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Wormholes Do Not Exist: They Are Mathematical Artifacts from an Incomplete Gravitational Theory

Espen Gaarder Haug

Norwegian University of Life Sciences, Ås, Norway
Email: espenhaug@mac.com

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Abstract

The Schwarzschild solution to the Einstein field equation leads to a solution that has been interpreted as wormholes. While many researchers have been sceptical about this interpretation, others have been positive about it. We show that wormholes are not mathematically allowed in the spherical metric of a newly-released unified quantum gravity theory known as collision space-time [1] [2] [3]. We, therefore, have reasons to believe that wormholes in general relativity theory are nothing more than a mathematical artefact due to an incomplete theory, but we are naturally open to discussions about this point. The premise that wormholes likely do not exist falls nicely into line with a series of other intuitive predictions from collision space-time where general relativity theory falls short, such as matching the full spectrum of the Planck scale for micro "black holes".

Keywords

Wormholes, General Relativity, Quantum Gravity, Collision Space-Time

1. Background

Flamm [4] had hinted at wormholes existing as early as 1916, but in 1935, Einstein and Rosen [5] seem to be the first to take the wormhole idea seriously and to try to accomplish some mathematical physics with it by utilizing general relativity theory [6] and the Schwarzschild metric [7] [8] given by:

$$ds^2 = -\frac{dr^2}{1-\frac{r_s}{r}} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \left(1-\frac{r_s}{r}\right)c^2dt^2 \tag{1}$$

Next, as often carried out, we set $c=1$ and $r_s = \frac{2GM}{c^2} = 2m$. Furthermore,

as suggested by Einstein and Rosen, we define a new variable $u^2 = r - 2m$, and then replace r with $r = u^2 + 2m$ in the Schwarzschild metric. From this, we get:

$$ds^2 = -4\left(u^2 + 2m\right)du^2 - \left(u^2 + 2m\right)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) + \frac{u^2}{u^2 + 2m}dt^2 \quad (2)$$

This is the result given by Einstein and Rosen in their 1935 paper, and they discuss the special case when $u = 0$. In this case, the $g_{4,4} = \left(1 - \frac{r_s}{r}\right)c^2dt^2 = \frac{u^2}{u^2 + 2m}dt^2$ term vanishes (as it becomes zero), while the other terms in the Schwarzschild solution are still well defined. This means the metric no longer is affected by change in time. This has been interpreted as at least a theoretical possibility for what is known as a wormhole: two points in space-time can possibly be connected with what is known as the Einstein Rosen bridge or in more popular terms, a "wormhole" or a Schwarzschild wormhole, since it is derived using the Schwarzschild metric. This indicates that two points can be close, and even if they are billions of light years apart, they can still be connected with the Einstein Rosen Bridge where it takes no time to travel between the two points. Wormholes have been investigated theoretically by a series of physicists [9]-[25]. Some think wormholes are a possibility, but others think of them more as a mathematical artefact coming out from the theory. The book *Lorentzian Wormholes* by Visser [26] gives a good overview of the topic.

In our view, despite the success in many predictions from general relativity and the Schwarzschild metric, we still think the theory has strong limitations. For example, the Schwarzschild metric cannot match up with the full Planck scale; see [27].

2. Are Wormholes Allowed in the Collision Space-Time Metric?

The spherical metric in collision-space time [3] [27] (three time dimensions and three space dimensions) that takes into account gravity is given by:

$$\begin{aligned} ds^2 = &-\frac{dr^2}{1 - \frac{2GM}{rc^2} - \frac{G^2M^2}{c^4r^2}} - r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \\ &+\left(1 - \frac{2GM}{rc^2} + \frac{G^2M^2}{c^4r^2}\right)c^2dt^2 + c^2t^2d\theta^2 + c^2t^2\sin^2\theta d\phi^2 \end{aligned} \quad (3)$$

Next, we will also be setting $c = 1$ for this metric. The radius where the escape velocity is c in this theory is $r_h = \frac{GM}{c^2}$ when $v_e = c$, see [28]. For notation purposes, we can set $m = \frac{GM}{c^2}$. Furthermore, we can also here define a new variable of $u^2 = r - m$. The choice of u is such that we end up with the dt^2 term vanishing; that is, we can replace r with $r = u^2 + m$ in the space-time metric above. From this, we get:

$$\begin{aligned} \mathrm{d}s^2 = &-\frac{4\left(u^2+m\right)^2}{u^2}\mathrm{d}u^2 - \left(u^2+m\right)^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right) \\ &+\left(\frac{u^2}{u^2+m}\right)^2\mathrm{d}t^2 + t^2\mathrm{d}\theta^2 + t^2\sin^2\theta\mathrm{d}\phi^2 \end{aligned} \tag{4}$$

If we next set $u=0$ then, as expected, the $\mathrm{d}t^2$ element disappears similarly to how it did in the Einstein-Rosen modified Schwarzschild metric, but in our metric, the $\mathrm{d}u^2$ term now goes on to be infinite or is actually mathematically undefined (a singularity). The fact that the $\mathrm{d}u^2$ term is no longer mathematically valid can be interpreted as no valid solution being available when the $\mathrm{d}t^2$ element disappears. The interpretation of this must be that wormholes are forbidden in our theory. This is in contrast to the Einstein-Rosen metric in which the $\mathrm{d}u^2$ term and other parts of the metric were well behaved even after the $\mathrm{d}t^2$ term vanished. One should not only look for what a theory predicts and what is confirmed by observations, but also for what it predicts that not has been observed and also sounds very unlikely. If a theory predicts that pink elephants fly back and forth between the moon and the earth, then I cannot prove they do not exist as one could always claim there are only a few of them hiding somewhere in a jungle, but since they have not been observed and, in addition, they have properties that seem extremely unlikely, a theory that shows they cannot exist would perhaps be preferable?

The "fact" (others should naturally check the derivations) that wormholes do not exist in our theory has little to do with the fact that we are using a six-dimensional theory (three space and three time dimensions). It is connected to the fact that Einstein abandoned relativistic mass. In his famous special relativity theory paper, Einstein [29] suggested relativistic mass in the end of his most famous 1905 paper, but got it wrong, while in 1899 Lorentz [30] [31] already gave a likely-correct relativistic mass formula ($m_r = m\gamma$). The fact that Einstein and some of his followers [32] [33] [34] [35] abandoned relativistic mass leads to an escape velocity in general relativity theory, which is identical to that of Newton mechanics $v_e = \sqrt{\frac{2GM}{r}}$. See [36] that formally shows why the escape velocity in general relativity is the same as that in Newton mechanics. On the other hand, we take into account relativistic mass as well as relativistic kinetic energy in our escape velocity and space-time metric, and our escape velocity is given by $v_e = \sqrt{\frac{2GM}{r} - \frac{G^2M^2}{c^2r^2}}$. This escape velocity we simply get from taking into account relativistic kinetic energy as well as relativistic mass in the Newton formulae:

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 - \frac{GM\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}}{r} = 0$$

$$1-\sqrt{1-\frac{v^2}{c^2}}-\frac{GM}{c^2 r}=0$$

$$\sqrt{1-\frac{v^2}{c^2}}=-\frac{GM}{c^2 r}+1$$

$$1-\frac{v^2}{c^2}=\left(-\frac{GM}{c^2 r}+1\right)^2$$

$$1-\frac{v^2}{c^2}=-\frac{2GM}{c^2 r}+\frac{G^2M^2}{c^4 r^2}+1$$

$$v=\sqrt{\frac{2GM}{r}-\frac{G^2M^2}{c^2 r^2}} \tag{5}$$

This different escape velocity is what leads to wormholes being forbidden in our theory. We could also have formulated a four-dimensional space-time metric and simply replaced the general relativity escape velocity with our full relativistic escape velocity. The Schwarzschild metric we can re-write in the form of escape velocity as:

