

Statistical Arbitrage Strategy in Multi-Asset Market Using Time Series Analysis

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Abstract

The statistical arbitrage strategy is one of the most traditional investment strategies. There are many theoretical and empirical studies until now. However, almost all of the statistical arbitrage strategies focus on the price difference (spread) between two similar assets in the same asset class and exploit the mean reversion of spreads, *i.e.* pairs trading. In this study, we extend the strategy to multiple assets in the multi-asset market. Although mean-reverting portfolios were derived based on a single criterion in related researches, we derive a mean-reverting portfolio by optimizing multiple mean-reversion criteria. We expect that a mean-reverting portfolio based on multiple indicators leads to a higher return/risk. We perform an empirical analysis in multi-asset market and show the profitability of our strategy.

Keywords

Statistical Arbitrage Strategy, Asset Allocation, Mean Reverting Portfolio

1. Introduction

Portfolio selection is one of the most important topics in mathematical finance. Modern portfolio theory has its genesis in the seminal works of Markowitz [1]. In Markowitz analysis, the investment return should be maximized for a given level of risk. Therefore, the main problem of portfolio selection is how to derive a portfolio with a higher return/risk. Several researchers have been built some models to maximize return/risk of the portfolio. For example, there are many studies based on methods such as machine learning [2] and uncertainty theory [3] in recent years. In addition to maximizing return/risk, there have been proposed methods for constructing portfolios based on various criteria. Risk-based portfolio that focuses only on risk such as risk parity [4], factor risk parity [5]

and complex valued risk parity [6] is a typical example.

Also, deriving a mean reverting portfolio is one of the most popular methods in portfolio selection [7] [8]. Traditionally, a mean reverting portfolio originated from pairs trading. There are many studies on the mean reversion of the price difference between two similar assets, *i.e.* spread until now. A broad range of investors from individual investors to institutional investors invest in pairs trading strategy exploiting the mean reversion of spread [9]. According to [10], there are many approaches to the pairs trading strategy such as stocks distance, time series model e.g. co-integration and stochastic control. However, since many related works of the pairs trading strategy focused on the spread only between similar stocks, the investment universe was stocks in a single asset class. In this study, we propose pairs trading strategy which invest on assets in different asset classes, by deriving the mean-reverting portfolio in not a single asset market but the multi-asset market. When the portfolio is far away to a certain extent from the average level, we take a position in the direction of mean reversion. Specifically, we construct the portfolio based on multiple criteria for the mean reversion defined based on different perspectives. By using the technique of a multi-objective optimization problem called Polynomial Goal Programming (PGP), we propose a fair approach to combine the quantitative criteria of the mean reversion. We aim to obtain the arbitrage opportunity between global asset classes in the multi-asset market.

The remaining sections of this paper are organized as follows. In Section 2, we briefly describe the related studies of the mean reverting portfolio using the time-series model. In Section 3, we introduce multiple indicators denoting the “goodness” of the mean reversion and a method of integrating the indicators called PGP. In Section 4, we describe the pairs trading strategy in the multi-asset market and in Section 5, we verify its effectiveness through empirical analysis with the actual financial market data. Finally, we conclude.

2. Related Work

Quantitative indicators of the mean reversion have been proposed in various forms. Here, we use three types of indicators describing the mean reversion: Predictability, Portmanteau Statistics and Crossing Statistics. Predictability indicates how close to the white noise in terms of the variance of time series [11] [12]. Portmanteau Statistics indicates how close to the white noise in terms of the correlation of time series [13]. Crossing Statistics indicates how many times the time series crosses the average level in the unit time interval [14]. As pairs trading strategies using predictability, portmanteau statistics, crossing statistics alone respectively, there are related researches investing on the implied volatility of the U.S. stocks [7] and U.S. stocks [8]. Although these researches are useful in that they evaluate the effectiveness of the single quantitative indicator of the mean reversion, they do not construct a mean reverting portfolio based on multiple perspectives. Our method combines Predictability, Portmanteau Statistics

and Crossing Statistics by solving the multi-objective optimization problem. We expect that a mean-reverting portfolio based on multiple indicators leads to a higher return/risk. However, there remains a problem that we have no idea how to combine the multiple indicators fairly. As a method of solving such multi-objective optimization problems fairly, a method called PGP is often used for portfolio optimization problems with higher-order moments [15]. In this study, we apply PGP to solve the multi-objective optimization problem. Furthermore, we extend the strategy to multiple assets in the multi-asset market. The comparison of our research and related work is summarized in **Table 1**.

