Open Access Library Journal
2023, Volume 10, e10753
ISSN Online: 2333-9721
ISSN Print: 2333-9705

Mathematical Model of Actuarial Valuation of Social Benefits in Latin America under IAS 19

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How to cite this paper: Cruz, E.D. and Query, J.T. (2023) Mathematical Model of Actuarial Valuation of Social Benefits in Latin America under IAS 19. *Open Access Library Journal*, **10**: e10753.
https://doi.org/10.4236/oalib.1110753

Received: September 16, 2023
Accepted: October 28, 2023
Published: October 31, 2023

Abstract

Development of a generic model for the valuation of Labor Liabilities under the international standards IAS 19 and FASB 87, for defined benefits, generally granted in Latin American countries, following [1]. The contribution of this work is to provide a detailed valuation tool for defined benefits that depend structurally on a salary and creditable service over time. Likewise, assess a defined benefit plan for termination of employment under 2 very similar current approaches, but with different liabilities [2].

Subject Areas

Sociology, Actuarial Sciences, Finance

Keywords

Defined Benefits, IAS 19, FASB 87, Benefit Method, Projected, Actuarial Models, Liabilities, Actuarial Costs

1. Description of Benefits

Generally, the benefits as in the case of Venezuela, in effect, are a function of two variables: on the one hand, the salary, and on the other hand, creditable service in the company [3]. Additionally, there is a parameter of goodness associated with the magnitude of the benefits that are going to be granted (see **Figure 1**).

In the most elementary case for the issue of retroactive benefits, the form of the benefit is as follows:

B_x : Benefit at age *x*.

$x - y$: Cumulative service between ages *x* e *y*.

S_x : Salary at age *x*.

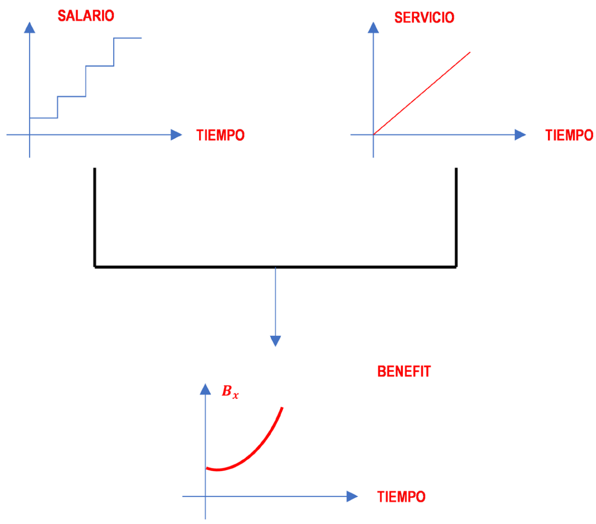

Figure 1. Main compensations variables.

$K\%$: Goodness parameter.

The benefit at age x is

$$B_x = (K\%)(S_x)(x - y) \tag{1}$$

Generally, $0 \le K \le 1$.

Generally, the benefit due to its structure presents **exponential growth,** derived from the change in salaries, following [3] and [4].

$$\hat{S}_x = S_{x-1}(1 + s) \tag{2}$$

Being s the rate of salary increases $0 \le s \le 1$.

In particular,

$$\hat{S}_x = S_o(1 + s)^{x-y} \tag{3}$$

S_o: Starting salary at age y.

Under the projected benefit method, which is indicated in IAS 19, the service accumulated to date is frozen and the salary is projected, as can be seen in the following graph (see Figure 2).

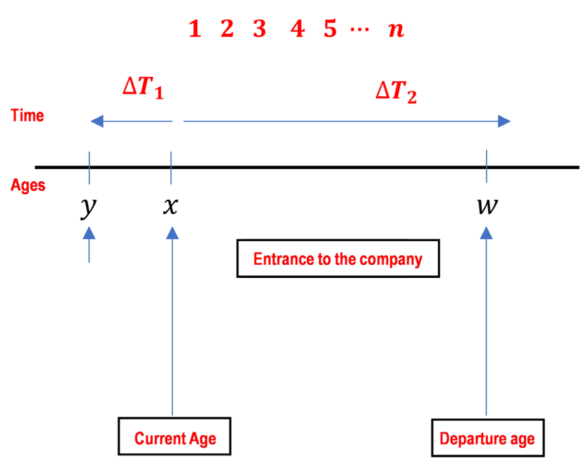

Figure 2. Time line of ages and services.

ΔT_1 : Current service $= x - y$.

ΔT_2 : Future service $= w - x$.

