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A Note on a One-Parameter Weibull Distributed Deteriorating Item EOQ Inventory Model with Varying Quadratic Demand and Delay in Payments

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Abstract

In this paper, an EOQ inventory model is developed for deteriorating items with variable rates of deterioration and conditions of grace periods when demand is a quadratic function of time. The deterioration rate considered here is a special type of Weibull distribution deterioration rate, *i.e.*, a one-parameter Weibull distribution deterioration rate and it increases with respect to time. The quadratic demand precisely depicts of the demand of seasonal items, fashion apparels, cosmetics, and newly launched essential commodities like android mobiles, laptops, automobiles etc., coming to the market. The model is divided into three policies according to the occurrence of the grace periods. Shortages, backlogging and complete backlogging cases are not allowed to occur in the model. The proposed model is well-explained with the help of a simple solution procedure. The three numerical examples are taken to illustrate the effectiveness of the EOQ inventory model along with sensitivity analysis.

Keywords

Economic Order Quantity (EOQ), One-Parameter Weibull Distribution Deterioration, Permissible Delay in Payments, Time-Dependent Quadratic Demand

1. Introduction

In recent three decades, most of the researches have been done on deteriorating items in inventory problems by a number of researchers. Items like seasonal fruits such as mango, grape and apple, vegetables like potatoes, carrots, etc. animal products like milk, meat, egg, fish etc., blood in blood banks, chemical and pharmaceutical products like medicines, drugs, volatile liquids and radio-active substances etc., deteriorate continuously due to the some natural phenomena like spoilage, decay and evaporation. The hardware, electronic items and essential commodities are not suitable for using in original purposes after their expiration periods. Such a type of physical phenomenon is known as deterioration. Therefore, it is always necessary to study the effect of deterioration on such items while formulating the models for deteriorating items. Ghare and Schrader [1] first formulated an Economic Order Quantity (EOQ) optimum policy for deteriorating items by using a negative exponential distribution. It is well known that the assumption of the demand pattern of the standard EOQ model is deterministic and is constant over an infinite planning horizon. However, most of the physical goods experience a steady demand pattern only for finite horizon of time during their life span. Furthermore, the nature of demand pattern is always time-dependent like constant, linear increasing or decreasing, exponential increasing or decreasing, etc. Therefore, some modification of the EOQ model is quite essential for future studies. In this regard, many researchers have been already done to accommodate the time-dependent demand pattern. An inventory replenishment no-shortage policy with constant rate of deterioration and linear tend in demand pattern over finite horizon of time was studied by Donaldson [2]. The inventory model developed related to the deteriorating items with deterioration as constant fraction of the on-hand inventory and demand as linear increasing pattern was formulated by Dave and Patel [3]. Later, Bahari-Kashani [4] presented a heuristic inventory model for determining the replenishment schedule for deteriorating items with linearly increasing demand rate subject to the constant deterioration. An inventory replenishment policy over a finite horizon for a deteriorating item having linear demand pattern and shortages was established by Goswami and Chaudhuri (1991) [5]. They determined the number of reorder points, the gap between two successive reorders and the shortage periods over a finite horizon of time in order to maintain the optimal average system cost. However, deterioration is independent of demand patterns and dependent on the distribution of time period. Therefore, constant rate of decay is no more lasting for the formulation of decaying inventory model. The EOQ model for deteriorating items where the distribution of the time to deterioration follows the two-parameter Weibull distribution was considered by Covert and Philip [6]. An optimal production lot size model with both the varying and constant rate of deterioration and no-shortages was presented by Mishra [7]. Nahmias [8], Raafat [9], Goyal and Giri [10] and Li *et al.* [11] reviewed the advances of deteriorating inventory literature. Singh *et al.* [12] established an optimal ordering policy for deteriorating items with inventory dependent demand and initial order quantity dependent deterioration. A three-parameter Weibull distributed deteriorated inventory model with quadratic demand and salvage value under partial backlogging was presented by Singh *et al.*

[13]. Singh *et al.* [14] also developed an ordering policy with varying deterioration rate, time-dependent trapezoidal-type demand rate with shortages. Kumar and Yadav [15] established an optimal inventory model for the advanced payment strategy on perishable item with maximum lifetime, customer return and preservation technology under shortages.

In business scenario, it is customary for customers to have a specified grace period before paying a supplier or producer. During this fixed period, the customer is not allowed to pay the interest, but if payment is not made before the end of the grace period, the supplier will set in motion to charge interest. This grace period is referred to as the delay period or the permissible delay period or the trade credit period, and during this period, the customer may sell the goods and earn interest on the revenue generated from the sales. In this context, Goyal [16] studied the economic order quantity under conditions of permissible delay in payments. In business, the unit selling price should be greater than the unit purchasing price. The ordering policies of deteriorating items under permissible delay in payments were studied by Aggarwal and Jaggi [17]. In their models, the demand rate and deterioration rate were assumed as constant. Jamal *et al.* [18] studied the inventory model to determine an optimal ordering policy for deteriorating items under permissible delay of payment and allowable shortage. Ouyang *et al.* [19] developed an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Musa and Sani [20] studied the ordering policies for the inventory model of delayed deteriorating items under permissible delay in payments. Furthermore, Khanra *et al.* [21], Singh and Pattnayak [22] developed the EOQ models for a deteriorating item under permissible delay in payment assuming the time varying demand rate and variable deterioration rate. Singh and Pattanayak [23] presented an optimal policy for a deteriorating item with varying deterioration rate and time-dependent demand rate and the delay in payment conditions. Singh *et al.* [24] presented a note on optimal model with time-dependent demand, three-parameter Weibull distribution deterioration, no-shortages and permissible delay in payment. Pant *et al.* [25] studied an optimal replenishment and preservation investment policy for deteriorating items with hybrid demand rate and trade credit schemes. Mohanty and Singh [26] established an inventory model for a deteriorating item with time-dependent cubic demand and variable deterioration under delay in payment conditions. A note on an order level optimal policy with varying two-phased demand and variable deterioration rate was developed by Mohanty *et al.* [27]. In the real life situation, the deterioration rate in the items increases with respect to time always. Deterioration rate in items are determined by different Weibll distribution and Gamma distribution, etc., in this respect, Pal and Ghosh [28] studied an optimal inventory policy with stock dependent demand and general rate of deterioration under conditions of grace periods in payments. They incorporated two different deterioration rates such constant deterioration rate and one parameter Weibull distribution deterioration rate as two special types of Weibull distribution deterioration rate in their model.

An EOQ optimal model varying with exponential-constant-exponential demand and shortages was introduced by Rout *et al.* [29]. Swain and Singh [30] studied a note on optimal model with time-dependent demand, time-proportional deterioration, shortages and conditions of permissible delay in payments.

Formulation of optimal policy for deteriorating items having one-parameter Weibull distribution deterioration and time-dependent quadratic demand has seldom been mentioned. So, in this model, an optimal EOQ model is developed for deteriorating items with one-parameter Weibull distribution deterioration, quadratic demand pattern and different grace periods. Here the assumed grace period is either less than or greater than or equal to the cycle time. Shortages, partial and complete backlogging are not allowed to occur. Three numerical examples are mentioned to illustrate the effectiveness of the proposed EOQ model with sensitivity analysis.

The rest of the paper is set according to different sections which are stated as follows. In section 2 describes the notations and fundamental assumptions taken for the construction of the model throughout this paper. In section 3, the mathematical analysis of the model and its computational solution procedure are described in order to minimize the system costs. The three numerical examples and the sensitivity analysis of several parameters of some selected example are discussed in section 4. Finally, concluding remarks and the future work on deteriorating inventory research are pointed out in section 5.

2. Notations and Assumptions

The following mathematical notations and assumptions are needed for the formulation of the model.

2.1. Notations

p	Purchase cost per unit ($)
c_o	Ordering cost per order ($)
h_s	Holding cost of the inventory system excluding interest charges; ($) per unit per year
I_e	Interest which can be earned, ($) per year
I_p	Interest charges, ($) per year
$R(t)=a+bt+ct^2$ $(a>0,b>0,c\neq 0)$	Demand is continuous and quadratic in nature with respect to time. If $c=0$, $c=b=0$, then the demand function changes into linear and constant function, respectively
$\theta(t)=\alpha t^{\alpha-1}(0<\alpha\ll 1)$	Deterioration rate which is one-parameter Weibull distribution type
$I(t)$	Positive inventory level during the time period $[0,T]$
μ	Grace period offered by supplier at the time of settlement of account

Continued

T	Cycle time (decision variable)
S_0	Size of inventory
S_0^*	Optimal size of inventory
$ASC_1(T)$	Average system cost per unit time ($) when $\mu < T$
$ASC_2(T)$	Average system cost per unit time ($) when $\mu > T$
$ASC(T)$	Minimum system cost per unit time ($)

2.2. Assumptions

1) The inventory system deals with one type of items.

2) The demand rate is related with quadratic function of the time during the cycle.

3) The deterioration rate follows one-parameter Weibull distribution.

4) All system costs (purchase, ordering and holding) are taken as constant.

5) The planning horizon is taken infinite with negligible delivery lead time.

6) The grace period is taken less than, greater than and equal to the cycle time in three policies, respectively.

7) The model does not consider shortages with partial as well as complete back-logging.

8) The replenishment is instantaneous.

3. Mathematical Formulation of the Model

In this section, a model is formulated for one-parameter Weibull distribution deterioration rate and quadratic demand rate with permissible delay in payment conditions when replenishment occurs. Figure 1 depicts the proposed inventory system with respect to time.

