Scientific Research Publishing

Open Journal of Applied Sciences, 2024, 14, 3202-3215
https://www.scirp.org/journal/ojapps
ISSN Online: 2165-3925
ISSN Print: 2165-3917

Evaluating Fund Performance Based on *Lp* Quantile Nonlinear Regression Model

Ying Sun, Fuming Lin*

College of Mathematical and Statistics, Sichuan University of Science and Engineering, Zigong, China
Email: sunying722@163.com, *215966774@qq.com

How to cite this paper: Sun, Y. and Lin, F.M. (2024) Evaluating Fund Performance Based on *Lp* Quantile Nonlinear Regression Model. *Open Journal of Applied Sciences*, **14**, 3202-3215.
https://doi.org/10.4236/ojapps.2024.1411211

Received: October 19, 2024
Accepted: November 22, 2024
Published: November 25, 2024

Copyright © 2024 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/
Open Access

Abstract

There is a substantial body of empirical research that has found the fund return distributions to exhibit pronounced peakiness, heavy tails, and skewness, deviating from a normal distribution. Addressing the limitations of the traditional Sharpe ratio, which assumes a normal distribution of returns and uses standard deviation to measure investment risk, this paper primarily employs the Value at Risk (VaR) based on *Lp* quantile to adjust excess returns of funds. This method offers superior robustness, is capable of capturing asymmetry and heavy-tailed characteristics, and is more flexible, providing a better description of the tail risk in fund returns. Empirical studies have shown that using the Sharpe ratio corrected with the *Lp* quantile is feasible for evaluating and ranking the performance of open-end funds.

Keywords

Sharpe Ratio, Expectile, *Lp* Quantile, VaR

1. Introduction

As global financial markets rapidly develop, funds play a crucial role in wealth management for investors. There are two ways in which funds operate: open-end funds and closed-end funds. Open-end funds are the mainstream in the international fund market, offering significant advantages over closed-end funds in terms of incentive and constraint mechanisms, liquidity, transparency, and investment convenience. This is also why open-end funds occupy a prominent position in investors' asset allocation. In this context, effectively evaluating the performance of funds is of great significance. From the perspective of investors, accurate fund performance evaluation helps them make more informed investment decisions. From the standpoint of fund managers, scientific performance evaluation methods can contribute to improving management effectiveness and enhancing market competitiveness.

Features of fund return distributions include skewness, heavy tails, and spikes, which are essential for investment analysis and risk assessment. Skewness describes the symmetry of the return distribution, with positive skewness usually implying potentially high returns, while negative skewness implies an increased risk of loss. Heavy tails indicate that extreme events are more likely to occur than conventionally expected, which puts investors at greater risk in the event of a market crash or economic crisis. In addition, the spike feature measures the level of return concentration, and a high spike may lead investors to misjudge risk. Currently, the research methods for evaluating fund performance mainly focus on the traditional Sharpe ratio [1], Treynor ratio [2], and Jensen's alpha [3]. These three indicators are classic methods for measuring fund performance. This article primarily uses the Sharpe ratio to study fund performance evaluation. When applying the traditional Sharpe index to measure fund performance, it assumes that the sequence of fund returns follows a normal distribution. However, in reality, most financial time series data often do not conform to the normal distribution assumption, showing more characteristics of sharp peaks and thick tails. The standard deviation in the traditional Sharpe ratio only represents the volatility of returns, failing to adequately reflect the more critical tail risk in financial markets. Therefore, it cannot fully measure the true risk of the fund.

In response to the limitations of using standard deviation to measure the traditional Sharpe ratio, scholars have employed methods such as Value at Risk (VaR) to adjust the traditional Sharpe ratio for risk. Peng and Wu [4] studied the use of VaR and Conditional VaR to adjust the traditional Sharpe indicator for a more reasonable risk portrayal. To fully consider the skewness, peak, and heavy-tailed nature of return distributions, Shi *et al.* [5] and Yu *et al.* [6] respectively considered using the Asymmetric Laplace distribution and Skewed-t distribution to fit the return distribution, and then used the VaR values calculated based on these distributions and models to correct the Sharpe index. Additionally, Su and Zhou [7] used the Asymmetric Power Distribution (APD) to fit the distribution of fund returns, proving that the modified Sharpe ratio based on APD standard deviation and VaR is very feasible for application in fund ranking and evaluation. Their research indicates that the Sharpe index adjusted by VaR not only provides a more scientific evaluation of a fund's overall returns, but also more accurately reflects the volatility characteristics of the fund's returns, thereby better revealing the fund manager's ability to control risk.

VaR is a widely used standardized tool for measuring financial risk, typically defined as the maximum potential loss at a given confidence level over a specific time period. It aids decision-makers in understanding the level of potential risk. Meanwhile, as a key risk management tool, VaR is widely applicable to various portfolios and market environments. In a diversified portfolio, VaR can conduct risk assessment for different asset classes such as stocks, bonds and derivatives, helping investors understand potential liquidity and credit risks. Under different market conditions, whether bull market, bear market or volatile market, VaR can effectively quantify potential losses and guide investors to adjust investment strategies to balance returns and risks. In addition, VaR also provides data support for risk management, helps develop hedging strategies, optimize capital allocation, and meet regulatory requirements, which enables investors and financial institutions to better identify and manage risks in a complex and volatile market environment, so as to make more informed decisions. However, traditional methods of measuring VaR, such as the variance-covariance method, historical simulation, and Monte Carlo simulation, each have their limitations. In response to these limitations, both domestically and internationally, numerous studies have adopted the expectile measure proposed by Newey and Powell [8] to estimate VaR. For instance, international researchers such as Kuan *et al.* [9], Taylor [10], and Kim and Lee [11] have, along with domestic scholars like Yao [12], Xie [13], and Hu *et al.* [14], developed different models based on expectile to explore the measurement of VaR for stocks or stock indices. These studies have confirmed that the expectile measure is more sensitive to risks in the tails. Particularly, in the research by Su and Zhou [15], it was shown that using the expectile measure to correct the traditional Sharpe ratio in the CARE model provides a more accurate assessment of extreme risk, better evaluating the performance of funds under downside risk.

Additionally, Chen [16] proposed the Lp quantile, a method that has gained popularity in recent years for estimating. The Lp quantile is obtained by minimizing the p-th power of the loss function. Especially when $1 < p < 2$, the Lp quantile is favored due to its ability to balance the robustness of quantile and the efficiency of the expectile. