards each other. Now the inside of the plates has the ϕ^2 topological “pressure” of the empty set inherited from its Hausdorff dimension. The outside will have at least the topological pressure of quantum particles which is the zero set represented by ϕ. The pressure difference between the outside and the inside of the two plates is given by the difference namely $\phi - \phi^2 = \phi^3$. In other words in the limit the exact Casimir topological pressure is simply ϕ^3. Recalling that Einstein spacetime $D = 4$ is a smooth spacetime while the core is $D = 4 + \phi^3$ indicating its fractal character we see that the extra ϕ^3 fractal part is equal to the limit of the topological Casimir pressure. To find the density of the Casimir energy in a way comparable to that of the dark energy density of the cosmos we have to transfer the result to a five dimensional K-K spacetime or a five dimensional anti de Sitter spacetime. This is easily done because it is nothing but the union of all thefour dimensional fusion algebra dimension function given by $d(1) = d(\in) = 1$ and $d(\alpha) = d(\beta) = \phi$ which means: [92]

$$D = 1 + 1 + 1/\phi + 1/\phi = 2(1 + 1/\phi) = 4 + 2\phi = 5 + \phi^3$$

The Casimir energy density is therefore given by the following ratio [35]-[51]

$$\gamma = \frac{\phi^3}{5 + \phi^3} = \phi^5/2 \simeq 1/22.$$

This is nothing but the familiar ordinary energy density of the universe that agrees with all previous analysis of the problem as well as accurate observations and measurements. Now this is only about 4.5 percent of the expected energy, so where is the rest is a natural question. However this question was answered some time ago. The bulk of the energy density is due to the quantum wave and is given by [33]-[51]

$$\gamma(\text{dark}) = (D=5)(\phi^2)/2 \simeq (21)/(22).$$

This is about 95.5 percent of the energy and is stored in the integer part of $D = 5+\phi^3$, namely $D = 5$. Consequently

$$\gamma(\text{dark}) = 5/5+\phi^3 \simeq 21/22.$$

The result is in complete agreement with that obtained using various methods. The fact that this 95.5 percent is located at the edge of the universe is explained within our five dimensional spacetime theory via Dvoretzky's theorem [59] [63]. On reflection the result is obvious. To see how obvious let us imagine that the two Casimir plates of the famous experiment are taken apart in two opposite directions until each one reaches the cosmological boundary. Thus instead of having ϕ^2 of the empty set between them they have now the entire topological pressure of the universe between them, that is to say $5+\phi^3$. Outside there is nothing to balance this pressure because on the other side there is zero pressure, which is natural for a one sided Möbius-like boundary. The topological density is thus $(5+\phi^3)/(5+\phi^3)=1$ which is Einstein's theoretical maximal topological energy density. To find the dark energy density we just have to subtract ϕ^3 of the Casimir density which is pushing intrinsically in the opposite direction and find that $\left[(5+\phi^3)-\phi^3\right]/(5+\phi^3)=0.954915$ percent is the relevant energy density. The analysis could be extended to find the Immirzi parameter connecting loop quantum gravity with string theory by looking at $(4)/4+\phi^3=\phi^6$ which is the value found using different methods.

4. The Mirror Symmetry and T-Duality Connecting Casimir Energy with Dark Energy

There is a fascinating history behind the subject of this section testifying to the unity of science and showing that at the end, physicists and mathematicians of all moulds must think and work together on the most fundamental level to be able to push the borders of science forward [95]. The most word economical way to explain mirror symmetry for someone—like the author—living between physics and mathematics, is to say mirror symmetry means that two Calabi-Yau manifolds with different topologies give the same conformal quantum field theory [95]. A more general and probably stronger statement of the same fact can be formulate as a duality which in our particular mirror symmetry case is Witten's T-duality [93]-[95].

Let me be more specific even at the risk of appearing elementary or even trivial. Let us start with Hardy's quantum entanglement. The exact solution for two quantum particles entanglement is given by the well known accurate result ϕ^5. Now quantum entanglement experiments may be performed on a billiard table rather than using the entire universe [96] [97]. However the inversion of ϕ^5 gives us the amazing fractal M-theory dimensionality of the entire universe, namely [94]

$$1/\phi^5 = 11+\cfrac{1}{11+\cfrac{1}{11+\cdots}}.$$

This is an implicit application of T-duality which is clear cut and in view of the experimental verification of Hardy's entanglement and the theoretical success of M-theory points clearly to being a physical reality [9] [12] [15] [30] [94]. It is really hard to believe that all these results, pertinent to the dark energy density of the cosmos $E(D)=(\phi^5/2)mc^2$ and the complimentary result of ordinary energy density $E(O)=1-E(D)=(5\phi^2/2)mc^2$ was not immediately noticed by the author that it means $E(O)=\left[\phi^3/(5+\phi^3)\right]mc^2$ and consequently implies mirror symmetry [95] and T-duality transformation of dark energy to ordinary measurable energy which is nothing but the intrinsic spacetime energy of our universe [94]. Even more astonishing was the initial failure of the present author to recognize that the $(22+k)/2=11+\phi^5$ factor is a super symmetric isomorphic length for a super symmetric Klein-Penrose fractal universe given by $(4+\phi^3)(5+\phi^3)$ and implies a topological Planck length equal this isomorphic radius and by mirror symmetry and T-duality ϕ^5 is a topological Planck energy [6] [7] [10] [15] [22] [23] [69] [94] [97]. The author is sure Sir Herman Bondi would have loved this conclusion. We urge the reader to study carefully Figures 2-4 of Ref. [94] to gain an intuitive grasp for the power and simplicity of Witten's T-duality connecting the quantum with the cosmic scale in the way presented here. We see here the interaction not only of physics and mathematics but also mathematics and engineering leading to a design of a Casimir-dark energy nano reactor [96].

5. Conclusions

To put things into perspective we should not forget that we always referred to the Casimir effect and dark energy as different forms of quantum vacuum zero point fluctuation. Nevertheless the fact that ϕ^3 is the intrinsic energy of spacetime and is equal to the global part of Hardy's quantum entanglement was not clear at all let alone that the Casimir energy density is equal to the ordinary energy density of the universe. Consequently we see that the Casimir energy density plus the energy of the quantum pre-wave in five dimensional spacetime leads to Einstein energy density. We see clearly that Casimir energy density and the quantum wave dark energy density lead to Einstein energy. This introduces to our modern physical and cosmological theory a hitherto unknown unit and elegance and all wrapped up in some of the most beautiful mathematical theorems like that of Dvoretzky and mirror symmetry of Witten's T-duality [93]-[95]. At the end we cannot find a rational reason for overlooking these results for a long time except Sir Herman Bondi's wise words about the nature of certain fundamental laws of nature.

It consists of three main layers [41]-[45]. We have first an infinite number of zero and empty sets with an average bi-dimension $\left\langle\left(-2;\phi^3\right)\right\rangle$. This is the outer circle representing quantum spacetime. Inside this we have the quantum wave given by the bi-dimension $\left(-1;\phi^2\right)$ which is the empty set. Finally inside the quantum wave as its inner eye, we have the zero set quantum particle with the bi-dimension $\left(O;\phi\right)$. The above picture also gives us an almost trivial resolution for quantum wave collapse. This is so because to "locate" QP we must somehow penetrate QW. Since QW is the empty set, the slightest touch would convert it to a non-empty set. Consequently QW disappears and metamorphose into QP. This is the observed mysterious state vector reduction which as the reader sees, is not mysterious at all within this topological set theoretical picture. The above picture gives us also an almost trivial resolution for quantum wave collapse. This is so because to "locate" QP we must penetrate somehow QW. Since QW is the empty set the slightest touch would convert it to a non-empty set. Consequently QW disappears and metamorphs into QP. This is the observed mysterious state vector reduction which as the reader sees, is not mysterious at all within this topological set theoretical picture.

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