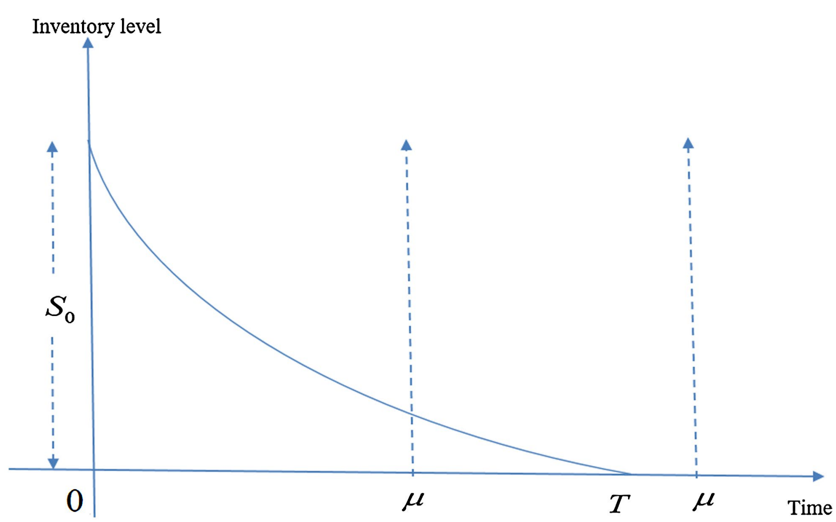

Figure 1. Graph of inventory depletion with time.

The proposed model derived under three different policies, viz. Policy I: the grace period μ is less than the cycle time T. Policy II: the grace period μ is greater than the cycle time T and Policy III: the grace period μ is equal to the cycle time T. The loss of utility of inventory is due to the combined effect of demand as well as deterioration function. For the beginning, *i.e.*, at time $t=0$, the order quantity is S_0 and replenishment occurs after each cycle time T.

The differential equation governing the inventory status during the time interval $[0,T]$ is given by

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t}+\theta(t)I(t)+R(t)=0,\quad 0\le t\le T, \tag{1}$$

where $\left(\theta(t)=\alpha t^{\alpha-1},\ 0<\alpha\ll 1\right)$ & $\left(R(t)=a+bt+ct^2,\ a>0,b\ne 0,c\ne 0\right)$.

Here, the integrating factor (IF) and the solution with the help of the boundary condition $I(T)=0$ are

$$IF=\mathrm{e}^{t^\alpha}, \tag{2}$$

and

$$\begin{aligned}I(t)=\Bigg[&aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{aT^{\alpha+1}}{\alpha+1}+\frac{bT^{\alpha+2}}{\alpha+2}+\frac{cT^{\alpha+3}}{\alpha+3}\\&-at-\frac{bt^2}{2}-\frac{ct^3}{3}-\frac{at^{\alpha+1}}{\alpha+1}-\frac{bt^{\alpha+2}}{\alpha+2}-\frac{ct^{\alpha+3}}{\alpha+3}\Bigg]\cdot\mathrm{e}^{-t^\alpha},\ 0\le t\le T,\end{aligned} \tag{3}$$

respectively, (by ignoring the terms containing the powers like $2\alpha,3\alpha,4\alpha,\cdots$ as $0<\alpha\ll 1$).

The initial status of inventory level (S_0) is calculated by putting $I(0)=S_0$ in Equation (3), *i.e.*,

$$S_0=aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{aT^{\alpha+1}}{\alpha+1}+\frac{bT^{\alpha+2}}{\alpha+2}+\frac{cT^{\alpha+3}}{\alpha+3}. \tag{4}$$

The average system cost ($ASC(T)$) of the system for each cycle comprises of the following cost components:

- Ordering cost (CO):

$$CO=c_0. \tag{5}$$

- Holding cost (CH) during the interval $[0,T]$:

$$CH=ph_s\int_0^T I(t)\,\mathrm{d}t=c_h\int_0^T I(t)\,\mathrm{d}t,$$

where $ph_s=c_h$, *i.e.*,

$$CH=c_h\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\frac{\alpha}{\alpha+1}\left(\frac{aT^{\alpha+2}}{\alpha+2}+\frac{bT^{\alpha+3}}{\alpha+3}+\frac{cT^{\alpha+4}}{\alpha+4}\right)\right], \tag{6}$$

(by ignoring the terms containing the powers like $2\alpha,3\alpha,4\alpha,\cdots$ as $0<\alpha\ll 1$).

- Deterioration cost (CD) during the interval $[0,T]$:

$$CD=p\left[S_0-\int_0^T\left(a+bt+ct^2\right)\mathrm{d}t\right]=c_p\left(\frac{aT^{\alpha+1}}{\alpha+1}+\frac{bT^{\alpha+2}}{\alpha+2}+\frac{cT^{\alpha+3}}{\alpha+3}\right). \tag{7}$$

Policy I: $\mu < T$.

1) Interest earned (EI_1) during $[0,T]$:

$$EI_1 = pI_e\int_0^T t\left(a+bt+ct^2\right)\mathrm{d}t = pI_e\left(\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}\right). \tag{8}$$

2) Interest charged (CI_1) during $[0,T]$:

$$\begin{aligned} CI_1 &= pI_p\int_0^T I(t)\mathrm{d}t \\ &= pI_p\Bigg[\left(aT+\frac{bT^2}{2}+\frac{cT^3}{3}+\frac{aT^{\alpha+1}}{\alpha+1}+\frac{bT^{\alpha+2}}{\alpha+2}+\frac{cT^{\alpha+3}}{\alpha+3}\right) \\ &\quad\times\left(T-\mu-\frac{T^{\alpha+1}-\mu^{\alpha+1}}{\alpha+1}\right)-\frac{a}{2}\left(T^2-\mu^2\right)-\frac{b}{6}\left(T^3-\mu^3\right)-\frac{c}{12}\left(T^4-\mu^4\right) \\ &\quad+\alpha\left\{\frac{a\left(T^{\alpha+2}-\mu^{\alpha+2}\right)}{(\alpha+1)(\alpha+2)}+\frac{b\left(T^{\alpha+3}-\mu^{\alpha+3}\right)}{2(\alpha+2)(\alpha+3)}+\frac{c\left(T^{\alpha+4}-\mu^{\alpha+4}\right)}{3(\alpha+3)(\alpha+4)}\right\}\Bigg], \end{aligned} \tag{9}$$

(by ignoring the terms containing the powers like $2\alpha, 3\alpha, 4\alpha, \cdots$ as $0<\alpha\ll 1$).

