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On the Change Rule of 3*x* + 1 Problem

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Abstract

This article introduces a change rule of $3x+1$ problem (Collatz conjecture), it's named LiKe's Rule. It's a map of $3x+1$ problem, and details the path of each step of the change: For any positive integer, change by the Collatz conjecture. 1) This positive integer will change to an odd number; 2) The odd number must change to a number of LiKe's second sequence $\{3^n-1 \mid n\in Z^+\}$; 3) Then this 3^n-1 will change to a smaller 3^n-1 and gradually decrease to 8 (that is 3^2-1) then back to 1 in the end. If we can determine each step, the Collatz conjecture will be true. This is certainly more valuable than 2^n (it might even explain 2^n). And to illustrate the importance of this rule, introduced some important funny corollaries related to it.

Keywords

$3x+1$ Problem, Collatz-Problem, Collatz Conjecture, LiKe's Rule, Number Theory

1. Introduction

$3x+1$ problem (Collatz conjecture) [1] says: If a positive integer x is odd then "multiply by 3 and add 1", while if it's even then "divide by 2", iterations of them, it will eventually reach the number 1. The Collatz function is as follows.

$$C(x)=\begin{cases}3x+1 & \text{if } x\equiv 1 \pmod 2\\ x/2 & \text{if } x\equiv 0 \pmod 2\end{cases}$$

As soon as this problem appeared, it became popular all over the world, and teachers and students in both primary and secondary schools and colleges were fascinated by it. For nearly a century, mathematicians, physicists, computer scientists and others have studied this. It covers a wide range of mathematical fields, such as Number Theory, Ergodic Theory, Dynamical Systems, Mathematical Logic and the Theory of Computation, Stochastic Processes and Probability Theory, and Computer Science. Although achieved certain results, such as: J. C. Lagarias [2] By the way of probability, researches show that it will take about 6.95212 log$n$ steps to reach 1; A.V. Kontorovich and J. C. Lagarias [3] raised a probabilistic Model:

$$\rho(n) := \frac{\log\left(\max_{k\geq 1}\left(T^{(k)}(n)\right)\right)}{\log n}$$

Terence Tao achieved a significant result in 2019 [4]. But in his paper his conclusion is almost all, not all. And he admits the law is unsustainable.

The problem seems unsolvable, and no one can crack its secrets. Richard Guy said “Don’t try to solve these problems!”, Paul Erdos said “Hopeless. Absolutely hopeless.” and “Mathematics is not yet ready for such problems.” and so on.

But through my research, I discovered a special rule (LiKe’s Rule), it reveals a clear path of change, that is: For any positive integer, if it is odd, multiply it by 3 and add 1; if it’s even, divides it by 2, iterations of them, it will convert to a number of $3^n - 1$ (LiKe’s second sequence, LiKe sequence appears in reference [5]), and it will convert to a smaller $3^n - 1$ then gradually decrease to 8 and back to 1. It is well known that all positive integers will change to 2^n and then be reduced to 1. But no one knows “how” and “why”. However, it is clear in LiKe’s Rule. The rule has attracted wide attention since it was published on the pre-print website in 2020 [6], and no one has found a counterexample for many years, existing studies adopted ideas similar to this paper [7]. So it is published in the journal in English today. The details are as follows.

2. Definition

$$L_0(1,3,5,7,\cdots,O_{0m}) = \left\{2n-1 \mid n \in Z^+\right\} \subset Z^+, O_{01} = 1;$$

$$L_1(5,11,17,\cdots,O_{1m}) = \left\{6n-1 \mid n \in Z^+\right\} \subset Z^+, O_{11} = 5;$$

$$L_2(17,35,53,\cdots,O_{2m}) = \left\{18n-1 \mid n \in Z^+\right\} \subset Z^+, O_{21} = 17;$$

$$L_3(53,107,161,\cdots,O_{3m}) = \left\{54n-1 \mid n \in Z^+\right\} \subset Z^+, O_{31} = 53;$$

$$\vdots$$

$$L_n\left(2*3^n-1, 2*2*3^n-1, 3*2*3^n-1, \cdots, O_{nm}\right)$$
$$= \left\{2*3^n i - 1 \mid n \in Z, i \in Z^+\right\} \subset Z^+, O_{n1} = 2*3^n - 1;$$

$$O_{n1}(1,5,17,53,\cdots,O_{n1}) = \left\{2*3^n - 1 \mid n \in Z\right\} \subset Z^+;$$

$$LK_2(2,8,26,80,\cdots) = \left\{3^n - 1 \mid n \in Z^+\right\} \subset Z^+;$$

$$LC(1,4,13,40,\cdots) = \left\{\left(3^n - 1\right)/2 \mid n \in Z^+\right\} \subset Z^+.$$

3. Theorem and Proof

3.1. Exclude the Even Numbers

The starting condition of the $3x+1$ conjecture is any positive integer, and the rule divides it into even and odd numbers. See Table 1.

Because all even Numbers divided by 2 are integers; if you divide an even number by 2, it's an integer again, and iterations of it. Eventually, all integers that are not powers of 2 can be converted to non-one odd Numbers (integers that are powers of 2 divided by 2^n will naturally return to 1). So in order to bypass the odd-even barrier, we only need to study the odd numbers $L_0\left\{2n-1 \mid n \in Z^+\right\}$.