$$c^2\mathrm{d}\tau^2=-\frac{\mathrm{d}r^2}{1-\frac{r_s}{r}}-r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)+\left(1-\frac{r_s}{r}\right)c^2\mathrm{d}t^2$$

$$c^2\mathrm{d}\tau^2=-\frac{\mathrm{d}r^2}{1-\frac{v_e^2}{c^2}}-r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)+\left(1-\frac{v_e^2}{c^2}\right)c^2\mathrm{d}t^2 \tag{6}$$

where v_e is the escape velocity, that again in general relativity theory is $v_e=\sqrt{2GM/r}=c\sqrt{r_s/r}$. Now ad hoc we simply replace this with the escape velocity we get when taking into account Lorentz's relativistic mass $v_e=\sqrt{\frac{2GM}{r}-\frac{G^2M^2}{r^2c^2}}$. Then it is naturally not really the Schwarzschild metric any more, but an ad hoc modified metric that takes into account Lorentz's relativistic mass, so this should be investigated further. This gives:

$$c^2\mathrm{d}\tau^2=-\frac{\mathrm{d}r^2}{1-\frac{2GM}{rc^2}+\frac{G^2M^2}{r^2c^4}}-r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)+\left(1-\frac{2GM}{rc^2}+\frac{G^2M^2}{r^2c^4}\right)c^2\mathrm{d}t^2$$

$$c^2\mathrm{d}\tau^2=-\frac{\mathrm{d}r^2}{1-\frac{r_s}{r}+\frac{r_s^2}{4r^2}}-r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)+\left(1-\frac{r_s}{r}+\frac{r_s^2}{4r^2}\right)c^2\mathrm{d}t^2 \tag{7}$$

Next, we set $c=1$ and replacing GM/r with m and r with u^2+m (bear in mind that the event horizon is now $R_h=\frac{GM}{c^2}$ and not the Schwarzschild radius $R_s=\frac{2GM}{c^2}$). Again $\frac{\mathrm{d}r^2}{1-\frac{r_s}{r}+\frac{r_s^2}{4r^2}}=\frac{4\left(u^2+m\right)^2}{u^2}\mathrm{d}u^2$ and

$\left(1-\frac{r_s}{r}+\frac{r_s^2}{4r^2}\right)c^2\mathrm{d}t^2=\left(\frac{u^2}{u^2+m}\right)^2\mathrm{d}t^2$, see the appendix, this gives:

$$c^2\mathrm{d}\tau^2=-\frac{\mathrm{d}r^2}{1-\frac{2GM}{rc^2}-\frac{G^2M^2}{r^2c^4}}-r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)+\left(1-\frac{2GM}{rc^2}+\frac{G^2M^2}{r^2c^4}\right)c^2\mathrm{d}t^2$$

$$c^2\mathrm{d}\tau^2=-\frac{4\left(u^2+m\right)^2}{u^2}du^2-\left(u^2+m\right)^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)+\left(\frac{u^2}{u^2+m}\right)^2\mathrm{d}t^2 \quad (8)$$

If setting is $u=0$ now, then the $\mathrm{d}t^2$ term vanishes (as in general relativity theory), but the $\mathrm{d}r^2$ term is now mathematically forbidden as it means dividing by zero; this is unlike the predictions from general relativity theory. In other words, wormholes are forbidden when we take into account Lorentz's relativistic mass. Further, when only working in 4D space-time, we do not seem able to make it fully consistent with the quantum world [2] [3], but this is outside the scope of this paper. Anyway it is clear that taking into account Lorentz's relativistic mass means wormholes are mathematically forbidden. This should not be seen in isolation, but combined with taking into account that Lorentz's relativistic mass means the Planck mass then fits all the properties of the Planck scale. With all the properties of the Planck scale, we mean such things as the Planck length, the Planck time, Planck acceleration, Planck density etc. For example, in general relativity theory, the Planck mass has a Schwarzschild radius twice the Planck length (do not fit the Planck length), and the gravitational acceleration at the Schwarzschild radius is one-fourth of the Planck acceleration, and the density is only one-eighth of the Planck mass density. When taking into account Lorentz's relativistic mass, all the properties of the Planck scale fit a micro black hole with mass size equal to the Planck mass; see Haug [27].

