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# Effect Due to the Particle Nature of the Doppler Shifted Radiation on the Dynamics of the Spherical Light Emitting Source

**Sameer Yadav**

Department of Physical Sciences, Indian Institute of Science Education and Research, Mohali, India
Email: sameerydv574@gmail.com

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## Abstract

The Doppler effect is a phenomenon of intrinsic kinematic character. This paper analyzes the kinematic Doppler effect for the case where the source is moving and the observer is at rest in the classical limit. The particle nature properties of radiation are considered and how it affects the dynamics of the Source has been studied. The dynamical and kinematical equations have been derived by considering this effect. It has been conclusively shown that a moving light-emitting source experiences a finite recoil momentum in the direction opposite to the direction of motion and come to rest in finite time.

## Keywords

Doppler Effect, Light Emission, Frequency Shift, Recoil Momentum

## 1. Introduction

The Doppler effect is a well-known phenomenon, a shift in the frequency of waves emitted from an object moving relative to an observer. The change in frequency of electromagnetic (EM) waves was first described by Christian Doppler [1] in his most famous paper Über das farbige Licht der Doppelsterne ("Concerning the Colored Light of Double Stars"), published in 1842. The hypothesis was tested by Buys Ballot in 1845 and confirmed the existence of the Doppler effect. This phenomenon allowed Edwin Hubble [2] to propose that the universe was expanding when he observed that all galaxies are redshifted. Later on, Sir William Huggins [3] confirmed the recessive motion of Sirius by showing the change in frequency of the recorded radiation from Sirius towards the red Hydrogen lines. As explained in elementary textbooks, the Doppler effect for sound waves is very well understood. However, several aspects are yet to be explained for the Doppler effect in the case of Light. For frequency shift of purely kinematic origin is intriguing. In the present paper, the effect of Doppler shifted Light on the dynamics of the source object is studied considering the particle nature of Light. The aim of this analysis is to show that in the inertial frame of an observer, the light emitting source will experience finite recoil momentum in the direction opposite to the direction of motion due to which the source will come to rest in finite time.

In the next section, the equation governing frequency shift due to axial Doppler shift is introduced, and then by considering particle properties of radiation the recoil momentum transferred to the source per unit time has been calculated. Afterward, equations governing the dynamics of the spherical light emitting source were derived. Thereafter, conclusions have been drawn from the final form of dynamical equations. Further, the equations governing the kinematics of the source have been derived. At last, we conclude with a derivation of the equation that gives the time that a spherical light emitting source will take to come to rest, hence justifying the hypothesis that any spherical light emitting source will come to rest in finite time.

## 2. Main Text

Considering the case when the observer (o) is at rest and the light emitting source (s) is moving. Let a Spherical Light Emitting Object of radius R traveling along negative x direction emanating electromagnetic waves isotropically. The origins of the cartesian coordinates, the observer's frame, and the spherical polar coordinates, centered at the source, coincide at the time = 0. The axial doppler shift in the frequency of the radiation traveling at speed $c$ emitted from a source moving with the velocity components $\left(-v_x, 0, 0\right)$ at the time of observation $t_0$ at an Azimuthal angle $\phi$ is,

$$\vartheta_0 = \frac{1}{1+\frac{v_x}{c}\cos\phi}\vartheta_s \tag{1}$$

where $\vartheta_s$ is the frequency of radiation produced by the source, $\vartheta_0$ is the doppler shifted frequency of the electromagnetic waves emanating at an angle $\phi$.

The nature of Light is twofold. This means that Light possesses both particle and wave properties. The theory of Light being a particle completely vanished until the end of the 19$^{\text{th}}$ century when Albert Einstein revived it [4]. In this analysis, the particle properties of the radiation are considered. When emitted, a photon imparts a recoil momentum on the Source object. If $n$ is the number of photons emitted per unit time per unit area from the surface of the spherical light emitting object, then the recoil momentum transferred to the light emitting source per unit time per unit area by the photons emitted along the direction,

$$\hat{r} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k} \tag{2}$$

is,

$$-n\frac{h}{\lambda_0}\hat{r}. \quad (3)$$

Since each dimension's dynamics work independently, the analysis of each axis has been done separately. Firstly, the equation governing the change in momentum of the source object per unit time along x direction has been studied,

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = \int_0^{\pi}\int_0^{2\pi} \frac{-nh\sin\theta\cos\phi}{\left(1+\frac{v_x}{c}\cos\phi\right)\lambda_s} R^2\sin\theta\mathrm{d}\phi\mathrm{d}\theta \quad (4)$$

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = \frac{-nhR^2}{\lambda_s}\frac{\pi}{2}\frac{2\pi}{\left(\frac{v_x}{c}\right)} \quad (5)$$

the recoil momentum transferred per unit time to the source object by the Doppler shifted photons in the observer's frame is non-zero, and the negative sign implies that the motion is retarding.

Now, analysing the dynamics of other axes. The change in momentum per unit time along y direction is,

$$\frac{\mathrm{d}p_y}{\mathrm{d}t} = \int_0^{\pi}\int_0^{2\pi} \frac{-nh\sin\theta\sin\phi}{\left(1+\frac{v_x}{c}\cos\phi\right)\lambda_s} R^2\sin\theta\mathrm{d}\phi\mathrm{d}\theta = 0 \quad (6)$$

and the change in momentum per unit time along z direction is,

$$\frac{\mathrm{d}p_z}{\mathrm{d}t} = \int_0^{\pi}\int_0^{2\pi} \frac{-nh\cos\theta}{\left(1+\frac{v_x}{c}\cos\phi\right)\lambda_s} R^2\sin\theta\mathrm{d}\phi\mathrm{d}\theta = 0. \quad (7)$$

The definite integral of the Equations (6) and (7) was calculated using the functions property of being symmetric.

