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Analytical Approximation for Treasury Bill Default Spreads, Profits and Losses Equations

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Abstract

In this work, we introduce expressions for the default spread calculation based on an approximation of the discount factor, for the specific case of Treasury Bills. Additionally, expressions for the profit and losses array are obtained supported by a pass-time yield correction. Some relevant limits are explored as well in order to illustrate the large range model applicability. Treasury Bills are especially relevant within the banking industry since the financial institutions usually hold the largest portfolio position in them.

Keywords

Risk Modelling, Credit Risk, Default Spread, Theta Effect

1. Introduction

The Value at Risk (*VaR*) has shown itself as the most useful and widely spread risk metric within the finance industry. Since the *VaR*'s first appearance in the portfolio analysis of JP Morgan in the late 1980s (Holton, 2002), the metric has spanned many different usages and applications in a bunch of risk disciplines. The *VaR* has centered on the Profit and Losses (*P&Ls*) concept. The *P&L* is a profit and losses profile where a price collection of financial scenarios are compared with a reference. These scenarios usually enclose price variation coming from risk factor variations (Jorion, 2001; Holton, 2014). Typically, the risk factors have been interest rates, foreign exchange rates, equity prices, ...but after the European debt crisis in 2009, it was clear the need to introduce credit risk factors into the *VaR* calculation, in order to add credit components to the business limits at the Front Office desks (Hsu, Saa-Requejo, & Santa-Clara, 2010; Jeanblanc & Lecam, 2008; Geske & Delianedis, 2001).

A collection of models has been developed to introduce such credit components into a risk model. Most of the work has been done at *Credit Default Swap* (CDS), since they are products that intrinsically enclose the credit risk, by specifically adding default corrections and recovery rate payments into their valuation (Brigo & Morini, 2005; Haugh, 2016). These models are mainly based on the determination of some financial parameters connected to a default event that end up defining a default probability. Indeed, the default probability is usually expressed in terms of a collection of variables called *hazard rates* (Brody, Hughston, & Macrina, 2007), obtained from some assumptions and fixed market data. For the specific case of bonds, a traditional method to include credit risk into the bond valuation is to add a constant spread into the discount curve aiming to decrease the value of the bond by the single addition of a spread, this spread is usually called *z-spread*. A more refined approach relies on the determination of a *default spread* curve, where the constant spread is substituted by a collection of spreads linked to a tenor structure (Dobner & Lindsey, 2013).

Especially, relevant among the bonds within the financial industry is the treasury bills since the largest position in the banks is usually held in this kind of instrument. Therefore, it is important to develop models with credit contribution that can be added to the daily *P*&*L* calculation (Arvanitis, Gregory, & Laurent, 1999; Horta, 2010; Malz, 2022; Gregory, 2012; Cooper & Mello, 1991; Bisetti, Li, & Yu, 2021). An industrial model for the *P*&*L* needs for simple implementation and high-speed performance, the *P*&*L* is calculated daily, so that, there are strict time limits to finish the calculation processes, particularly in a tier-one bank where the number of instruments and positions is extremely large. Analytical models fulfill the performance and simplicity requirements, thus any analytical model adds value to the general *P*&*L* calculation process.

In this paper, we introduce an analytical model for the pricing of zero coupon bonds with credit contribution. The first section introduces the model and the approximation that drives the whole paper, along with an analytical solution for the default spread, and a set of limit cases for this solution. Section 2 contains the *P*&*L* expression and the correction that we have introduced to better fit the market data. Section 3 comparisons among the model results and *P*&*L* from the market are discussed. Finally, we summarize the most relevant conclusions arising from the work.

2. Theoretical Model

In a given zero coupon bond, the possible cashflows can be divided into two different kinds. The flows come from a default event and from non-default events. Let's assume the probability distribution for these events is a time-continuous Poisson distribution where at any time $t \in \{0,T\}$ a default can happen, where T is given by the maturity instrument. For a zero coupon bond that basically means that either a default cashflow will happen or any flow at all, except at maturity where only a non-default flow can happen. Additionally, as the default may occur at any time the default contribution to the price must be expressed in a time continuous based description. Figure 1 shows this cashflow diagram, the default flows are represented by the dashed arrows, as we have already pointed out, these flows may happen at any time. At maturity a non defaulted cashflow may happen, where the notional is given back. Therefore, in a given discrete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with risk neutral measure the zero coupon bond price can be expressed as:

$$P = \mathcal{E}^{Q}\left(\mathrm{e}^{-rT} 1_{\tau<T} + R\int_0^T \mathrm{e}^{-rt} 1_{\tau=t}\right) = \mathrm{e}^{-(r+D)T} + R\int_0^T \mathrm{e}^{-rt} \mathrm{d}Q \tag{1}$$

where r is risk free rate at maturity, T is maturity time, P is the market price, $1_A(x)$ is an indicator function being one for x belonging to A and zero otherwise, and the default probability defined as $Q(\tau < t) = 1 - \mathrm{e}^{-D(t)t}$. At this point, we bring the approximation that will allow us to end up finding an analytical solution for the default spread. It will be assumed through the paper that the discount factor is approximately constant through the whole instrument life period. $\mathrm{e}^{-rt} \approx cnst$. Assuming a constant discount factor is comparable to assuming a $df \approx 1 + \epsilon$ being ϵ a small number. Then, Equation (1) can be expressed as:

$$P \approx \mathrm{e}^{-(r+D)T} + R\mathrm{e}^{-rT}\int_0^T \mathrm{d}Q \tag{2}$$

As the discount factor is assumed to be constant, we can pick any value from $t = 0$ to $t = T$, and for certain credit risk desired properties in the model, that we will describe later, we have set up $t = T$. Now the integral can be trivially integrated and we obtained:

$$P = \mathrm{e}^{-(r+D)T} + R\left(1 - \mathrm{e}^{-DT}\right)\mathrm{e}^{-rT} \tag{3}$$

At this point, it should be pointed out that similar expressions have been postulated based on some different assumptions (Schaefer, 2012). However, what the best of our knowledge, we could find a deeper analysis and approach as the one described in this work.

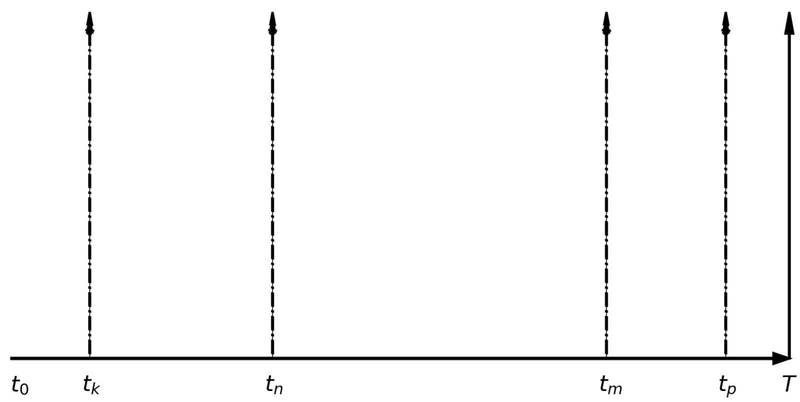

Figure 1. Cashflows representation in a zero coupon bond for default events (dashed arrows) and non-default events (regular arrows). Source: from the authors.

2.1. Model Solution

In order to find a default spread solution (D) for Equation (3), we can first find a solution for $R = 0$, in that case the equation reduces to:

$$P = \mathrm{e}^{-(r+D)T} \tag{4}$$

We can define here the yield to maturity as the rate that perfectly reproduces market price at maturity, that is:

$$P = \mathrm{e}^{-cT} \tag{5}$$

From Equation (4) and Equation (5), it is trivially found that the default spread fulfills $D = c - r$. Then the next step is to set a solution candidate adding a recovery rate contribution. Indeed, the pricing equation can be solved by our candidate $D = c - r + \phi$, where c is the yield curve that fulfills $P = \mathrm{e}^{-ct}$ by construction, r is the risk-free curve, and ϕ is a function that must be found by solving the pricing equation. After some algebra the default spread equation can be analytically expressed as:

$$D = c - r - \frac{1}{T}\log\left(\frac{1 - R\mathrm{e}^{-(c-r)T}}{1 - R}\right) \tag{6}$$

Equation (6) is the main result on this paper, some interesting conclusions stem from this equation. Firstly, the equation is a development from the very basic definition of default spread, where is defined as the difference between the yield curve and the risk-free curve. The second term in the equation holds the contribution coming from the $R \neq 0$. Indeed, for $R = 0$ follows:

$$D = c - r - \frac{1}{T}\log\left(\frac{1}{1}\right) = c - r - \frac{1}{T}\log(1) = c - r \tag{7}$$

So basically, for instruments where the recovery rate is not added to the model the default spread really is a spread between two curves, the discount curve and the yield curve. We should keep in mind that a $R = 0$ does not necessarily mean that the instrument has defaulted, since the default spread is still different that zero, $c \neq 0$. Indeed, we should keep in mind that the default spread depends on the intrinsic bond components and not only on the issuer creditworthiness, then R is considered a model parameter in this work. Additionally, Equation (6) has an asymptote in $R = 1$, however, the model is correctly defined at $R = 1$ as it can be seen at Equation (3). So taking $R = 1$ at Equation (3) results in:

$$P = \mathrm{e}^{-rT} \tag{8}$$

This result shows the reason because we have chosen the discount curve value at maturity. Upon fulfill this condition the price P is still $P = \mathrm{e}^{-cT}$ by definition, so only T-Bills which the yield curve equals the discount curve can be compatible with a recovery rate value equal one, $R = 1$. Therefore, only T-Bills with $R = 1$ have no credit risk since the price only depends on the interest rate curve. Another interesting point in Equation (6) is the condition:

$$Re^{(c-r)T} > 1 \tag{9}$$

When this condition is fulfilled log is not well-defined within the $\mathbb{R}$, since the overall argument in the equation becomes negative. The condition actually is a market limit for the R since the recovery rate must fulfill $Re^{(c-r)T} < 1$, this results in the expression:

$$R < e^{-(c-r)T} \tag{10}$$

or the other way around the spread cannot be wider than:

$$c - r = -\frac{1}{T}\log(R) \tag{11}$$

The condition Equation (9) results in the pricing default contribution been larger than the premium contribution, then, the price gets favored by a default event to happen. Finally, an interesting approximation can be obtained upon imposing that credit spread is small enough to expand the exponential at Equation (6), doing so, the default spread can be expressed as:

$$D \approx c - r - \frac{1}{T}\log\left(1 - \frac{R(c-r)T}{1-R}\right) \tag{12}$$

This equation can be expanded again into the logarithm Taylor expansion resulting in a very well known expression that defines the default spread first approximation for credit default spread:

$$D = \frac{c-r}{1-R} \tag{13}$$

Figure 2 shows the default spread calculated by the Equation (6), a set of different credit spread was chosen in order to illustrate the feature equations varying with the recovery rate. Firstly, a large credit spread resolves in an overall larger default spread. Indeed, the default spread for a given credit spread is larger than the default spread for a smaller credit spread for every recovery rate. Secondly, the default spread not only shows an asymptotic behaviour getting close to $R = 1$, but it exhibits a cut off for the recovery rate given from Equation (9).

2.2. P&L Based on Sensitivities

In this section, we describe the daily $P\&L$ calculation, based on the risk factors from the risk free curve and the ones coming from the default spread curve defined by Equation (6). In that way, it is taken into account rate and credit risk. In order to simulate the daily $P\&L$, various approaches can be taken, one of the most common ones due to its simplicity is the Taylor expansion, where the $P\&L$ is calculated expanding the price difference in the risk factors. Taking derivatives on Equation (6) gives:

$$\begin{aligned} \frac{\partial P}{\partial r} &= -tP \\ \frac{\partial P}{\partial D} &= (1-R)\frac{\partial P}{\partial r} \end{aligned} \tag{14}$$

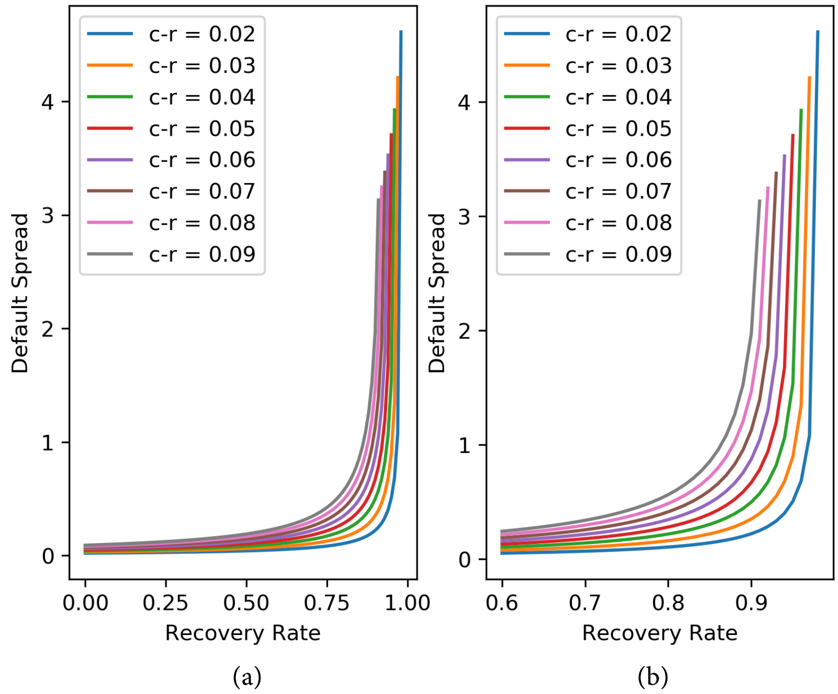

Figure 2. (a) Shows the default spread for a set of credit spreads varying the recovery rate in the interval (0, 1). (b) Shows a closer detail to the recovery rate upper limit for same configuration that in (a). Source: from the authors.

Then the $P\&L$ can be cast as:

$$\Delta_{P\&L} = \frac{\partial P}{\partial r}\Delta r + \frac{\partial P}{\partial D}\Delta D = -tP\Delta r - (1-R)tP\Delta D \tag{15}$$

where differences come from subtracting the risk factor values for two consecutive days. An interesting limit can be reached by plugging approximation in Equation (13) into the $P\&L$ is obtained:

$$\begin{aligned}\Delta_{P\&L} &= \frac{\partial P}{\partial r}\left(\Delta r + (1-R)\Delta D\right) = \frac{\partial P}{\partial r}\left(\Delta r + (1-R)\frac{\Delta c - \Delta r}{1-R}\right) \\ &= \cdots = \frac{\partial P}{\partial r}\Delta c = \frac{\partial P}{\partial c}\Delta c\end{aligned} \tag{16}$$

Equation (16) is the result from calculating the $P\&L$ considering the yield curve as the risk factor instead the default spread and the risk free curve. The practitioner could be tempted at this point to avoid the use of a curve with a tenor structure, and stick to the actual yield value instrument. This $P\&L$ approximation could be a very good approximation but it lacks from the features to be a risk factor, since the yield is fully related to the life time of an actual instrument and a risk factor must be an abstract concept fully defined through whole history, as the tenors within a curve are. In addition, the Equation (15) holds two independent risk contributions explicitly, that is specially useful in terms of risk measurement, whereas Equation (16) only exhibits the yield dependence.

P&L Theta Correction

Through the sections above, we have shown the model is based on the definition of a yield curve that together with the maturity perfectly fits the T-Bill prices. However, the yield curve brings an interesting dynamic in term of $P\&L$ calculation. Let assume that for a given T-Bill two consecutive prices are identical, in this case the $P\&L$ equals zero, but the yield will suffer a variation just by the one day difference in the year fraction. By using the yield to maturity as the risk factor in a $P\&L$ calculation by Taylor expansion the $P\&L$ results in a non-zero value. In order to compensate this effect a correction can be added into the $P\&L$. Our second main contribution in this paper is the definition of this correction. The starting point for our definition is based on the fact that the yield difference stems from the time to maturity difference between two consecutive days and the price variation by the pass of time. Then the correction can be understood as a time sensitivity, indeed, the full $P\&L$ adding the time correction can be expressed:

$$\begin{aligned}\Delta_{P\&L} &= \frac{\partial P}{\partial r}\Delta r + \frac{\partial P}{\partial D}\Delta D + \frac{\partial P}{\partial t}\Delta T = \cdots \\ &= -tP\Delta r - \left(1-R\right)tP\Delta D + \left(cP - RD\mathrm{e}^{-rT}\right)\Delta T\end{aligned} \tag{17}$$

Keeping in mind T is time to maturity, ΔT is thus well defined since T is reduced in one, from one day to another, and basically maturity date comes one day closer by passing one day. Therefore $\Delta T = -1$, module business days, since the closer to the maturity the shorter the year fraction is. The idea behind this time sensitivity is to consider the price difference induced by the pass of time as coming from a risk factor variation, and thus in order to describe its $P\&L$ contribution a new sensitivity must be defined, leading to the expression Equation (17).

In the following section, Equations (6) and (17) are going to be tested against actual market $P\&L$ values for a given set of T-Bills. Two different statistical metrics are used in order to address our model accuracy.

3. P&L Comparison

In this section, the results from the model are discussed, **Table 1** and **Table 2** contain the comparison between the daily $P\&L$ calculation based on the model and the $P\&L$ from the market data values for a one year window from 12/03/2019 to 12/01/2022. In order to measure these differences three different magnitudes are evaluated. Spearman correlation, the Kolmogorov Smirnov test in order to address the equivalence of both statistical distributions and a self-defined $P\&L$ explanation measure (E_{PL}). For the calculation, it has been used public price data obtained from Reuter's data base from 24 different Spanish T-Bills, which maturities expand through one year starting at 2019. The risk free interest rates have been obtained from Bundesbank Bills (Bu-Bills) data prices from Reuters data source as well, the Bu-Bills are issued every month with one year maturity, the German government bills are used as risk free curve due to the well known excellent German government creditworthiness, and they have been interpolated by a cubic spline method, and finally a $R = 0.4$ has been chosen for the whole

Table 1. Metric calculations with no theta correction.

No Theta Correction				
Instrument	Days Alive	P&L Expl	Corr Spear	KS Stat
ES0L01912069	3	0.499999707	0.866025404	0.666666667
ES0L02001177	31	0.531369376	0.904481282	0.35483871
ES0L02002142	51	0.524647408	0.827878078	0.450980392
ES0L02003066	66	0.612703836	0.876348448	0.484848485
ES0L02004171	95	0.582203017	0.886438228	0.410526316
ES0L02005087	110	0.656309424	0.928112417	0.327272727
ES0L02006127	135	0.73104161	0.948189959	0.340740741
ES0L02007109	155	0.75589121	0.971909857	0.258064516
ES0L02008149	180	0.795170883	0.958058691	0.327777778
ES0L02009113	200	0.806058676	0.940366646	0.335
ES0L02010095	220	0.818251898	0.959586288	0.277272727
ES0L02011135	245	0.842956377	0.964316175	0.265306122
ES0L02012042	258	0.835042939	0.956512459	0.313953488
ES0L02101159	225	0.867545638	0.960293302	0.257777778
ES0L02102124	205	0.894605415	0.959437564	0.273170732
ES0L02103056	190	0.901039202	0.960966256	0.247368421
ES0L02104161	161	0.823464417	0.965396329	0.273291925
ES0L02105077	146	0.741647914	0.963715319	0.260273973
ES0L02106117	121	0.64698241	0.954978578	0.26446281
ES0L02107099	101	0.624471928	0.955030541	0.277227723
ES0L02108139	76	0.644311187	0.948102061	0.276315789
ES0L02109103	56	0.691809508	0.9630943	0.267857143
ES0L02110085	36	0.666313853	0.972830415	0.25
ES0L02111125	11	0.746808533	0.972727273	0.272727273

Source: from the authors.

Table 2. Metric calculation with theta correction.

Theta Correction				
Instrument	Days Alive	P&L Expl	Corr Spear	KS Stat
ES0L01912069	3	0.222198513	0.866025404	0.333333333
ES0L02001177	31	0.794818833	0.922271705	0.193548387
ES0L02002142	51	0.784719648	0.822867261	0.156862745
ES0L02003066	66	0.830371412	0.880596951	0.196969697
ES0L02004171	95	0.794372927	0.895871947	0.147368421
ES0L02005087	110	0.827979279	0.931278387	0.109090909
ES0L02006127	135	0.865239997	0.948953975	0.118518519

Continued

ES0L02007109	155	0.883256267	0.97275213	0.077419355
ES0L02008149	180	0.905718232	0.957475388	0.116666667
ES0L02009113	200	0.904082106	0.942031913	0.105
ES0L02010095	220	0.9100448	0.961324171	0.077272727
ES0L02011135	245	0.918946744	0.964233713	0.081632653
ES0L02012042	258	0.911253128	0.95747878	0.073643411
ES0L02101159	225	0.929059725	0.961428986	0.084444444
ES0L02102124	205	0.94381675	0.95928918	0.07804878
ES0L02103056	190	0.945045191	0.962041765	0.1
ES0L02104161	161	0.915330409	0.96480378	0.074534161
ES0L02105077	146	0.885989642	0.962623642	0.068493151
ES0L02106117	121	0.848810098	0.955263213	0.074380165
ES0L02107099	101	0.828311611	0.955520232	0.079207921
ES0L02108139	76	0.852830147	0.949647583	0.092105263
ES0L02109103	56	0.881935166	0.961383288	0.089285714
ES0L02110085	36	0.863806931	0.973216714	0.055555556
ES0L02111125	11	0.926093689	0.972727273	0.090909091