So he;

1) $B_x = (K)(S_x)(x-y)$ Cumulative benefit at age x (4)

2) $\hat{B}_r = (K)(sr)(x-y)$ Cumulative benefit at age renelintervalo: (x, w) (5)

An interesting variable is the difference in profits accumulated in the interval $(x, x+1)$ which we will call b_x

$$\hat{b}_x = \hat{B}_{x+1} - B_x \quad (6)$$

$\hat{B}_{x+1}$: Accumulated profit in $x+1$.

B_x : Estimated profit in x.

b: Annual rate of profit increase.

$\hat{b}_x$: Estimated change in profit between $(x, x+1)$.

(See **Figure 3**)

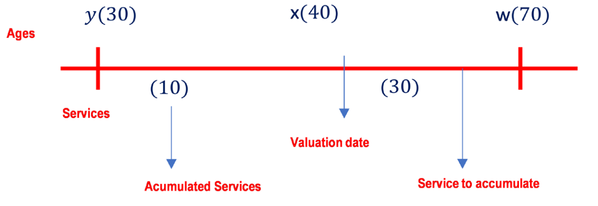

Figure 3. Time line of accredited service.

$$\hat{b}_x = \hat{B}_{x+1} - B_x \quad (7)$$

$$= B_x(1+b) - B_x \quad (8)$$

$$= B_x[(1-b)-1] \quad (9)$$

$$= B_x b \quad (10)$$

Being $0 \le b \le 1$, the rate of increase of profit.

In the case of retroactive defined benefits, the following follows

Being:

$$\hat{b}_x = \hat{B}_{x+1} - B_x \quad (11)$$

$$B_x : \text{Beneficio en } x \quad (12)$$

$$B_x = KS_x(x-y) \quad \text{and} \quad (13)$$

$$\hat{B}_{x+1} = KS_{x+1}(x+1-y) \quad (14)$$

Then, the change being $K = ctte$ would be given by:

$$\hat{b}_x = (k)S_{x+1}(x+1-y) - KS_x(x-y) \quad (15)$$

$$= (K)S_x(1+s)(x+1-y) - KS_x(x-y) \quad (16)$$

$$= KS_x[(1+s)(x+1-y)-(x-y)] \quad (17)$$

$(1+s)(x+1-y)$; **service adjusted by the wage rate.**

Expanding the bracket, we see:

$$\left[(1+s)(x+1-y)-(x-y)\right]=x(s)-y(s)+(1+s) \quad (18)$$

$$=s(x-y)+(1+s) \quad (19)$$

Then, $\hat{b}=KS_x\left(s(x-y)+(1+s)\right)$; **which represents the level of profit earned between**

$$(x,x+1) \quad (20)$$

2. Actuarial Liability Models

Under a job termination scheme, that is, at any time between the valuation date and an age w previously defined as the mandatory age of departure, the following can be displayed.

(See **Figure 4**)

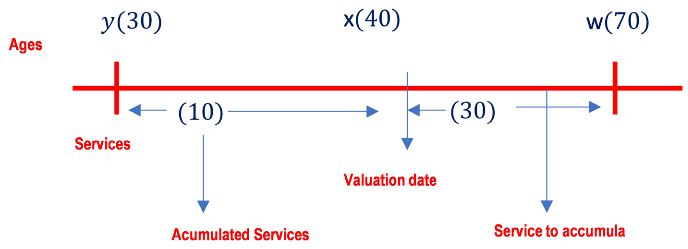

Figure 4. Time line years of services.

The worker in each future year between 40 years and 70 years can terminate his employment relationship. Suppose for pedagogical reasons only **3 causes of departure, death, turnover, leaving** the company, that is, there are 3 contingencies given by tables that statistically reflect death, turnover and final departure.

q_x^m : Mortality rate at age *x*.

q_x^r : Turnover rate at age *x*.

q_x^s : Withdrawal rate at *w*.

It is important to note that eventually there are at least (2) models, one a little more expensive than the other in terms of labor cost based on the following concepts (see **Figure 5**):

MODEL 1 (MULTI-DECREASE)

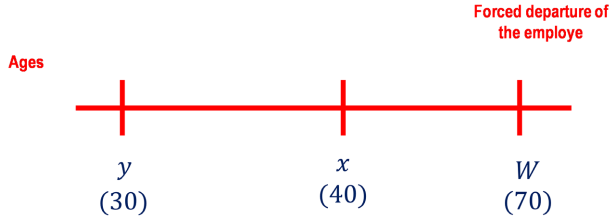

Figure 5. Ages time line.