3.2. The Odd Change of Odd Numbers

For all L_0, multiply by 3 and add 1 (expressed as $3*O+1$) must be an even, so $(3*O+1)/2$ is regarded as a one-step operation in this paper, so the Collatz function $C(x)$ can be expressed as the following equation too.

$$T(x)=\begin{cases}(3x+1)/2 & \text{if } x \equiv 1(\operatorname{mod} 2) \\ x/2 & \text{if } x \equiv 0(\operatorname{mod} 2)\end{cases}$$

Since an even number will become an odd number, we can only study the odd number that will only become another odd number, so we can get the following theorem 1.

Table 1. Change rule of even numbers.

Even/E	$E/2$	$E/2^2$	$E/2^3$	$E/2^4$	$E/2^n$
2	1				
2^2	2	1			
6	3				
2^3	4	2	1		
10	5				
12	6	3			
14	7				
2^4	8	4	2	1	
18	9				
20	10	5			
22	11				
24	12	6	3		
26	13				
28	14	7			
30	15				
2^5	16	8	4	2	…
34	17				
E_x	…	…	…	…	…

Theorem 1: For all L_0, perform the operation of $(3*O+1)/2$, If the result is odd too, repeat $(3*O+1)2$. Iterations of them, will eventually reach the sequence $O_{n1}\{2*3^n-1 \mid n\in Z\}$ and the sequence $L_n\{2*3^n i-1 \mid n\in Z, i\in Z^+\}\subset Z^+$. And no certain odd number can be shifted to L_∞.

Proof:

Obviously, for all odd Numbers in $L_0\{2n-1 \mid n\in Z^+\}$, according to the $(3*O+1)/2$ mathematical operations. Half will become to $L_1\{6n-1 \mid n\in Z^+\}$ except 1, $O_{11}=5$;

Then for all odd Numbers L_1, calculated by the formula of $(3*O+1)/2$ too, The result is half translates to $L_2\{18n-1 \mid n\in Z^+\}$ except 5, $O_{21}=17$;

Iterations of them, see Table 2, it is easy to see, after the n-th calculation we will get $O_{n1}\{2*3^n-1 \mid n\in Z\}$ and the sequence $L_n\{2*3^n i-1 \mid n\in Z, i\in Z^+\}\subset Z^+$.

That is all odd numbers are shifted to O_{n1} and L_n by $(3*O+1)/2$.

And

$$\lim_{n\to\infty}\left(\frac{1}{2}\right)^n=0.$$

Table 2. Change rule of odd numbers.

Odd/O	$(3*O+1)/2$	$(3*O+1)/2$	$(3*O+1)/2$	$(3*O+1)/2$	…
1	2				
3	<u>5</u> ($2*3^1-1$)	8 (3^2-1)			
5					
2^3-1	**11**	<u>17</u> ($2*3^2-1$)	26 (3^3-1)		
9	14				
11	**17**	26			
13	20				
2^4-1	**23**	**35**	<u>53</u> ($2*3^3-1$)	80 (3^4-1)	
17	26				
19	**29**	44			
21	32				
23	**35**	**53**	80		
25	38				
27	**41**	62			
29	44, $d_1=2*3$	$d_2=2*3^2$	$d_3=2*3^3$	$d_4=2*3^4$	
2^5-1	**47**	**71**	**107**	<u>161</u> ($2*3^4-1$)	…
33	50				
O_x	…	…	…	…	…

So for a certain odd number, it mustn't change to L_∞.

Theorem 1 is proved. □

Except that the odd number is half after each operation, it is easy to get:

1) The tolerance of L_n after the nth calculation is:

$$d_n = 2*3^n.$$

2) The smallest odd number of L_n after the nth calculation is:

$$O_{n1} = 5 + \sum_2^n 4\times 3^{n-1} = 2*3^n - 1.$$

3) The corresponding odd number of O_{n1} in the L_0 is:

$$O_x = 2^{n+1} - 1.$$

It can also conclude that no odd number can change to L_∞.

3.3. The Even Change of Odd Numbers

All even numbers can be turned into odd numbers, but the even numbers which transit from odd numbers can't without consideration. So what's the rule for odd numbers like this? See Theorem 2.

Theorem 2: Odd numbers that will translate to even numbers will change to O_{n1} and L_n by $T(x)$.

To prove theorem 2, we need to prove the following Lemma first.

Lemma 2.1: In the odd sequence $L_0\{2n-1 \mid n\in Z^+\}$, except 1, half of L_0 (3, 7, 11, …, $4n+3$) will become to $L_1\{6n-1 \mid n\in Z^+\}$; the remaining half (9, 13, 17, …, $4n+1$) will become to even numbers, and translate to 1 or L_1 finally.

Proof:

See Figure 1.

The general term formula of (3, 7, 11, ⋯, $4n+3$) is $4n+3$ (1)

The general term formula of (5, 9, 13, 17, ⋯, $4n+1$) is $4n+1$ (2)

Multiply Equation (2) by 3 plus 1 then divide by 2, get

$$\left[3(4n+1)+1\right]/2 = 6n+2.$$

Divide by 2 is $3n+1$.

Case I:

$3n+1$ are odd numbers (n is even number).

The odd type of $3n+1$ is $\{6n+1 \mid n\in Z^+\}$.

Because $4x+3=6n+1$.

Get $x = 3n/2 - 1/2$.