Source: from the authors.

calculation as the most common standard market value. Data in **Table 1** is calculated with no theta correction, in the other hand, data in **Table 2** is calculated fully adding the theta correction by Equation (17). Firstly, we will discuss individually the results from both tables, afterwards, both results will be compared to understand whether the theta correction results in a relevant improvement in terms of $P\&L$ reproduction.

A relevant value reference for the spearman correlation and the KS-test to address the calculation as good results are the levels fixed by the *BASEL III* committee at the *Fundamental Review of the Trading Book* (*FRTB*). In the guide a set of rules is introduced to the banks in order to order the wholesale trading of a bank. Roughly speaking, two kind of models can be used to calculate the capital reservoir, the standard approach and the advanced approach, the requirements to apply the advance approach for the daily $P\&L$ (usually resulting in a capital reduction) in terms of correlation and KS-test are $\rho_s = 0.8$ and $KS = 0.09$, respectively. These values are to be the application level to the so called *P&L attribution*. The E_{PL} is defined in the following expression:

$$E_{PL}\left(r, r_{ref}\right) = 1 - \frac{\sum_{i=1}^{n}\left(\left|r^i - r_{ref}^i\right|, \left|r_{ref}^i\right|\right)^{-}}{\sum_{i=1}^{n}\left|r_{ref}^i\right|} \tag{18}$$

The P&L explanation is by definition bounded between [0, 1], it is worth to notice that the E_{PL} is not an error, since an error is itself not bounded. Indeed, let assume a $P\&L$ value well below average r_{small} the maximum contribution from this value to the E_{PL} is by definition r_{small}, independently of the difference between the calculation and the reference value. So the *P*&*L* explanation is aiming to collect the total amount of *P*&*L* that the calculation is able to reproduce, no more, no less. The tables are organized in five columns, the first column is the ISIN code of the T-Bill, the second is the number of days the bill is alive though the time window, and the rest columns contain the three metrics. As can be noticed, the results are organized from top to bottom by decreasing maturities. At this point, it is relevant to point out that the T-Bills mostly are a one-year maturity instrument that is rerolled. Then, in a one-year window, not all the instruments are equally significant, since some of the T-Bills will be only one month or less alive. More precisely, in a one-year window, only one T-Bill will be alive for the whole year.

Result Discussion

In this section, the results from the model with and without theta correlation are comparing with the actual *P*&*L* values obtained from market prices.

Table 1 does not contain the theta correction. Generally speaking, the model performs very well in terms of correlation and *P*&*L* explanation. Indeed, 23 correlations out of 24 T-Bills are above 0.8, which is our given confident level following the *FRTB* level for the P&L attribution as we have already introduced. The E_{PL} does not exhibit that good levels but it is still in a regime of acceptable results, the T-Bills with market values above 100 days keep levels around ~0.6, which they are acceptable. On the other hand, the Kolmogorov-Smirnov test clearly underperforms, being unable to reach better values than 0.25 faraway from the reference value of 0.09.

Table 2 shows the results for the model adding the theta correction. Firstly, the correlation levels are still above the reference level, again 23 out of 24 bills reach the value 0.85, so the theta correction keeps the same correlation levels. However, for the E_{PL} the theta correction plays a relevant role, since every single E_{PL} for every T-Bill improves. Furthermore, the T-Bills with more than 100 days alive in the period show explanation levels above 0.82, reaching a maximum value of 0.94. Although the theta correction clearly improves the E_{PL} results, the lager improvement is obtained for the KS-test. Indeed, 12 out of 24 T-Bills fulfill the test, and 17 T-Bills exhibit KS values below 0.11, so there close to the test threshold.

