About the Nature of De Broglie Wave

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Abstract

In this article, it is shown that if a particle is at rest and has some harmonic oscillation, then a traveling particle with a constant speed corresponds to a De Broglie Wave. In 1924, De Broglie proposed that particles of matter exhibit wave like behavior, the expression for which became fundamental in the creation of wave mechanics (quantum mechanics). The main equation for this is the Schrödinger equation, which in the case of free motion of a particle becomes the expression for the De Broglie wave. This hypothesis is supported by many early experiments on the diffraction of electrons, and later the development of modern physics. However until now, the origin of this hypothesis remains unclear. We show that the expression for the De Broglie wavelength can be obtained from a more general well-known concept, that any particle at rest has a corresponding harmonic oscillation that transforms into a wave of a moving particle with the help of relativistic change of coordinates (the Lorentz transformations). In this way, a De Broglie wave can be explained as a spacetime wave.

Subject Areas

Fundamental Physics, Particle Physics, Quantum Mechanics, Theoretical Physics

Keywords

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1. Introduction

In modern physics, it is believed that any particle has a harmonic oscillation associated to it with frequency ω:

$$E = \hbar\omega, \tag{1}$$

(Plank's hypothesis) [1], where E is the energy of the particle and $\hbar$ is Plank's

constant. De Broglie's hypothesis says that the free movement of particles with a certain energy E and impulse p is defined through the wave function ψ corresponding to the De Broglie wave:

$$\psi(\boldsymbol{r},t) = A\exp\left[j(\boldsymbol{kr} - \omega t)\right], \tag{1'}$$

where

$$\omega = \frac{E}{\hbar}, \; \boldsymbol{k} = \frac{\boldsymbol{p}}{\hbar}, \; \hbar = \frac{h}{2\pi}. \tag{1''}$$

The De Broglie wavelength is given by:

$$\lambda = \frac{h}{p}.$$

The nature of this wave holds an unexplained role and De Broglie considered it a pilot wave.

It will be shown later that if we review this matter from a broader perspective, there is an oscillation for every particle at rest:

$$A\exp\left[j\omega_0 t\right].$$

Then from the Lorentz transformation, a De Broglie wave corresponds to any particle traveling in a constant speed v. This approach has methodological value since it demonstrates that instead of using the two hypotheses (1) and (1'), one can just use (1)—Plank's hypothesis.

2. Basic Relations

Let us define a four-dimensional wavevector with the following components:

$$k_{1,2,3} = k_{x,y,z}, \; k_4 = \frac{j\omega}{c} \quad [2]$$

Then, the wave's phase (1') is given by:

$$\boldsymbol{kr} - \omega t = k_i x_i \quad \text{where } x_1 = x; x_2 = y; x_3 = z; x_4 = jct.$$

To make it simpler, let us analyze a one-dimensional motion along the z axis with a constant speed v *i.e.* this is the speed of the reference frame K (where the particle stays at rest) relative to K'. Then, it is agreed that for the general formulas after a four-vector transformation, we have [2]:

$$k_4 = \frac{k_4' - j\dfrac{v}{c}k_1'}{\left(1 - \dfrac{v^2}{c^2}\right)^{\frac{1}{2}}}, \tag{2}$$

$$k_1 = \frac{k_1' + j\dfrac{v}{c}k_4'}{\left(1 - \dfrac{v^2}{c^2}\right)^{\frac{1}{2}}}, \tag{3}$$

$$k_4 = j\frac{\omega_0}{c}, \; k_4' = j\frac{\omega}{c}, \; k_1 = 0. \tag{4}$$

From (3), considering (4), we get:

$$k_1' = -\frac{jv}{c}k_4'. \tag{5}$$

From (2), considering (5), we get:

$$k_4' = \frac{k_4}{\left(1-\dfrac{v^2}{c^2}\right)^{\frac{1}{2}}}. \tag{6}$$

From (6), we furthermore get that:

$$\omega = \frac{\omega_0}{\left(1-\dfrac{v^2}{c^2}\right)^{\frac{1}{2}}}. \tag{6'}$$

The wavelength, λ, can be obtained from the next relation:

$$\lambda = \frac{2\pi}{k_1'}.$$

When by considering (5) and (4), it can be shown that:

$$\lambda = \frac{2\pi}{\dfrac{v}{c^2}\omega}.$$

Let us note that from (1"), we get:

$$\lambda = \frac{h}{\dfrac{v}{c^2}E}.$$

But the impulse p relates to the energy E through the following relation [2]:

$$p = \frac{v}{c^2}E.$$

So, we finally have:

$$\lambda = \frac{h}{p},$$

which matches the De Broglie wavelength [1].

3. Conclusions

It can be shown that if a particle at rest has some harmonic oscillation, then a traveling particle corresponds to a De Broglie wave that can be explained as a wave in spacetime as a consequence of relativistic transformations of Lorentzian coordinates. This shows that instead of using the two hypotheses (1) and (1'), one can just use (1)—Plank's hypothesis.

Sometimes, quantum mechanics is called "wave mechanics", but from our point of view, it would be better to call it "oscillation mechanics", as the wave possesses a secondary effect.

4. Postscript

By defining the phase velocity v_p and the group velocity v_g of the wave:

$$A\exp\left[j\left(kx-\omega t\right)\right],$$

$$v_p = \frac{\omega}{k},\ \ v_g = \frac{\partial\omega}{\partial k}.$$

The following is given for the De Broglie wave:

$$v_p = \frac{c^2}{v},\ \ v_g = v\ \text{and}\ v_g \cdot v_p = c^2.$$

The interesting thing is that this relation is satisfied by electromagnetic waves in open space and in a waveguide [3].

Accidentally, or not, this result may be found in a future study.

Conflicts of Interest

The author declares no conflicts of interest.

References

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