Where x is an integer when n is odd (n is 1/2 of the integer).

So half of $6n+1$ will translate to $4n+3$ (or $4n-1$).

The other half of $6n+1$ is $12n+1$.

Multiply them by 3 and plus 1 then divide by 2, get

$\left[3(12n+1)+1\right]/2 = 18n+2$ (n is positive integer).

$18n+2$ are even numbers.

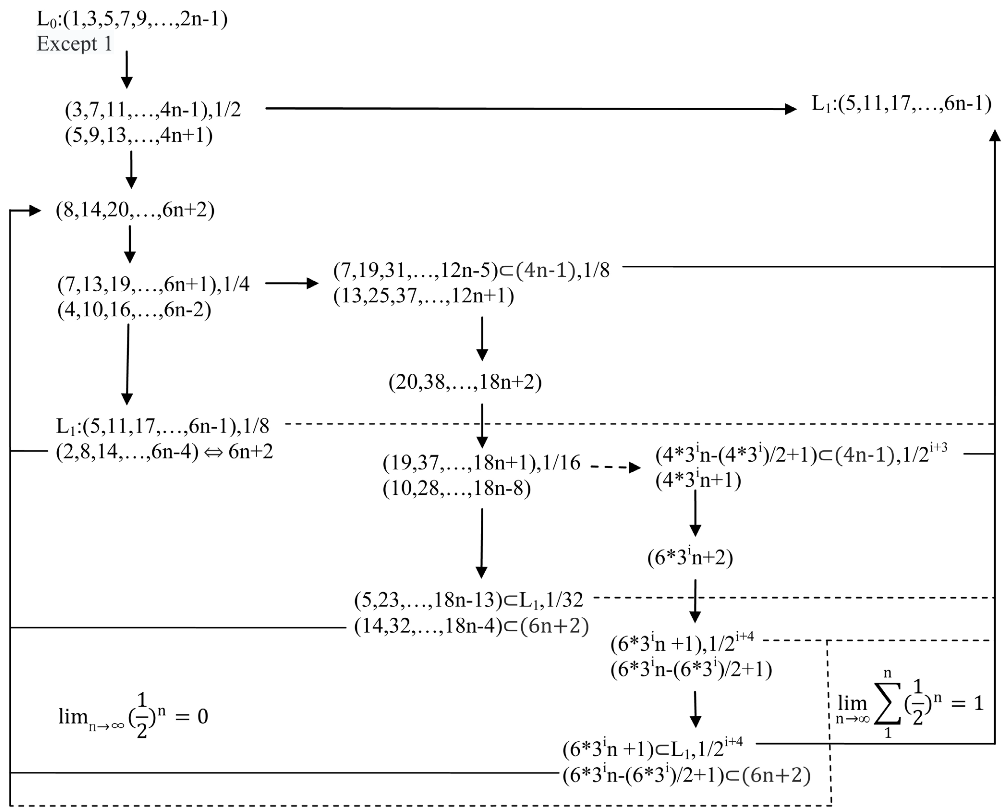


Figure 1. The path of $L_0 \to L_1$.

So in (5, 9, 13, 17, …, $4n+1$), a half (9, 17, 25, …, $8n+1$) will become to even numbers and shift to odd numbers divide by 2. And a half these odds $(7,19,31,\cdots,12n-5)\subset(3,7,11,\cdots,4n+3)$; The other half (13, 25, 37, …, $12n+1$) will become to even numbers and reduced.

That is to say, half of L_0 change to L_1, the other half change to even numbers. Divide these even numbers by 2 (named reduce), 1/2 are odd numbers and half of them $\{12n-5 \mid n\in Z^+\}$ will change to L_1 (1/8 of L_0); the other half $\{12n+1 \mid n\in Z^+\}$ will change to even numbers $\{18n+2 \mid n\in Z^+\}$ (1/8 of L_0, see case II); the other 1/2 are even numbers $\{6n-2 \mid n\in Z^+\}$ (1/4 of L_0, see case II).

Case II:

$3n+1$ are even numbers (n is odd number).

The even type of $3n+1$ is $\{6n-2 \mid n\in Z^+\}$.

Divide these even numbers by 2 (reduce), a half $\subset L_1$, the other half $\subset\{6n+2 \mid n\in Z^+\}$ except 2 (2 change to 1), loop computation, we will know they will change to 1 or L_1.

Similarly, the change of $\{18n+2 \mid n\in Z^+\}$ is same as $\{6n-2 \mid n\in Z^+\}$

Iterations of them, we will get:

The numbers in L_0, only some change to 1, 5, the others, $1/2+1/4+1/8+\cdots+1/2^n$ will change to L_1.

And

$$\lim_{n\to\infty}\sum_1^n\left(\frac{1}{2}\right)^n=1=100\%.$$

So L_0 must translate to 1 or L_1 finally.

Lemma 2.1 is proved. □

Similarly: See Figure 2, When $(3*O+1)/2$ is calculated for the odd sequence L_1, half of them will be converted to the odd sequence L_2; The other half will converted to 2^n, 5 or L_2 after some $T(x)$ also.

So repeat the process over and over again, we will get

$L_0\to L_1\to L_2\to L_3\to\cdots\to L_n$, finally to O_{n1} and L_n.

Theorem 2 is proved. □

3.4. Reduce to the Sequence O_{n1}

From Theorem 1 and Theorem 2 we know that all odd numbers in $L_0\{2n-1\mid n\in Z^+\}$ will change to L_1 except 1, and L_1 will change to 1 and L_2 except 5. With the increase of n, the sequence $L_n(O_{n1},O_{n2},O_{n3},\cdots,O_{nm})$. And only the first term of L_n is not continuous, so when $\lim_{n\to\infty}\text{count}(O_{nm})/\text{count}(Z^+)=0$, they will change to $O_{n1}\{2*3^n-1\mid n\in Z\}$ which is consist of the first item of all L_n and L_n in the end. So, is there some number can change to L_∞?

Of course, the answer is no. The proof of theorem 2 is not only can get $L_0\to L_1\to L_2\to L_3\to\cdots\to L_n$, and finally to O_{n1} and L_n; In fact, the discussion of even numbers also involves the process of reduction, that is, L_0 must reduce to the L_0 in the process of changing to *n*-level L_n (e.g. $L_0\to L_1\to L_0\to L_1\to L_2\to\cdots\to L_n$, see Figure 1, which L_0 is $6n+1$), but the proof of theorem 2 only involves the reduction of even numbers without determining its relationship with the increase of sequence. It is the reduction of

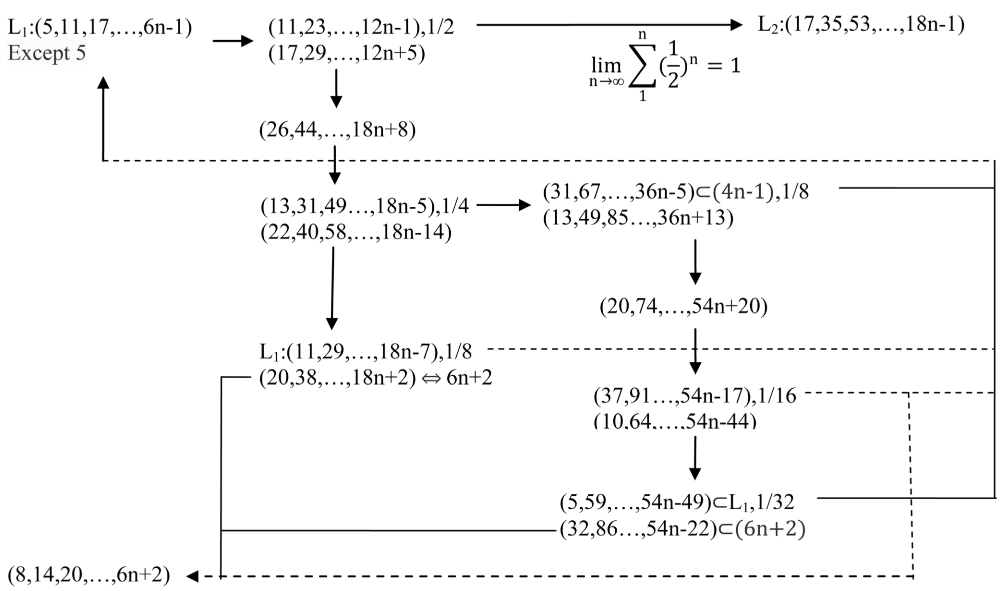


Figure 2. The path of $L_1\to L_2$.

even numbers (divided by 2) that guarantees a certain reduce, so how can we prove that the reduction of even numbers guarantees that no number will go to L_∞ infinity?

Theorem 3: All given odd numbers (1, 3, 5, 7, …, *O*) must convert to items in the sequence O_{n1} by $T(x)$ calculation.

Proof:

See Figure 1.

For L_0, it not only 1/2 change to 1 or L_1, but half of L_0 change to evens $\{6n+2 \mid n \in Z^+\}$, and then we know that for $6n+2$, every time divide by 2, half of them are even; And half of the odd numbers of $6n+2$ divided by 2 are $(5, 11, 17, 23, \cdots) \in L_1$ and half $\in \{18n+2 \mid n \in Z^+\}$, $18n+2$ and $6n+2$ belong to the same set of numbers and change exactly the same.

That is to say, see Figure 3. The times of increase or reduce and their corresponding relationship of any number is self-evident (change the period regularly, similar to quasicrystal):

1/2 of L_0 increase to L_1 directly; 1/2 reduce one time, 1/4 reduce two times, …, $1/2^n$ reduce *n* times; The numbers which don't reduce, 1/2 (that is 7, 19, 31, …) change to L_1, 1/2 (that is 13, 25, 37, …) $\in L_0$, repeat these process and change to 1 or L_1 ultimately. So, all L_0 must change to 1 or L_1 and no cycle.

In a similar way, in each case of $L_1 \to L_2 \to L_3 \to \cdots \to L_n$, there must be a 1/2 reduce to the smaller L_0 (different starting number and a larger interval) and then change to the L_n.