3. Conclusion

In the Schwarzschild solution (Einstein-Rosen), we are able to allow the $\mathrm{d}t^2$ term to vanish, while the other parts of the Schwarzschild metric are still well defined and well behaved; this has led to speculations about wormholes being possible in general relativity theory. In our collision space-time theory, the metric does not allow wormholes as demonstrated in this paper. In addition, we have previously shown that our metric matches up with all properties of the Planck scale for micro black holes, while general relativity theory and the Schwarzschild metric can only match one or two properties of the Planck scale for micro black holes. In addition, our metric is consistent with a new universe equation that seems more logical [28]. Most importantly, our new metric also seems to be consistent with a quantum gravity theory that unifies gravity with quantum mechanics.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix: Derivations

When taking into account Lorentz relativistic mass then the $\mathrm{d}t^2$ term is given by

$$\left(1-\frac{2GM}{c^2r}+\frac{G^2M^2}{c^4r^2}\right)\mathrm{d}t^2 \tag{9}$$

After replacing $c=1$ and $\frac{GM}{c^2}$ with m and $r=u^2+m$ (bear in mind the horizon radius is now $R_h=\frac{GM}{c^2}$ and not $R_s=\frac{2GM}{c^2}$), we get

$$\begin{gathered}\left(1-\frac{2GM}{c^2r}+\frac{G^2M^2}{c^4r^2}\right)\mathrm{d}t^2\\ \left(1-\frac{2m}{r}+\frac{m^2}{r^2}\right)\mathrm{d}t^2\\ \left(1-\frac{m}{r}\right)\left(1-\frac{m}{r}\right)\mathrm{d}t^2\\ \left(1-\frac{m}{r}\right)^2\mathrm{d}t^2\\ \left(1-\frac{m}{u^2+m}\right)^2\mathrm{d}t^2\\ \left(\frac{u^2}{u^2+m}\right)^2\mathrm{d}t^2\end{gathered} \tag{10}$$

Next, we set $u=0$ and we now see the $\mathrm{d}t^2$ term vanish as it is multiplied by zero.

Furthermore related to $\mathrm{d}r^2$ term we now get:

$$\begin{gathered}\frac{1}{1-\frac{2m}{r}+\frac{m^2}{r^2}}\mathrm{d}r^2\\ \frac{1}{1-\frac{2m}{r}+\frac{m^2}{r^2}}4u^2\mathrm{d}u^2\\ \frac{1}{\left(1-\frac{m}{r}\right)^2}4u^2\mathrm{d}u^2\\ \frac{1}{\left(1-\frac{m}{u^2+m}\right)^2}4u^2\mathrm{d}u^2\\ \frac{1}{\left(\frac{u^2}{u^2+m}\right)^2}4u^2\mathrm{d}u^2\\ \frac{4\left(u^2+m\right)^2}{u^2}\mathrm{d}u^2\end{gathered} \tag{11}$$

when we set $u = 0$, we see this leads to division by zero, which is mathematically undefined, in other words, a singularity (pole). That is our solution cannot be valid for when $\mathrm{d}t^2$ vanishes as this would mean we have to move infinite in space (without time going by), which is impossible and in line with that nothing can move faster than the speed of light. In other words, wormholes cannot exist in our spherical metric, and can likely not exist at all in our new unified quantum gravity theory [4]. This is in strong contrast to general relativity theory where this is a mathematical possibility as the $\mathrm{d}u^2$ terms and other terms are well defined and valid even after the $\mathrm{d}t^2$ term vanishes there.