Using Equations (5)-(9), the average system cost ($ASC_1(T)$) of the integrated inventory model per unit time is calculated by

$$ASC_1(T) = \frac{1}{T}\left[CO+CH+CD+EI_1-CI_1\right]. \tag{10}$$

The objective of the present study is to determine the minimum value of the average system cost of the model by optimizing the cycle time T. For the optimality, the necessary and sufficient conditions of the corresponding average system cost ($ASC_1(T)$) are given below:

Necessary conditions:

$$\begin{aligned} \frac{\mathrm{d}\left[ASC_1(T)\right]}{\mathrm{d}T} &= \frac{1}{T}\Bigg[\left(a+bT+cT^2\right)\Bigg[c_h\left(T+\frac{\alpha T^{\alpha+1}}{\alpha+1}\right)+pT^\alpha \\ &\quad+pI_p\left(1+T^\alpha\right)\left(T-\mu-\frac{T^{\alpha+1}}{\alpha+1}+\frac{\mu^{\alpha+1}}{\alpha+1}\right)-pI_eT\Bigg]-ASC_1(T)\Bigg] \\ &= 0. \end{aligned} \tag{11}$$

Now solving Equation (11), the optimal value of T as T_1^* is obtained. The corresponding optimal average system cost of the system and EOQ are found by substituting the value of T_1^* in Equations (10) and (4), respectively.

Sufficient conditions:

It must satisfies

$$\begin{aligned} \frac{\mathrm{d}^2\left[ASC_1(T)\right]}{\mathrm{d}T^2} &= \frac{1}{T}\Bigg[(b+2cT)\Bigg[c_h\left(T+\frac{\alpha T^{\alpha+1}}{\alpha+1}\right)+pT^\alpha \\ &\quad+pI_p\left(1+T^\alpha\right)\left(T-\mu-\frac{T^{\alpha+1}}{\alpha+1}+\frac{\mu^{\alpha+1}}{\alpha+1}\right)-pI_eT\Bigg] \\ &\quad+\left(a+bT+cT^2\right)\Big[c_h\left(1+\alpha T^\alpha\right)+\alpha pT^{\alpha-1} \end{aligned}$$

$$+\alpha pI_pT^{\alpha-1}\left(T-\mu-\frac{T^{\alpha+1}}{\alpha+1}+\frac{\mu^{\alpha+1}}{\alpha+1}\right)+p\left(I_p-I_e\right)\Bigg]$$
$$-\frac{2\mathrm{d}\left(ASC_1\left(T\right)\right)}{\mathrm{d}T}\Bigg] \tag{12}$$
$$>0.$$

Policy II: $\mu > T$.

1) Interest earned (EI_2) during $\left[0,T\right]$:

$$\begin{aligned} EI_2 &= pI_e\left[\int_0^T t\left(a+bt+ct^2\right)\mathrm{d}t+\left(\mu-T\right)\int_0^T\left(a+bt+ct^2\right)\mathrm{d}t\right] \\ &= pI_e\left[\frac{aT^2}{2}+\frac{bT^3}{3}+\frac{cT^4}{4}+\left(\mu-T\right)\left(aT+\frac{bT^2}{2}+\frac{cT^3}{3}\right)\right]. \end{aligned} \tag{13}$$

2) Interest charged (CI_2) during $\left[0,T\right]$:

$$CI_2 = 0. \tag{14}$$

Using Equations (5)-(7) and (13)-(14), the average system cost ($ASC_2\left(T\right)$) of the integrated inventory model per unit time is calculated by

$$ASC_2\left(T\right)=\frac{1}{T}\left[CO+CH+CD+EI_2-CI_2\right]. \tag{15}$$

For the optimality, the necessary and sufficient conditions of the corresponding average system cost ($ASC_2\left(T\right)$) are given below:

Necessary conditions:

$$\begin{aligned} \frac{\mathrm{d}\left[ASC_2\left(T\right)\right]}{\mathrm{d}T} &= \frac{1}{T}\Bigg[\left(a+bT+cT^2\right)\left[c_h\left(T+\frac{\alpha T^{\alpha+1}}{\alpha+1}\right)+pT^{\alpha}\right] \\ &\quad -pI_e\left[\frac{bT^2}{2}+\frac{2cT^3}{3}+\left(\mu-T\right)\left(a+bT+cT^2\right)\right]-ASC_2\left(T\right)\Bigg] \\ &= 0. \end{aligned} \tag{16}$$

Now solving Equation (16), the optimal value of T as T_2^* is obtained. The corresponding optimal average system cost and EOQ are found by substituting the value of T_2^* in Equations (15) and (4), respectively.