As,

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} \neq 0, \frac{\mathrm{d}p_y}{\mathrm{d}t} = 0, \frac{\mathrm{d}p_z}{\mathrm{d}t} = 0 \quad (8)$$

implies that the net momentum is not conserved. The momentum only along the transverse directions is conserved. Due to the non-zero value of recoil momentum transferred per unit time by the Doppler shifted photons in the observer's frame to the source object, it is evident that the source object which is emanating light continuously will come to rest in finite time.

The kinematics of the spherical light emitting source having mass $M$ and radius $R$ having initial velocity components $(-v_s, 0, 0)$ at the time $t_{0s}$ are governed by the following equations,

$$v_x^2 = v_s^2 - \frac{nhR^2\pi^2\vartheta_s}{M}(t - t_{0s}) \quad (9)$$

$$v_y = 0 \quad (10)$$

$$v_z = 0. \quad (11)$$

The source with initial velocity components $(-v_s, 0, 0)$, having retardation

$\frac{-nhR^2}{M\lambda_s}\frac{\pi^2}{\left(\frac{v_x}{c}\right)}$, come to rest in time $T$ given by,

$$T = t_{0s} + \frac{M}{nhR^2\pi^2\vartheta_s} v_s^2 . \tag{12}$$

## 3. Conclusion

In the above theoretical analysis, the effect of the particle nature of Doppler (kinematic) shifted light on the dynamics of the spherical light emitting source has been studied. The light emitted by a source in relative motion with respect to an observer will have a Doppler shifted frequency and wavelength, which implies a shift in the photon energy and momentum, respectively. The fundamental law of momentum conservation infers that the emitted photons will impart a recoil momentum on the source, which is given by Equation (3). The recoil momentum transferred by the redshifted and the blueshifted photons is not equal, which leads to the transfer of finite recoil momentum in the direction opposite to the direction of the source velocity. Due to this transferred recoil momentum, the source object will experience a retarding force in the observer's frame of reference. The derived kinematics equations predict that with the passage of time, the velocity of source will decrease, and the source will come to rest in the time given by the Equation (12). Considering the particle properties of the emitted light leads us to the equations governing the kinematics of the source and confirms the hypothesis that any spherical light emitting source will come to rest in finite time. This new insight will help us improve our understanding of the effect of the Doppler shift on the dynamics of the light emitting source.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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## Appendix.

Calculation of Antiderivative of Equation (3),

$$I=\int_0^{\pi}\int_0^{2\pi}\frac{-nh\sin\theta\cos\phi}{\left(1+\frac{v_x}{c}\cos\phi\right)\lambda_s}R^2\sin\theta\,\mathrm{d}\phi\,\mathrm{d}\theta$$

Since $\theta$ and $\phi$ are two independent variables, the antiderivative can be calculated separately.

Firstly, solving for $\theta$,

$$I_1=\int_0^{\pi}\sin^2\theta\,\mathrm{d}\theta$$

Using trigonometric identity,

$$\sin^2\theta=\frac{1-\cos 2\theta}{2}$$

$$I_1=\left.\left(\frac{\theta-\frac{\sin(2\theta)}{2}}{2}\right)\right|_0^{\pi}$$

$$I_1=\frac{\pi}{2}$$

Now solving for $\phi$,

$$I_2=\int_0^{2\pi}\frac{\cos\phi}{1+\frac{v_x}{c}\cos\phi}\mathrm{d}\phi$$

Writing $\cos\phi$ as $\frac{1}{\left(\frac{v_x}{c}\right)}\left(\frac{v_x}{c}\cos\phi+1\right)-\frac{1}{\left(\frac{v_x}{c}\right)}$

$$I_2=\frac{1}{\left(\frac{v_x}{c}\right)}\int_0^{2\pi}1\mathrm{d}\phi-\frac{1}{\left(\frac{v_x}{c}\right)}\int_0^{2\pi}\frac{1\mathrm{d}\phi}{1+\frac{v_x}{c}\cos\phi}$$

Simplifying using trigonometric/hyperbolic identities,

$$I_2=\frac{2\pi}{\left(\frac{v_x}{c}\right)}-\frac{1}{\left(\frac{v_x}{c}\right)}\int_0^{2\pi}\frac{-\sec^2\left(\frac{\phi}{2}\right)}{\left(\frac{v_x}{c}\right)\tan^2\left(\frac{\phi}{2}\right)-\tan^2\left(\frac{\phi}{2}\right)-\frac{v_x}{c}-1}\mathrm{d}\phi$$

Substituting $y=\frac{\sqrt{\frac{v_x}{c}-1}}{\sqrt{-\frac{v_x}{c}-1}}\tan\left(\frac{\phi}{2}\right)$, $\mathrm{d}\phi=\frac{2\sqrt{-\frac{v_x}{c}-1}}{\sqrt{\frac{v_x}{c}-1}\sec^2\left(\frac{\phi}{2}\right)}\mathrm{d}y$ and solving will give,

$$I_2=\frac{2\pi}{\left(\frac{v_x}{c}\right)}+\frac{2}{\left(\frac{v_x}{c}\right)\sqrt{-\frac{v_x}{c}-1}\sqrt{\frac{v_x}{c}-1}}\arctan\left.\left(\frac{\sqrt{\frac{v_x}{c}-1}\tan\left(\frac{\phi}{2}\right)}{\sqrt{-\frac{v_x}{c}-1}}\right)\right|_0^{2\pi}$$

$$I_2 = \frac{2\pi}{\left(\frac{v_x}{c}\right)}$$

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = \frac{-nhR^2}{\lambda_s}\frac{\pi}{2}\frac{2\pi}{\left(\frac{v_x}{c}\right)}$$