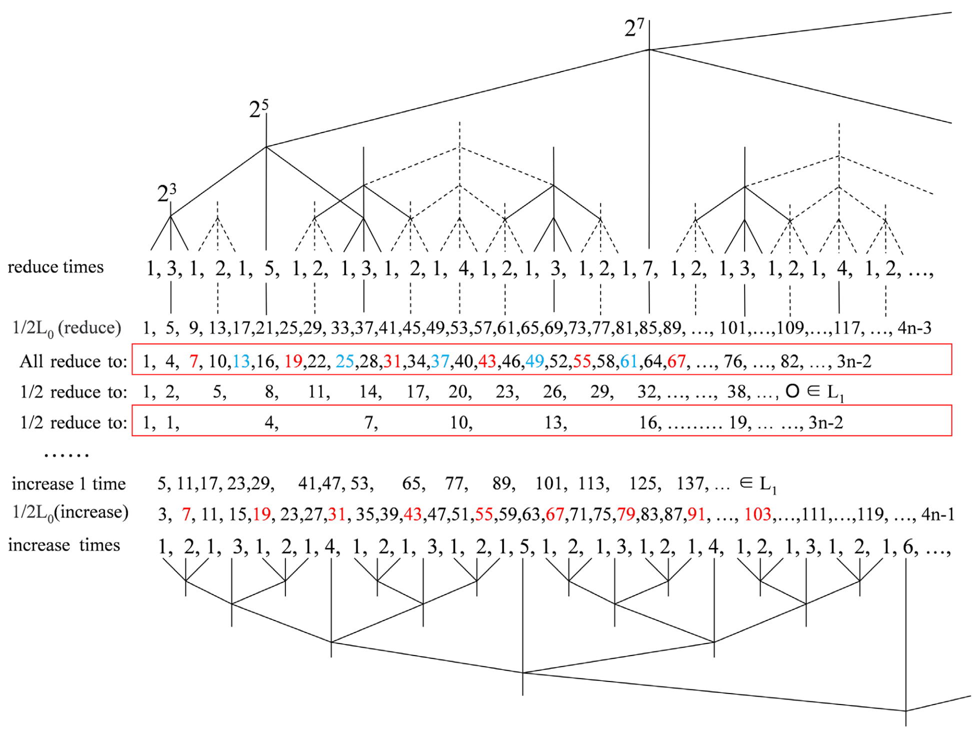


Figure 3. The times of reduce and increase of L_0.

Induction can be obtained:

$L_n \subset \cdots \subset L_2 \subset L_1 \subset L_0$, L_n are thinning out in odd numbers.

And this corresponding relationship in **Figure 3** also ensures no certain odd number can change to L_∞ .

So for all certain odd number, it must change to a number in L_n .

And only the first item O_{n1} of L_n is not sustainable.

So all the odd numbers must change to $O_{n1}\{2*3^n-1 \mid n\in Z\}$ in the end.

Theorem 3 is proved. □

In other words, for an infinite number of positive integers, it can change to any L_n , but as n gets bigger and bigger, there are fewer and fewer numbers that can change to L_n . But for a certain positive integer, it can only be changed to a certain L_n , then reduce to L_0 and then changed to a new L_n , and after several iterations, it must stoped at one O_{n1} (It's not probability anymore). Of course, **Figure 3** also explicitly points out the classic variation path of 2^n: Positive integer $\to L_n \to \left(2^{2n}-1\right)/3 \to 2^{2n-1} \to 1$.

3.5. O_{n1} Back to 1

So far, according to the proof process of theorem 3, it can be confirmed that L_n must change to 1, but it requires strong logical thinking to understand, so this section will use another clever method to prove.

Theorem 4: The number in $O_{n1}\{2*3^n-1 \mid n\in Z\}$ must change to another number in $O_{n1}\{2*3^n-1 \mid n\in Z\}$ by the $T(x)$ calculation.

Proof:

Because O_{n1} is odd number.

So According to theorem 2 and Theorem 3, O_{n1} must change to another O_{n1} .

Theorem 4 is proved. □

Theorem 5: If O_{n1} ($2*3^n-1$) changes to O_{m1} ($2*3^m-1$) by $T(x)$ conditions, there must have $m<n$.

Proof:

See **Table 3**.

Table 3. The variation of $2*3^n-1$.

From	O_{n1}	$T(x)$	$C(x)$	$T(x)$	$C(x)$
3	5	8	4	2	1
7	17	26	13	20	10
15	53	80	40		
31	161	242	121	182	91
63	485	728	364		
127	1457	2186	1093	1640	820
255	4373	6560	3280		
$2^{n+1}-1$	$2*3^n-1$	$3^{n+1}-1$	$\left(3^{n+1}-1\right)/2$	$\left(9^n-1\right)/4$	$\left(9^n-1\right)/8$

For $2^{n+1}-1$, keep doing $C(x)$ calculation can get:

$$2^{n+1}-1 \xrightarrow{T(x)} 3*2^n-1 \xrightarrow{T(x)} 3^2*2^{n-1}-1 \xrightarrow{T(x)^{(n-1)}} 3^{n+1}-1$$
$$\xrightarrow{C(x)} (3^{n+1}-1)/2 \xrightarrow{T(x)} (9^n-1)/4 \xrightarrow{C(x)} (9^n-1)/8$$

Which shows that, $2*3^n-1$ not only comes from $2^{n+1}-1$; And it will change to $3^{n+1}-1$, $(3^{n+1}-1)/2$, $(9^n-1)/4$, $(9^n-1)/8$. Among them:

$3^{n+1}-1$ (or 3^n-1), its form is consistent with $2^{n+1}-1$, it's more distinctive. So it is named "LiKe's second sequence";

$(3^{n+1}-1)/2$ named "LiKe-Collatz number", the secret is revealed below;

$(9^n-1)/8$ named "LiKe's nine nine", as you read through, you will find it's an interesting numbers.