Sufficient conditions:

It must satisfies

$$\begin{aligned} \frac{\mathrm{d}^2\left[ASC_2\left(T\right)\right]}{\mathrm{d}T^2} &= \frac{1}{T}\Bigg[\left(b+2cT\right)\left[c_h\left(T+\frac{\alpha T^{\alpha+1}}{\alpha+1}\right)+pT^{\alpha}\right] \\ &\quad +\left(a+bT+cT^2\right)\left[c_h\left(1+\alpha T^{\alpha}\right)+\alpha pT^{\alpha-1} \\ &\quad -pI_e\left(a+bT-\mu\left(b+2cT\right)\right)\right]-\frac{2\mathrm{d}\left(ASC_2\left(T\right)\right)}{\mathrm{d}T}\Bigg] \\ &> 0. \end{aligned} \tag{17}$$

Policy III: $\mu = T$.

For time $\mu = T$, both the average system costs $ASC_1\left(T\right)$ and $ASC_2\left(T\right)$ are same and the respective system cost $ASC(T)$ is determined by putting $\mu = T$ in either Equation (10) or (15). The EOQ in three policies can be calculated from Equation (4) by providing the corresponding value of T.

4. Computaional Algorithms and Numerical Examples

4.1. Computaional Algotithm

The aim of the classical optimization model is to minimize the average system cost. The working procedure is dependent on the following steps.

Step 1: Perform (i)-(iv)

(i) Assign the values of the system parameters with their proper units in Policy I.

(ii) Evaluate the first-order partial derivative of the average system cost with respect to the decision variable T and equate it to zero. Then, solve for T_1 from equation $\frac{\mathrm{d}\left[ASC_1(T)\right]}{\mathrm{d}T} = 0$.

(iii) Check the convexity of the objective function, *i.e.* $\frac{\mathrm{d}^2\left[ASC_1(T)\right]}{\mathrm{d}T^2} > 0$.

(iv) Calculate $ASC_1\left(T_1^*\right)$ by putting $T = T_1^*$.

Step 2: Perform (i)-(iv)

(i) Assign the values of the system parameters with their proper units in Policy II.

(ii) Evaluate the first-order partial derivative of the system cost with respect to the decision variable T and equate it to zero. Then, solve for T_2 from equation $\frac{\mathrm{d}\left[ASC_2(T)\right]}{\mathrm{d}T} = 0$.

(iii) Check the convexity of the objective function, *i.e.* $\frac{\mathrm{d}^2\left[ASC_2(T)\right]}{\mathrm{d}T^2} > 0$.

(iv) Calculate $ASC_2\left(T_2^*\right)$ by putting $T = T_2^*$.

Step 3: Perform (i)-(iii)

(i) If both $\mu < T_1^*$ and $\mu > T_2^*$ are satisfied, then $ASC\left(T^*\right)$, the optimal average system cost, is obtained by comparing the values of $ASC_1\left(T_1^*\right)$ and $ASC_2\left(T_2^*\right)$. Or

(ii) If $\mu < T_1^*$ is true and $\mu > T_2^*$ is false, then $ASC(T)$, the optimal average system cost, is obtained from $ASC_1\left(T_1^*\right)$. Or

(iii) If $\mu < T_1^*$ is false and $\mu > T_2^*$ is true, then $ASC(T)$, the optimal average system cost, is obtained from $ASC_2\left(T_2^*\right)$.

Step 4. Finally, calculate the respective EOQ.

4.2. Numerical Examples

The proposed study has been illustrated with three numerical examples with the appropriate units of the system parameters:

Example 1: Policy I and Policy II:

Let $c_h = \$0.12/\text{year}$, $c_o = \$200/\text{order}$, $p = \$20/\text{unit}$, $I_e = 0.13/\text{year}$, $I_p = 0.15/\text{year}$, $a = 240$ units/year, $b = 120$ units/year, $c = 16$ units/year, $\alpha = 0.002$ and $\mu = 0.4$ year.

Solving Equation (11), we get $T_1^* = 0.759103$ year and the corresponding average system cost $ASC\left(T_1^*\right) = \$5507.36$ provided the sufficient condition

$$\left.\frac{\mathrm{d}^2\left(ASC(T)\right)}{\mathrm{d}T^2}\right|_{T=T_1^*} = 1411.47 > 0.$$

Similarly, solving Equation (16), we get $T_2^* = 0.353253$ year and the corresponding average system cost $ASC\left(T_2^*\right) = \$5632.74$ provided the sufficient condition $\left.\frac{\mathrm{d}^2\left(ASC(T)\right)}{\mathrm{d}T^2}\right|_{T=T_2^*} = 5930.24 > 0$.

Here both $\mu < T_1^*$ and $\mu > T_2^*$ are satisfied, then $ASC\left(T^*\right)$, the optimal average system cost is obtained by comparing the values of $ASC\left(T_1^*\right)$ and $ASC\left(T_2^*\right)$. Hence the optimal average system cost, cycle time and EOQ are $ASC\left(T_1^*\right) = \$5507.36$, $T_1^* = 0.759103$ year and $S_0^* = 437.664$ units, respectively.