3. Mean Reverting Portfolio

When N assets exist at the time of t , $\mathbf{y}_t = \{y_{1,t}, \dots, y_{N,t}\}$ denotes the log prices at the time. When $\mathbf{w} = \{w_1, \dots, w_N\}$ denotes the weight vector of each assets, the portfolio can be described as below.

$$z_t = \mathbf{w}^T \mathbf{y}_t \tag{1}$$

Logarithmic return of the portfolio r_t can be described as below.

$$r_t = z_t - z_{t-1} = \mathbf{w}^T (\mathbf{y}_t - \mathbf{y}_{t-1}) \tag{2}$$

The problem in this study is to determine the weight \mathbf{w} in which the portfolio z_t in the Equation (1) is mean reverting. In other words, the object is to calculate the weight as much mean reverting as possible between multiple assets.

3.1. Indicators of the Mean Reversion

We introduce multiple indicators which show the goodness in terms of the mean reversion of the portfolio z_t . Specifically, we introduce (1) Predictability, (2) Portmanteau Statistics, (3) Crossing Statistics to quantify the mean reversion. We start by defining the k th order (lag- k) autocovariance matrix for a stochastic process \mathbf{y}_t as

$$\mathbf{M}_k := \text{Cov}(\mathbf{y}_t, \mathbf{y}_{t+k}) = \mathbb{E} \left[(\mathbf{y}_t - \mathbb{E}[\mathbf{y}_t]) (\mathbf{y}_{t+k} - \mathbb{E}[\mathbf{y}_{t+k}])^T \right]. \tag{3}$$

Note that \mathbf{M}_0 represents the covariance matrix.

3.1.1. Predictability

Predictability shows how the time series is close to the white noise in terms of

Table 1. Comparison of our research and related work.

Paper	Criteria	Investment assets
Cuturi and d'Aspremont [2017]	Pred, Port, Cross	Implied volatility of U.S. stocks (single asset)
Zhao and Palomar [2018]	Pred, Port, Cross	U.S. stocks (single asset)
Our research	Pred, Port, Cross Pred + Port + Cross	Global futures (multi-asset)

a. Pred, Port and Cross represent Predictability, Portmanteau Statistics, and Crossing Statistics respectively.

the variance. We consider the following stationary time-series.

$$y_t = \hat{y}_{t-1} + \varepsilon_t \tag{4}$$

\hat{y}_{t-1} denotes the predicted value of y based on the information of the time of $t-1$. The simplest example of Equation (4) is AR (1) model representing $\hat{y}_{t-1} = \alpha y_{t-1}$. ε_t denotes the white noise which is independent from \hat{y}_{t-1} and whose variance is σ_ε^2 . Taking the variance of Equation (4), $\sigma_y^2 = \sigma_{\hat{y}}^2 + \sigma_\varepsilon^2$. σ_y^2 denotes the variance of y_t and $\sigma_{\hat{y}}^2$ denotes the variance of \hat{y}_{t-1} . Predictability is defined as follows.

$$\text{predictability} = \frac{\sigma_{\hat{y}}^2}{\sigma_y^2} \tag{5}$$

From the definition of predictability, predictability means that the smaller, the more mean reverting and vice versa. Here, we assume \hat{y}_{t-1} can be modeled by the following VAR (1) model. Notice that we can extend to VARMA model because VARMA (p, q) model can be reduced to VAR (1) model [16].

$$y_t = Ay_{t-1} + e_t \tag{6}$$

where e_t is the white noise.