In Table 3, all of $2*3^n-1$, $3^{n+1}-1$, $(3^{n+1}-1)/2$, $(9^n-1)/4$ and $(9^n-1)/8$. are equivalent, they named step numbers. And every five step numbers make up a group named "LiKe's step". As long as any number of LiKe's step cannot be converted to a larger one, theorem 5 will be proved. But obviously $(3^{n+1}-1)/2$ is the easiest one, because:

$(3^{n+1}-1)/2$: 1, 4, 13, 40, 121, 364, 1093, 3280, 9841, 29,524, …

Observe carefully, it is not difficult to find:

$$1=3*0+1;$$
$$4=3*1+1;$$
$$13=3*4+1;$$
$$\vdots$$

So if one item is *x*, the next item must be $3x+1$.

So the numbers of $(3^{n+1}-1)/2$ form named "LiKe-Collatz number". Thus, the LiKe-Collatz sequence can be expressed as:

$$(3x+1,\ 9x+4,\ 27x+13,\ 3^4x+40,\ 3^5x+121,\ \cdots,\ 3^nx+(3^n-1)/2)$$

(among them $x=(3^i-1)/2$).

The general term formula is: $3^nx+(3^n-1)/2$.

And according to the $T(x)$, all changes of $3x+1$ are in Table 4.

It can be seen that the general formula of all the changes of $3x+1$ is:

$$\frac{3^jx+a}{2^i}\quad (i,j>0).$$

Accordingly,

$$\frac{3^jx+a}{2^i}\neq 3^nx+(3^n-1)/2.$$

So $3x+1$ will never translate to a bigger ones such as $9x+4$, $27x+121$, 3^4x+40, 3^5x+121, and so on.

So according to Theorem 4: $3x+1$ will change to a smaller LiKe-Collatz number. So $2*3^n-1$ must change to a smaller $2*3^m-1$ ($m<n$).

Theorem 5 is proved. □

Table 4. All the changes of LiKe-Collatz number.

$3x+1$ even: up odd: below	$\frac{3x+1}{2}$	$\frac{3x+1}{4}$	$\frac{3x+1}{8}$	… …
			$\frac{9x+7}{8}$	… …
		$\frac{9x+5}{4}$	$\frac{9x+5}{8}$	… …
			$\frac{27x+19}{8}$	… …
	$\frac{9x+4}{2}$	$\frac{9x+4}{4}$	$\frac{9x+4}{8}$	… …
			$\frac{27x+16}{8}$	… …
		$\frac{27x+14}{4}$	$\frac{27x+14}{8}$	… …
			$\frac{81x+46}{8}$	… …

Corollary 1(conclusion): For all given positive integer, it must converted to the number of $O_{n1}\{2*3^n-1\mid n\in Z\}$ by $T(x)$ calculation, then the O_{n1} will convert to a smaller O_{m1} and gradually decrease to O_{11} then back to 1 in the end.

Proof:

See Figure 4.

First, all even Numbers will translate to odd numbers $L_0\{2n-1\mid n\in Z^+\}$;

According to Theorems 1, 2, and 3, all odd numbers will change to $O_{n1}\{2*3^n-1\mid n\in Z\}$;

According to theorem 4, O_{n1} must translate to another O_{m1} again after the $T(x)$ calculation;

According to theorem 5, the $O_{m1}<O_{n1}$;

Repeat this process over and over again, and eventually, it will translate to O_{11} and back to 1.

So, for all given positive integer, it must change to 1 after $T(x)$ calculation.

Q.E.D!

This whole process is the LiKe's Rule, Figure 4 makes it clear, it shows all the changes path of positive integers under $3x+1$ problem, and how change to 1 without exception.

With this, we fully understand the "LiKe's Rule" and can get another memorable expression, as shown in Figure 5.