In any case, once the worker reaches 40, he can leave at any time in the interval (40.70) according to the triplet of probabilities, that is, $\left(q_x^m, q_x^r, q_x^w\right)$ if he dies, the benefit is paid to him or if his relationship ends or it comes out at the end of a mandatory way at w 70 years old. In any case, where it comes out and for whatever contingent reason, the benefit is paid to you. $\{m, r, w\}$

In general, the structure of the model is as follows:

PBO_x: Pasivo actuarial en x

$$PBO_x = \sum_{k=0}^{w-1} B_k P_x^T V^{k+1} \tag{21}$$

$$\text{Being; } V = (1+i)^{-1} \text{ e } i = \text{the interest rate} \tag{22}$$

Updated expected value of benefit in $(x, w-1)$ con $s = w$

$$P_x^T = \left(kp_x^{(m,r,s)}\right)\left(q_{x+r}^{(m,r,s)}\right) \tag{23}$$

Probability of leaving for all causes in $(x, r, w-1)$.

The above represents the exit for any cause in the interval (40, 70). We follow the reference [4].

MODEL 2 (MULTIDECREMENT)[1]

It is very similar to the previous model, however, by design, once w(70)the maximum valuation age is reached, the accumulated amount of the benefit is paid, obviously because it did not come out before, **this emulates a mixed endowment in insurance,** where both death as the survival of the insured.

3. Comparative Analysis of the 2 Models

In summary, to understand the operation of the 2 models, the probabilistic structure of both is shown below (see Table 1).

Table 1. Insurance mathematical models.

MODEL I	$v^T \quad T \le n$ $0 \quad T > n$	$\bar{A}^1_{x:\overline{n}}$ [2]
MODEL II	$v^T \quad T \le n$ $v^n \quad T > n$	$\bar{A}_{x:\overline{n}}$ [3]

T: It is a random variable that describes the future life span, at age *x*.

v: discount factor.

n: time to reach w.

$$v = (1+i)^{-1}; \text{ being } i \text{ the interest rate} \tag{24}$$

A^1: Single premium or expected present value equivalent to ($PBOU$)[1].

[1]The second model, rewards the employee's permanence in the company even $(w-1)$ without a probability of leaving the company, recognizes a final benefit for job survival.

[2]Its structure is similar to a term insurance for $n$periods, in which case $n = w - x$.

[3]It corresponds to a mixed insurance where leaving is rewarded at any time, in the period $(x, w-1)$, and if it did not leave, survival in the company is paid in w.

A^2: Single Premium or Expected Present Value ($PBOU$)^2.

4. Test Results with Both Models

Below are the results of each of the aforementioned models with and without rewarding survival to the final age of departure *w*.

Annexes I-II show the tables used to describe mortality by age, rotation and probability of departure.

As expected, Model II is generally more expensive than Model I and in this sense, the contribution of our paper does not go beyond that; however, many times the benefit plans offered to workers are not clearly written in actuarial terms or, in other words, there **is no easy actuarial derivation from the text of the benefit plan,** which is generally a legal or intentional description that in the best of the cases it can have several interpretations; depending on which one is adopted, it will have a greater or lesser impact on the liability or actuarial obligation.

5. Description of the Population

The actuarial assumptions and hypotheses of the actuarial valuation are summarized below:

1) Interest rate: 7.49%.

2) Salary increase rate: 4.88%.

3) Population rates: **See Annex I** and **Annex II**.

4) Real rate of interest on salary.

(See Table 2)

Table 2. Demographic summary.

SUMMARY DEMOGRAPHIC				
TO THE 31/03/2023				
EVALUATED POPULATION				
LEGAL ENTITY	FEM	MAS	TOTAL	%
EMPRESA ABC	97	232	329	100.00%
TOTAL	**97**	**232**	**329**	**100.00%**
WEIGHTING OF ACTIVE PERSONNEL DATA BY PAYROLL				
LEGAL ENTITY	POPULATION	AVERAGE AGE (years)	AVERAGE SERVICE (years)	AVERAGE SALARY ($US)
EMPRESA ABC	329	34.40	6.60	3725.02
TOTAL	**329**	**34.40**	**6.60**	**3725.02**

From the previous tables (see Table 2), it is observed:

1) A total of 329 employees with 70.5% men and the rest women.

2) The average age in the order of 34.4 years.
3) An antiquity of 6.60 years of service.
4) An average reference salary in US dollars in the order of 3725.02.

6. Analysis of the Results

Below **are the results of the application of both models** to the same data from the ABC Company (see Tables 3-5).

MODEL I

Table 3. Actuarial obligations: MODEL I.

	ACTUARIAL OBLIGATIONS		
Population	**PBO**	**Service Cost**	**Interest Cost**
329	535784.16	102165.75	40130.23
329	**535784.16**	**102165.75**	**40130.23**

MODEL II

Table 4. Actuarial obligations: MODEL II.

	ACTUARIAL OBLIGATIONS		
Population	**PBO**	**Service Cost**	**Interest Cost**
329	1.094839.36	169466.76	82003.47
329	**1.094839.36**	**169466.76**	**82003.47**

As can be seen in Table 3 and Table 4, the liability difference is substantial, depending on the approach taken. Obviously, the demographics of the company, in terms of the number of employees by age, gender and magnitude of the benefit determine its magnitude.