Multiplying VAR (1) of Equation (6) by w^T , we can get $w^T y_t = w^T Ay_{t-1} + w^T e_t$. Taking its variance, the term on the left hand side is $w^T M_0 w$ and the first term on the right hand side $w^T Ay_{t-1}$ is $w^T AM_0 A^T w$. Since $A = M_1^T M_0^{-1}$ according to the property of VAR (1) model, the first term on the right hand side is $w^T AM_0 A^T w = w^T M_1^T M_0^{-1} M_0 (M_1^T M_0^{-1})^T w = w^T M_1^T M_0^{-1} M_1 w$.

Therefore, predictability of VAR (1) model is as follows.

$$\text{predictability}(w) = \frac{w^T M_1^T M_0^{-1} M_1 w}{w^T M_0 w} \tag{7}$$

3.1.2. Portmanteau Statistics

Portmanteau Statistics indicator shows how the time series is close to the white noise in terms of the correlation. We consider stationary time-series with lag- p .

$$y_t = \hat{y}_{t-1} + \dots + \hat{y}_{t-p} + \varepsilon_t \tag{8}$$

Portmanteau statistics are defined as follows.

$$\text{portmanteau} = \sum_{i=1}^p \rho_i^2 \tag{9}$$

where ρ_i is the i th order autocorrelation, and defined as $\mathbb{E}[z_t z_{t+i}] / \mathbb{E}[z_t^2]$.

Autocorrelation of the white noise is zero and portmanteau ≥ 0 by its definition. The time series is close to the white noise when portmanteau is close to 0. Therefore, we can get a mean reverting portfolio by minimizing Portmanteau statistics. For a mean reverting portfolio $z_t = w^T y_t$, we can get the expression for Portmanteau statistics as

$$\text{portmanteau}(w) = \sum_{i=1}^p \left(\frac{w^T M_i w}{w^T M_0 w} \right)^2 \tag{10}$$

3.1.3. Crossing Statistics

Crossing Statistics indicator counts how many times the time series crosses the average level in the time interval T .

For a stationary Gaussian process, the crossing statistics is defined as follows.

$$\text{crossing} = \frac{1}{T-1} \sum_{t=2}^T \mathbf{1}_E(y_t) \tag{11}$$

where $\mathbf{1}_E(y_t)$ denotes an indicator function that returns 1 when $E = \{y_t y_{t-1} \leq 0\}$, or 0 otherwise.

For a centered stationary Gaussian process, we can get the expression for Crossing Statistics as

$$\text{crossing} = \frac{1}{\pi} \arccos(\rho_1). \tag{12}$$

In order to get a spread having many zero-crossing, we minimize ρ_1 . Therefore, for mean reverting portfolio $z_t = \mathbf{w}^T \mathbf{y}_t$, we define the crossing statistics as

$$\text{crossing}(\mathbf{w}) = \frac{\mathbf{w}^T \mathbf{M}_1 \mathbf{w}}{\mathbf{w}^T \mathbf{M}_0 \mathbf{w}} \tag{13}$$

3.2. Formulation by Polynomial Goal Programming

In this section, we determine the optimal weights of the portfolio, integrating three mean reversion indicators introduced in the previous section by PGP. Goal programming (GP) was first proposed in [17]. GP is a technique that is often useful in assisting us to find good solutions to optimization problems with multiple objectives.

The GP has many extensions and applications. For example, Liu and Chen proposed an uncertain goal programming and [18] constructs expected value goal programming model and chance-constrained goal programming model for the bicriteria solid transportation problem.

PGP method is originally proposed in [19] for another extension of GP. The PGP method was introduced in the portfolio optimization problem including higher moments [20]. The PGP method has the benefit that it can normalize sub-objective functions and reflect the preferences of investors.

The first step in PGP is to get the optimal values pred^* , port^* , cross^* by solving independent minimizing problems whose objective functions are Equation (7), Equation (10), and Equation (13). We can get the optimal weights of the portfolio by solving the problem as follows.

$$\arg \min_{\mathbf{w}} \left| \frac{d_1}{\text{pred}^*} \right|^{\lambda_1} + \left| \frac{d_2}{\text{port}^*} \right|^{\lambda_2} + \left| \frac{d_3}{\text{cross}^*} \right|^{\lambda_3} \tag{14}$$

where $d_1 = \text{predictability}(\mathbf{w}) - \text{pred}^*$

$d_2 = \text{portmanteau}(\mathbf{w}) - \text{port}^*$

$d_3 = \text{crossing}(\mathbf{w}) - \text{cross}^*$

where $\lambda_1, \lambda_2, \lambda_3$ are variables of investor preferences for predictability, port-

manteau statistics, and crossing statistics. We show the conceptual figure of three indicators of the mean reversion and PGP in **Figure 1**.

4. Investment Strategy

In this study, we propose the pairs trading strategy by deriving the mean reverting portfolio based on the three mean reversion indicators introduced in the previous section.

Specifically, the pairs trading strategy includes following three steps.

Step 1

For investing on multi-assets, we select the order p that minimize AIC of VAR (p) model under the condition that p is equal to or less than the full-order selected in advance.

Step 2

We derive the mean reverting portfolios by minimizing the functions, which are 1) Predictability, 2) Portmanteau Statistics, 3) Crossing Statistics, 4) the multi-objective function integrated with Predictability, Portmanteau Statistics, and Crossing Statistics.

Step 3

We calculate the spread from the moving average of past return of the portfolio derived in Step 2. We get the position when the spread is ± 1 standard deviation farther from the average. As a loss cut, we unwind the position if the spread is ± 2 standard deviations farther from the average.

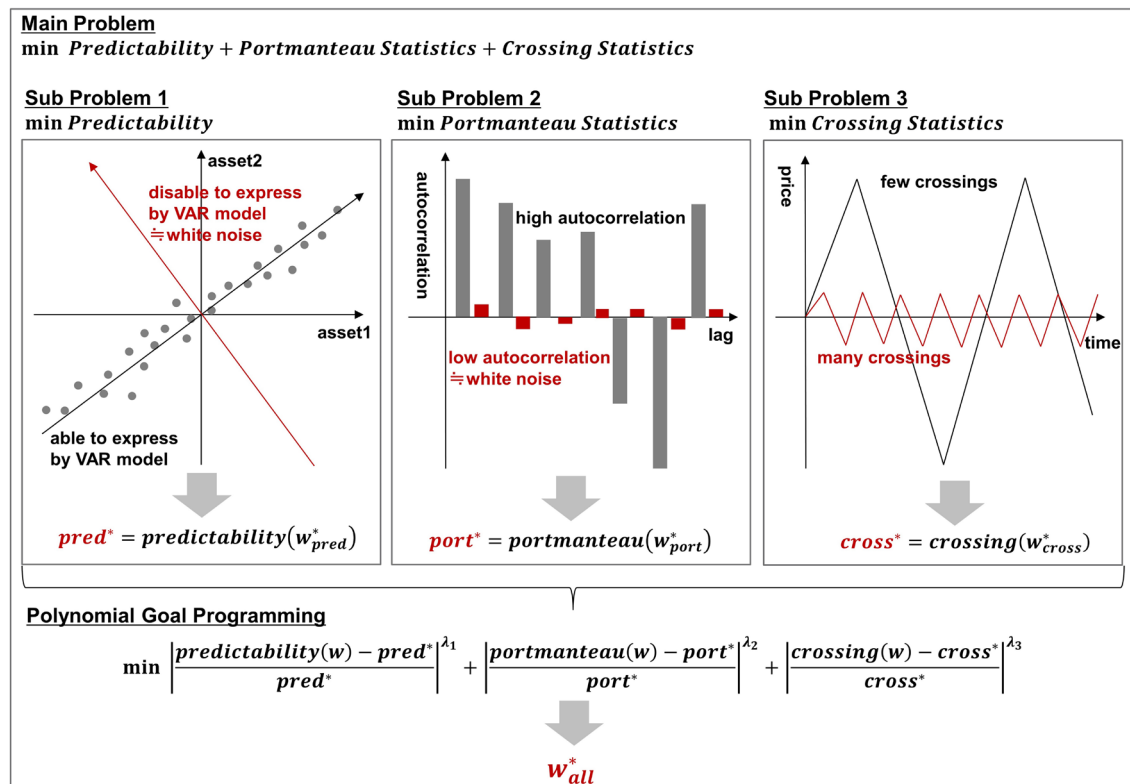


Figure 1. Three indicators of the mean reversion and those integration by PGP.

5. Empirical Results

This section describes the empirical study with real market data.

5.1. Datasets

We test our method using real market data from global futures. We show the investment universe in **Table 2** and the performance statistics in **Table 3**. The data are retrieved from Bloomberg and adjusted Friday-closed weekly return is employed. The trading experiment is carried out from March 23th, 2007 to August 30th, 2019. Sample size during the period is 650.

Table 2. Investment assets.

Investment assets								
Equity future (16 assets)	S & P500 (SP)	NAS DAQ (NQ)	CA (PT)	GB (Z)	FR (CF)	DE (GX)	EU (VG)	ES (IB)
	NL (EO)	NO (OI)	CH (SM)	NIKKEI (NK)	TOPIX (TP)	HK (HI)	AU (XP)	SG (QZ)
Bond future (13 assets)	US2Y (TU)	US5Y (FV)	US10Y (TY)	US20Y (US)	AU3Y (YM)	AU10Y (XM)	CA10Y (CN)	DE2Y (DU)
	DE5Y (OE)	DE10Y (RX)	DE30Y (UB)	GB10Y (G)	JP10Y (JB)			

a. words in parentheses denote tickers.

Table 3. Summary of statistics of investment assets.

Performance statistics	SP	NQ	PT	Z	CF	GX	VG	IB
Return(% , Ann)	8.4	15.1	5.2	5.8	5.9	6.9	5.2	2.8
Risk(% , Ann)	18.0	19.2	17.1	17.8	21.1	22.2	21.9	23.4
Return/risk	0.46	0.79	0.30	0.33	0.24	0.31	0.24	0.12
Maximum drawdown (%)	-56.8	-51.6	-49.1	-49.3	-58.8	-57.8	-61.6	-54.5
Performance statistics	EO	OI	SM	NK	TP	HI	XP	QZ
Return(% , Ann)	6.4	7.5	5.8	5.8	3.4	7.9	3.7	4.4
Risk(% , Ann)	21.0	23.1	18.3	22.7	21.5	22.7	16.8	18.9
Return/risk	0.30	0.32	0.32	0.26	0.16	0.35	0.22	0.23
Maximum drawdown (%)	-64.2	-64.4	-57.6	-60.4	-58.5	-59.5	-55.6	-60.2
Performance statistics	TU	FV	TY	US	YM	XM	CN	DU
Return(% , Ann)	1.2	2.9	4.3	5.8	1.3	3.7	3.8	1.0
Risk(% , Ann)	1.2	3.5	5.6	10.0	2.5	6.7	5.4	1.1
Return/risk	1.01	0.84	0.77	0.58	0.51	0.55	0.71	0.91
Maximum drawdown (%)	-2.4	-5.9	-8.8	-17.7	-5.6	-13.0	-10.0	-3.1

Performance statistics	OE	RX	UB	G	JB
Return(% Ann)	3.2	5.6	9.2	5.0	2.0
Risk(% Ann)	3.0	5.5	12.6	6.4	2.4
Return/risk	1.03	1.02	0.72	0.78	0.85
Maximum drawdown (%)	-6.0	-7.2	-18.3	-10.0	-5.8

5.2. Parameters Settings

We determine the weights of the optimal portfolio on condition that the full-order equals to 5 and data in the past 52 weeks are used for the model selection. Portfolio leverage is determined so that ex-ante risk of the portfolio equals to 5% calculated by the covariance matrix based on the data in the past 52 weeks. Note that model of predictability is VAR (1) model for the integration of indicators. It is decided to unwind the position every quarter even though we have the position. If we have a loss cut, we don't have the position in the quarter. All preferences of predictability, portmanteau statistics, and crossing statistics in PGP equal to 1. Moving average of portfolio return is calculated based on data in past 13 weeks and average and standard deviation of the spread are calculated based on data in past 52 weeks. We call the period from getting the position to unwinding the position a strategy.

5.3. Results

We show the performance summary of the portfolio based on the indicators of the mean reversion in [Table 4](#) and the cumulative return in [Figure 2](#).

The performance statistics in [Table 4](#) are defined as follows. Return, risk, return/risk in the investment period are derived from the data in the period we have a position. It is better that return and return/risk are larger and risk is smaller. Winning percentage of the strategy represent the percentage of the number of strategies whose return is positive to the number of all strategies. It is better that winning percentage is larger. Maximum drawdown of the strategy represents the value of the lowest return of all strategies, indicating that it is better that its absolute value is smaller. The proportion of the investment period represents the percentage of the period that we have a position to the entire period. Average investment period is average weeks in a strategy.

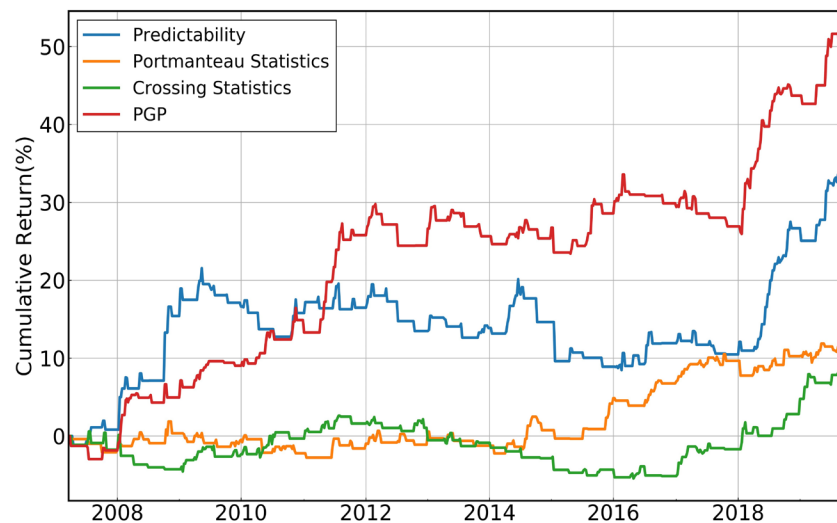
The return/risk and winning percentage of the strategy in [Table 4](#) show that PGP is better than the other single indicator and it suggests the effectiveness of combining multiple indicators of the mean reversion. According to [Figure 2](#), PGP can integrate all methods effectively although periods that each single method is valid are different.

6. Conclusion

In this study, we propose pairs trading strategy where we derive the mean reverting portfolio in the multi-asset market by using the time series model. We

Table 4. Performance statistics of mean-reverting portfolios.

	PGP	Predictability	Portmanteau	Crossing
Return in investment period (% Ann)	11.9	9.9	3.1	4.3
Risk in investment period (% Ann)	6.9	8.9	4.5	4.8
Return/risk in investment period	1.73	1.11	0.70	0.90
Winning percentage of the strategy (%)	59.8	57.3	50.6	54.8
Maximum drawdown of the strategy (%)	-2.5	-4.4	-1.7	-2.0
Proportion of the investment period (%)	28.0	24.5	30.0	27.7
Average investment period (weeks)	1.9	1.8	2.4	2.1

**Figure 2.** Comparison of cumulative return.

derive the portfolios based on predictability, which is measured in terms of the variance, portmanteau statistics, which is measured in terms of the correlation, crossing statistics, which represents how many times the time series crosses the average level, and the indicator integrated by a method called PGP. We get the empirical results that the return/risk and winning percentage of the strategy are best in the case of PGP, and it suggests that it is effective to combine multiple indicators of the mean reversion for deriving the mean reverting portfolio in the multi-asset market.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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