Example 2: Policy I:

Let $c_h = \$0.12/\text{year}$, $c_o = \$200/\text{order}$, $p = \$20/\text{unit}$, $I_e = 0.13/\text{year}$, $I_p = 0.15/\text{year}$, $a = 240$ units/year, $b = 120$ units/year, $c = 16$ units/year, $\alpha = 0.8$ and $\mu = 0.2$ year.

Solving Equation (11), we get $T_1^* = 0.254092$ year and the corresponding average system cost $ASC\left(T_1^*\right) = \$1673.48$ provided the sufficient condition

$$\left.\frac{\mathrm{d}^2\left(ASC(T)\right)}{\mathrm{d}T^2}\right|_{T=T_1^*} = 22229.8 > 0.$$

Similarly, solving Equation (16), we get $T_2^* = 0.233225$ year and the corresponding average system cost $ASC\left(T_2^*\right) = \$1700.88$ provided the sufficient condition $\left.\frac{\mathrm{d}^2\left(ASC(T)\right)}{\mathrm{d}T^2}\right|_{T=T_2^*} = 5930.24 > 0$.

Here $\mu < T_1^*$ is true and $\mu > T_2^*$ is false, then $ASC\left(T^*\right)$, the optimal average system cost is obtained from $ASC\left(T_1^*\right)$. Hence the optimal average system cost, cycle time and EOQ are $ASC\left(T_1^*\right) = \$1673.48$, $T_1^* = 0.254092$ year and $S_0^* = 118.961$ units, respectively.

Example 3: Policy II:

Let $c_h = \$0.12/\text{year}$, $c_o = \$200/\text{order}$, $p = \$20/\text{unit}$, $I_e = 0.13/\text{year}$, $I_p = 0.15/\text{year}$, $a = 240$ units/year, $b = 120$ units/year, $c = 16$ units/year, $\alpha = 0.08$ and $\mu = 0.5$ year.

Solving Equation (11), we get $T_1^* = 0.479376$ year and the corresponding average system cost $ASC\left(T_1^*\right) = \$5000.03$ provided the sufficient condition

$$\left.\frac{\mathrm{d}^2\left(ASC(T)\right)}{\mathrm{d}T^2}\right|_{T=T_1^*}=4464.0>0.$$

Similarly, solving Equation (16), we get $T_2^*=0.276463$ year and the corresponding average system cost $ASC\left(T_2^*\right)=\$4789.29$ provided the sufficient condition $\left.\frac{\mathrm{d}^2\left(ASC(T)\right)}{\mathrm{d}T^2}\right|_{T=T_2^*}=11773.8>0$.

Here $\mu<T_1^*$ is false and $\mu>T_2^*$ is true, then $ASC\left(T^*\right)$, the optimal average system cost is obtained from $ASC\left(T_2^*\right)$. Hence the optimal average system cost, cycle time and EOQ are $ASC\left(T_2^*\right)=\$4789.29$, $T_1^*=0.276463$ year and $S_0^*=130.559$ units, respectively.

5. Sensitivity Analysis

In order to examine the implications of the change in the values of the parameters, the sensitivity analysis will be a great help in decision-making. With the help of Example 1 given in the preceding section, the sensitivity analysis of several parameters has been done and the results are summarized in Table 1 and Table 2, respectively.

The following observations can be made from Table 1.

(i) Changes in carrying cost (c_h) does not have any significant effect in the present value of the average system cost $ASC\left(T^*\right)$ and result increasing in both T_1^* and T_2^*.

(ii) Changes in ordering cost (c_o) have moderately effect in the present value of the average system cost $ASC\left(T^*\right)$ and result decreasing in both T_1^* and T_2^*.

(iii) Changes in purchase cost (p) have highly effect in the present value of the average system cost $ASC\left(T^*\right)$ and result increasing in both T_1^* and T_2^*.

(iv) Changes in earned interest (I_e) have moderately effect in the present value of the average system cost $ASC\left(T^*\right)$ and result increasing in T_1^* and decreasing in T_2^*.

(v) Changes in chargeable interest (I_p) have highly effect in the present value of the average system cost $ASC\left(T^*\right)$ and result decreasing in T_1^* and remaining constant in T_2^*.

(vi) Changes in constant (a) have moderately effect in the present value of the average system cost $ASC\left(T^*\right)$ and result decreasing in T_1^* and increasing in T_2^*.

(vii) Changes in constants ($b, c\ \&\ \alpha$) have moderately effect in the present value of the average system cost $ASC\left(T^*\right)$ and result increasing in both T_1^* and T_2^*.

(viii) Changes in grace period (μ) have moderately effect in the present value of the average system cost $ASC\left(T^*\right)$ and result decreasing in both T_1^* and T_2^*.

Table 1. Sensitivity analysis of time cycle and optimal cost.