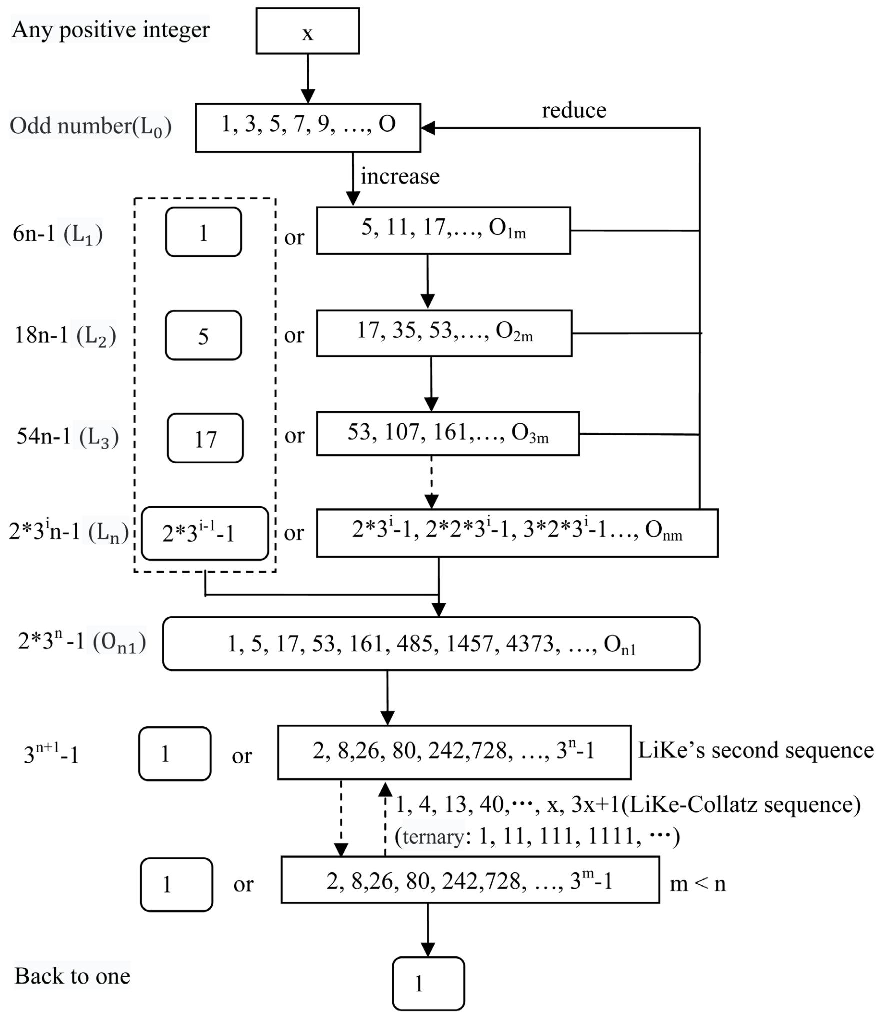


Figure 4. The route map of $3x + 1$ problem.

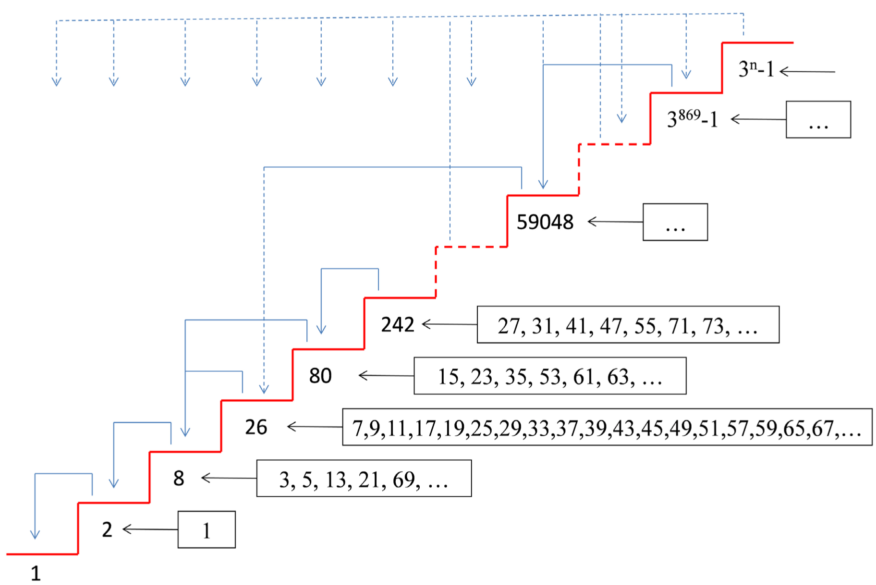


Figure 5. The change rule of Collatz conjecture: LiKe's rule.

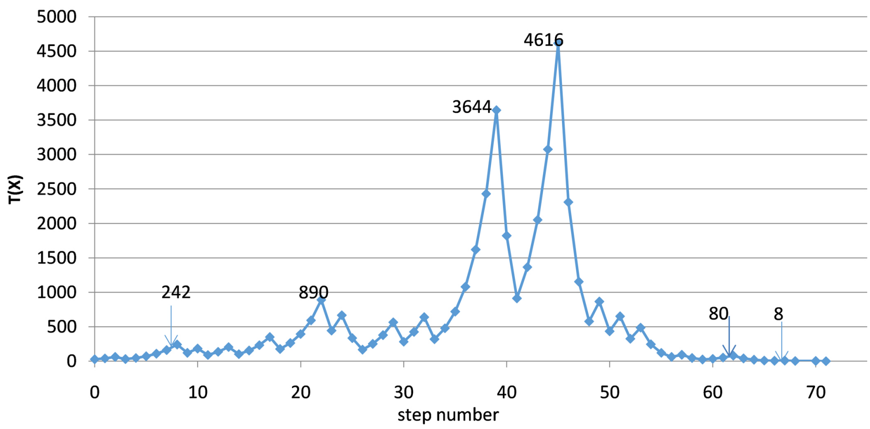
Figure 5, also known as the LiKe's step, all step numbers are equivalent. it's not hard to find that it contains all odd numbers; Obviously, all positive integers are going to convert to odd numbers; and for any odd number, it must convert to a number of LiKe's second sequence (2, 8, 26, 80, …, $3^n - 1$) by $T(x)$ operation; then $3^n - 1$ will convert to a smaller $3^n - 1$ and gradually decrease to 8 then reach the number 1 eventually. Of course, if you're good at studying, you might be able to figure out the pattern for every sequence that goes to $LK_2\left\{3^n - 1 \mid n \in Z^+\right\}$.