Table 5. Actuarial differentials.

MMBs (Million of Bs)	MODEL I	MODEL II	Δ%
PBO	**535**	**1094**	**48.90%**

The liability of Model I is less than half that of Model II.

In the end, what is important for this company is that the liability almost doubles, therefore, it is very important to clarify the issue of survival in the company.

In other words, whether or not to exhaustively recognize the survival of the employee in the company in the end is decisive in the magnitude of the cost. The foregoing is reduced, as explained above, by adding the term (*Dw*, *Dx*)[4] as a mixed insurance in actuarial commutative notation.

[4]Actuarial commutative functions help simplify calculations in general $D_x = (v^x)(l_x)$.

7. Conclusions

Indeed, in almost all employment termination benefit plans, it seems obvious that the corresponding benefit will be paid at any time and for any cause of departure.

However, what we want to highlight is that if it is explicitly clear that the benefit is going to be paid for any of the two causes (m, r) in theory to close the probabilistic model or a third cause is added, which is the separation of the employee upon reaching the maximum exit age (w) or, a final exit benefit is recognized in the company. The point is simply that both approaches generate different costs that could be substantially different, depending on the distribution of the company's demographics in terms of age and years of service and course of the creditable benefit accumulation system.

The use of a probability or exit rate of the company in (w) or the explicit recognition of the terminal benefit of survival in the company, are essentially 2 different types of benefits with different actuarial liabilities.

Because the legal text of some benefits may not be very clear, the difficulty arises in translating it into the actuarial formulation to evaluate the plan. Hence, the importance of clarifying very well with the company what the scope is in terms of the condition of the benefit.

As was clearly seen in the analysis of the results, the two possible versions are exhaustive and the differential cost can be important, to clearly distinguish if there is a survival benefit in the end, in the case that the 3 possible causes: {death (m), rotation (r), exit (s)} are only considered a payment with employer responsibility for (m and r) and not for (s), it is determining in the corresponding actuarial liability.

Conflicts of Interest

The authors declare no conflicts of interest.

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Annexes

Annex I

MODEL I

x	$q_x(m)$	$q_x(m)$	$q_x(crp)$	$q_x(crp)$	$q_x(srp)$	$q_x(srp)$
20	0.00103	0.00103	0.10110	0.10110	0.29537	0.29537
25	0.00125	0.00125	0.05700	0.05700	0.21775	0.21775
30	0.00142	0.00142	0.03214	0.03214	0.16053	0.16053
35	0.00159	0.00002	0.00103	0.01812	0.11834	0.00674
40	0.00205	0.00003	0.00058	0.01022	0.08724	0.00497
45	0.00271	0.00003	0.00032	0.00576	0.06431	0.00366
50	0.00393	0.00005	0.00018	0.00324	0.04741	0.00270
55	0.00573	0.00007	0.00010	0.00183	0.03495	0.00199
60	0.00908	0.00011	0.00059	0.00103	0.02577	0.00146
65	0.01380	0.00017	0.00033	0.00058	0.01899	0.00108
70	0.02057	0.00026	0.00016	0.17452	0.01419	0.00080

x: Age.

$q_x(m)$: Probability of death.

$q_x(crp)$: Probability of departure with employer responsibility (rotation).

$q_x(srp)$: Probability of departure withoutemployer responsibility (rotation).

Annex II

MODEL II

x	$q_x(m)$	$q_x(m)$	$q_x(crp)$	$q_x(crp)$	$q_x(srp)$	$q_x(srp)$
20	0.00103	0.00103	0.10110	0.10110	0.29537	0.29537
25	0.001252	0.001252	0.05700	0.05700	0.21775	0.21775
30	0.001422	0.001422	0.03214	0.03214	0.16053	0.16053
35	0.001594	0.001594	0.01812	0.01812	0.11834	0.11834
40	0.002051	0.002051	0.01022	0.01022	0.08724	0.08724
45	0.002719	0.002719	0.00576	0.00576	0.06431	0.06431
50	0.003938	0.003938	0.00324	0.00324	0.04741	0.04741
55	0.005737	0.005737	0.00183	0.00183	0.03495	0.03495
60	0.009085	0.009085	0.00103	0.00103	0.02577	0.02577
65	0.013807	0.013807	0.00058	0.00058	0.01899	0.01899
70	0	-	0.17452	0.17452	0.82547	0.82547

x: Age.

$q_x(m)$: Probability of death.

$q_x(crp)$: Probability of departure with employer responsibility (rotation).

$q_x(srp)$: Probability of departure without employer responsibility (rotation).

In this case, at the age of 70, the sum of all exit probabilities gives 1 with an explicit benefit of recognition in the event that the employee has not died or left with employer responsibility at (20, 69).