Additionally, it can be observed that the ending T-Bills, that are at the beginning of the tables, show overall worse results that the beginning T-Bills, that are at the end of the tables. However, these metric values are clearly no statistical representative since the instruments are alive a very short time period. Nevertheless, it seems a trend within both result tables. For the sake of completeness, and to further illustrate the theta correction effect over the *P*&*L*, we show both the *P*&*L* from the model and the market prices with and without theta correction in Figure 3(a) shows the comparison between the model (blue) and the market

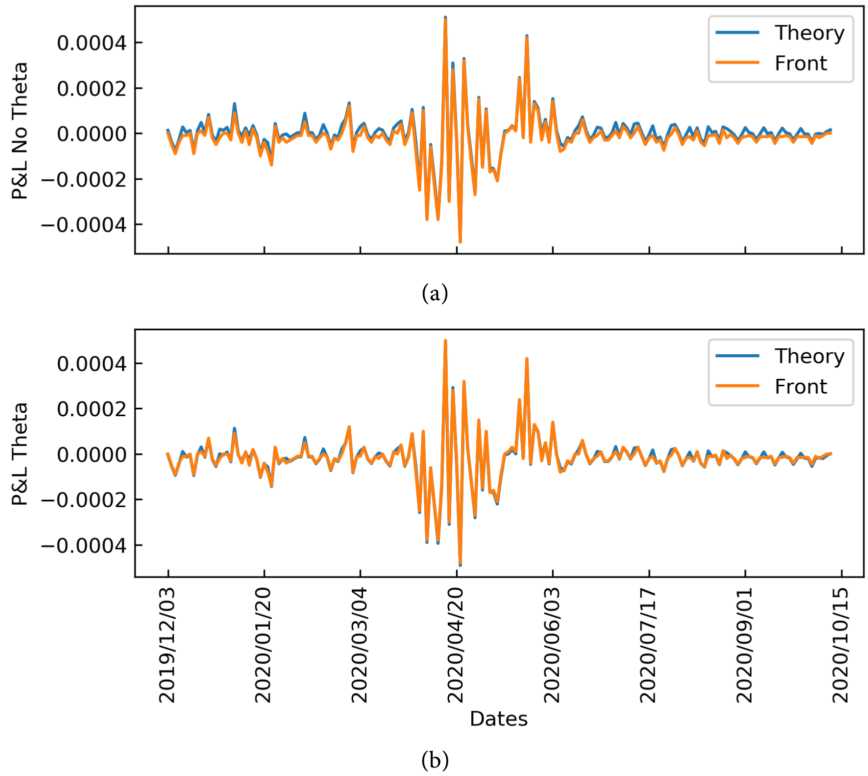

Figure 3. (a) Historic P&L without theta correction for the T-Bill ES0L02012042 from 2019/12/03 to 2020/10/15. (b) Historic P&L with theta correction for the T-Bill ES0L02012042 from 2019/12/03 to 2020/10/15. Source: from the authors.

data (orange) with no theta correction for the T-Bill named *ES0L02012042*, which is the bill with the largest calculated period. Roughly speaking the central historic period (2020/04/03-2020/06/03) that encloses COVID-19 is fairly well reproduced, the largest differences lay on the final and initial periods, where the time effect plays a larger role, since the T-Bill gets closer to maturity and the price is almost constantly the par-value (Zhu, Wu, Chern, & Sun, 2013). Indeed, in **Figure 3(b)** same magnitudes are represented, the model results hold in this case the theta correction. It can be observed how the theoretical curve (blue) fits better the market curve, especially in the final period where the T-Bill price is mostly driven by the pull-to-par effect as we have already discussed. We have shown through this section the model performance with and without theta correction by statistical metrics and explicit *P&L* comparison, illustrating how the model is able to accurately reproduce the market prices and that theta correction plays a very relevant role to do so.

4. Conclusion

In conclusion, we have built up an analytical model based on a realistic approximation for the calculation of default spread for zero coupon bonds, short-term, and near zero coupon bonds, together with closed expressions for the *P&L* calculation based on sensitivities estimations. Additionally, a correction has been added devoted to dealing with the time effect stemming from the yield curve definition, and model limits and further approximations have been discussed as well. We have performed a thorough comparison with realistic market data for 24 different Spanish T-Bills. The model has shown a very good performance specially adding the theta correction.

Further considerations and developments can be done based on the stated model. The solution introduced in this work can be used as a starting point to obtain an approximate analytical solution for the default spread, sensibilities and *P*&*L* of a general bond with credit contribution.

Finally, we consider that the work is very relevant for any practitioner within the finance industry that could need to introduce credit risk in their *P*&*L* calculation, based on the model's simplicity, its low implementation cost and the usual large investment position on T-Bills in a bank portfolio.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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