Parameters	Decreasing value of parameters	$ASC(T_1^*)$	T_1^*	$ASC(T_2^*)$	T_2^*	Remarks	Optimal solution	% Change in optimal solution
c_h	0.180	5514.27	0.748277	5636.60	0.352292	$T_2^* < \mu < T_1^*$	5514.27	+0.12
	0.150	5510.83	0.753606	5634.17	0.352772	$T_2^* < \mu < T_1^*$	5510.83	+0.06
	0.126	5508.06	0.757989	5633.03	0.353157	$T_2^* < \mu < T_1^*$	5508.06	+0.01
	0.114	5506.66	0.760224	5632.45	0.353350	$T_2^* < \mu < T_1^*$	5506.66	−0.01
	0.090	5503.86	0.764780	5631.31	0.353737	$T_2^* < \mu < T_1^*$	5503.86	−0.06
	0.060	5500.33	0.770650	5629.88	0.354223	$T_2^* < \mu < T_1^*$	5500.33	−0.12
c_o	300	5626.38	0.925228	5888.03	0.429894	$\mu < T_1^*$	5626.38	+2.16
	250	5569.80	0.843313	5766.61	0.393649	$T_2^* < \mu < T_1^*$	5569.80	+1.13
	210	5520.39	0.776209	5660.71	0.361732	$T_2^* < \mu < T_1^*$	5520.39	+0.23
	190	5494.04	0.741834	5604.08	0.344545	$T_2^* < \mu < T_1^*$	5494.04	−0.24
	150	5437.38	0.670757	5481.31	0.307005	$T_2^* < \mu < T_1^*$	5437.38	−1.27
	100	5357.08	0.575580	5302.33	0.251579	$T_2^* < \mu < T_1^*$	5357.08	−2.72
c_p	30	8112.22	0.645473	8135.88	0.290316	$T_2^* < \mu < T_1^*$	8112.22	+47.72
	25	6812.01	0.692622	6890.44	0.317188	$T_2^* < \mu < T_1^*$	6812.01	+25.11
	21	5768.73	0.743683	5885.45	0.345055	$T_2^* < \mu < T_1^*$	5768.73	+6.89
	19	5245.73	0.775877	5379.35	0.362067	$T_2^* < \mu < T_1^*$	5245.73	−2.32
	15	4196.04	0.861127	4357.89	0.405382	$\mu < T_1^*$	4196.04	−23.81
	10	2873.67	1.04175	3056.44	0.490839	$\mu < T_1^*$	2873.67	−47.82
I_e	0.1950	…	…	5553.97	0.339004	…	…	…
	0.1625	…	…	5593.61	0.345898	…	…	…
	0.1365	5491.98	0.785116	5624.96	0.351743	$T_2^* < \mu < T_1^*$	5491.98	−0.27
	0.1235	5522.15	0.736488	5640.50	0.354784	$T_2^* < \mu < T_1^*$	5522.15	+0.26
	0.0975	5576.71	0.667522	5671.33	0.361125	$T_2^* < \mu < T_1^*$	5576.71	+1.25
	0.0650	5637.54	0.608371	5709.35	0.369579	$T_2^* < \mu < T_1^*$	5637.54	+2.36
p	0.2250	…	…	5632.74	0.353253	…	…	…
	0.1875	…	…	5632.74	0.353253	…	…	…
	0.1575	5495.58	0.788754	5632.74	0.353253	$T_2^* < \mu < T_1^*$	5495.58	−0.21
	0.1425	5518.35	0.732758	5632.74	0.353253	$T_2^* < \mu < T_1^*$	5518.35	+0.19
	0.1125	5555.80	0.694109	5632.74	0.353253	$T_2^* < \mu < T_1^*$	5555.80	+0.87