4. Funny Corollaries

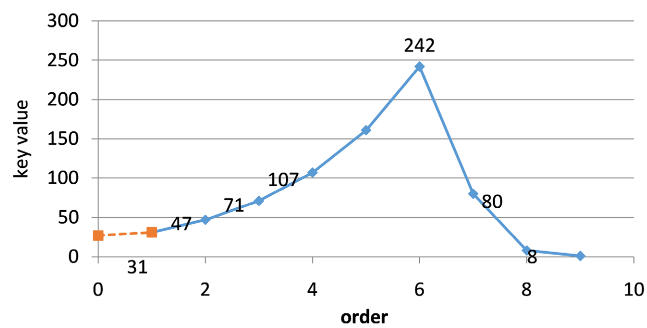
4.1. Simple Rule

Taking 27 as an example, the change path of it's Collatz ($T(x)$) is shown in **Figure 6(a)**, and **Figure 6(b)** is the change path according to LiKe's Rule.

As can be seen from **Figure 6**, the original variation diagram of 27 (a) is disordered, and it is difficult to see the change rule of 27. The new **Figure 6(b)** is simply and clearly shows the changing trend of 27: after the calculation of $T(x)$ it will change to 31 ($3^i * 2^{n-i} - 1$) and 161 ($2 * 3^4 - 1$), it rises to 242 ($3^5 - 1$), and finally decreases to 8 and returns to 1. Thus, as long as we use the LiKe's rule to draw, the change of all positive integers will have a curve that goes up and then down (only one peak), the change rule will appear on the paper.



(a)



(b)

Figure 6. Comparison of old and new change charts of 27.

4.2. Ternary

Look at the "LiKe-Collatz" sequence:

(1, 4, 13, 40, 121, 364, 1093, 3280, 9841, 29,524, 88,573, 265,720, 797,161, …).

Let write them in ternary terms:

(1, 11, 111, 1111, 11,111, 111,111, 1,111,111, …).

That is to say, in the ternary case, the change map is: Positive integer→1…1→…→11→1.

This is an amazing result, isn't it more profound than (2, 2^2, 2^3, 2^5, …)? And it's a far bigger concern worth researching about.

4.3. Family Number

Look closely at the "LiKe's second sequence":

(2, 8, 26, 80, 242, 728, 2186, 6560, 19,682, 59,048, 177,146, 531,440, 1,594,322, …).

If you're intuitive, you can also see that all of 3^n-1 can be subdivided into $3^{2n-1}-1$ and $3^{2n}-1$ forms, such as 2, 26, 242, which are in the odd form $3^{2n-1}-1$; And 8, 80, 728 are in the even form $3^{2n}-1$. Both $3^{2n-1}-1$ and $3^{2n}-1$ can change to $\left(9^n-1\right)/4$, so 3^n-1 is called "Family number". $3^{2n}-1$ and $3^{2n-1}-1$ are called mothers and fathers respectively, both of them will changes to a same number with 3^n-1 form (children), and the number (sex unknown) must be combined with another 3^n-1 form to form a new family. For example, 177,146 (father) and 531,440 (mother) will both change to 132,860 and then go through the same steps to 2186 (father). This number will combine with 6560 (mother) to form a new family, which is very interesting.

This leads to a very interesting math game: Looking for children. Such as, the child of 26 and 80 is 8, …, the child of $3^{183}-1$ and $3^{184}-1$ is 3^7-1, …, the child of $3^{869}-1$ and $3^{870}-1$ is $3^{10}-1$ and so on. In a word, find who has a larger child became a popular pursuit.

4.4. Nine Nine Go to One

Look carefully at the number in the form of $\left(9^n-1\right)/8$.

(1, 10, 91, 820, 7381, 66,430, 597,871, 5,380,840, 48,427,561, …).

It is not too difficult to find this:

$$1=9^0;$$

$$10=9^1+9^0;$$

$$91=9^2+9^1+9^0;$$

$$820=9^3+9^2+9^1+9^0;$$

$$7381=9^4+9^3+9^2+9^1+9^0;$$

$$\vdots$$

They are all the sum of the powers of 9 (LiKe's nine nine), So according to LiKe's rule, $\left(9^n-1\right)/8$ is only going to get smaller and smaller until it goes to

1. This is in line with an old Chinese saying: Nine Nine go to One, and happened to coincide with "the BOOK of Changes"!

5. Conclusion

This paper gives a brief overview of "Collatz conjecture" and introduces a very important mathematical concept—LiKe's Rule. The rule states that: For any positive integer, if it is odd, multiply it by 3 and add 1; If it's even, divides it by 2, iterations of them, it will convert to a number of $3^n - 1$, and it will convert to a smaller $3^n - 1$ then gradually decrease to 8 and back to 1. Through detailed mathematical analysis, the paper proves that the power of 2 in positive integer can be directly reduced to 1; any even number that is not a power of 2 will change to an odd number; all odd numbers must convert to 2^n or LiKe second sequence $LK_2\left\{3^n - 1 | n \in Z^+\right\}$ by increase and reduce; $3^n - 1$ goes down again and again and then goes back to 1. Compared with the 2^n, LiKe's Rule not only explains why and provides a new path, but also points out the specific change process, which has a profound impact on the research of $3x + 1$ Problem. As long as we can determine the change at each step, we will prove the Collatz conjecture. In addition, some interesting and important inferences such as "family number" and "Nine Nine go to One" are obtained.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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