Continued

	0.0750	5590.95	0.572989	5632.74	0.353253	$T_2^* < \mu < T_1^*$	5590.95	+1.51
a	360	…	…	7954.23	0.335601	…	…	…
	300	…	…	6793.87	0.344116	…	…	…
	252	5724.92	0.798964	5865.03	0.351372	$T_2^* < \mu < T_1^*$	5724.92	+3.95
	228	5288.43	0.724812	5400.41	0.355162	$T_2^* < \mu < T_1^*$	5288.43	−3.97
	180	4401.80	0.621861	4470.77	0.363090	$T_2^* < \mu < T_1^*$	4401.80	−20.07
	120	3274.97	0.534887	3307.89	0.373716	$T_2^* < \mu < T_1^*$	3274.97	−40.53
b	180	5845.23	0.508466	5820.75	0.303183	$T_2^* < \mu < T_1^*$	5845.23	+6.13
	150	5691.09	0.597118	5730.39	0.325380	$T_2^* < \mu < T_1^*$	5691.09	+3.33
	126	5547.45	0.716034	5652.94	0.347110	$T_2^* < \mu < T_1^*$	5547.45	+0.72
	114	5465.02	0.811407	5612.17	0.359731	$T_2^* < \mu < T_1^*$	5465.02	−0.76
	90	…	…	5525.72	0.389684	…	…	…
	60	…	…	5406.07	0.440066	…	…	…
c	24	5533.08	0.703288	5639.05	0.349455	$T_2^* < \mu < T_1^*$	5533.08	+0.46
	20	5520.68	0.728499	5635.91	0.351329	$T_2^* < \mu < T_1^*$	5520.68	+0.24
	16−8	5510.11	0.752441	5633.3856	0.352864	$T_2^* < \mu < T_1^*$	5510.11	+0.04
	15.2	5504.56	0.766087	32.10	0.353644	$T_2^* < \mu < T_1^*$	5504.56	−0.05
	12	5492.85	0.797965	5629.53	0.355231	$T_2^* < \mu < T_1^*$	5492.85	−0.26
	8	…	…	5626.29	0.357265	…	…	…
α	0.0030	5501.07	0.751624	5622.34	0.351889	$T_2^* < \mu < T_1^*$	5501.07	−0.11
	0.0025	5504.22	0.755344	5627.54	0.352570	$T_2^* < \mu < T_1^*$	5504.22	−0.05
	0.0021	5506.73	0.758348	5631.70	0.353116	$T_2^* < \mu < T_1^*$	5506.73	−0.01
	0.0019	5507.99	0.759859	5633.78	0.353390	$T_2^* < \mu < T_1^*$	5507.99	+0.01
	0.0015	5510.49	0.762901	5637.95	0.353940	$T_2^* < \mu < T_1^*$	5510.49	+0.05
	0.0010	5512.60	0.766739	5643.15	0.354629	$T_2^* < \mu < T_1^*$	5512.60	+0.11
μ	0.60	5605.42	0.895051	5496.51	0.356863	$T_2^* < \mu < T_1^*$	5605.42	+1.78
	0.50	5552.86	0.820342	5564.64	0.355044	$T_2^* < \mu < T_1^*$	5552.86	+0.82
	0.42	5515.82	0.770286	5619.12	0.353609	$T_2^* < \mu < T_1^*$	5515.82	+0.15
	0.38	5499.24	0.748452	5646.36	0.352898	$T_2^* < \mu < T_1^*$	5499.24	−0.14
	0.30	5470.39	0.711202	5700.81	0.351489	$T_2^* < \mu < T_1^*$	5470.39	−0.67
	0.20	5443.30	0.676839	5768.85	0.349752	$T_2^* < \mu < T_1^*$	5443.30	−1.16

Here "…" indicates infeasible solution in Table 1.

The following observations can be made from Table 2.

(i) When the grace period (μ) increases from 0.0100 to 0.5000, Policy I holds whereas from 0.6000 to 1.0020 Policy II holds.

(ii) When the grace period (μ) increases from 1.0030 to above, the cycle time T_1^* and average system cost $ASC_1\left(T_1^*\right)$ result infeasible solution in Policy I.

Table 2. Sensitivity analysis of grace period and optimal costs.

Increase in value of parameter (μ)	$ASC\left(T_1^*\right)$	T_1^*	$ASC\left(T_2^*\right)$	T_2^*	Remarks
$\mu = 0.0100$	5422.52	0.643557	5898.05	0.346521	Policy I
$\mu = 0.1000$	5427.14	0.656220	5836.86	0.348040	Policy I
$\mu = 0.2000$	5443.30	0.676839	5768.85	0.349752	Policy I
$\mu = 0.3000$	5470.39	0.711202	5700.81	0.351489	Policy I
$\mu = 0.4000$	5507.36	0.7591.03	5632.74	0.353253	Policy I
$\mu = 0.5000$	5552.86	0.820342	5564.64	0.355044	Policy I
$\mu = 0.6000$	5605.42	0.895051	5456.51	0.356863	Policy II
$\mu = 0.7000$	5663.59	0.984103	5428.35	0.358710	Policy II
$\mu = 0.8000$	5725.94	1.08980	5360.16	0.360586	Policy II
$\mu = 1.0000$	5857.32	1.38222	5223.69	0.364429	Policy II
$\mu = 1.0001$	5857.39	1.38241	5223.62	0.364431	Policy II
$\mu = 1.0010$	5857.98	1.38417	5223.01	0.364449	Policy II
$\mu = 1.0020$	5858.64	1.38613	5222.32	0.364468	Policy II
$\mu = 1.0030$	…	…	5221.64	0.364488	…
$\mu = 1.0040$	…	…	5220.96	0.364507	…

Here "…" indicates infeasible solution in Table 2.

6. Conclusions

In the real-market situation, it is commonly observed that the demand of the items like seasonal fruits like mango, apple, grapes, etc., vegetables, and sea fish like hilsa etc. varies with respect to time. From the beginning to the end of the season, the behavior of the demand pattern is a quadratic function of time. This trend in demand pattern is also valid for essential commodities like newly manufactured fashionable items and newly launched automobiles, android mobiles, laptops, etc. In the present paper, an EOQ model for deteriorating items with varying one-parameter Weibull distribution deterioration, quadratic demand and the condition of grace periods is proposed. We think that such type of time-dependent demand pattern, time-varying deterioration and conditions of grace period inventory model is quite realistic in our day-to-day life which builds the solid foundation for future research on inventory models. The results are provided to illustrate with the help of three numerical examples for additional insights and the performance of optimal average total cost with change in values of the different parameters are shown by the execution of sensitivity analysis.

The present problem model can be extended for research and study to many practical situations. The proposed model can be extended by introducing more generalized demand patterns. We could extend the present model with several features like two-warehouse systems, quantity discounts; several delays in payment conditions etc. We could also extend the model to different Weibull distributed models and Gamma distributed models by changing its deterioration rate.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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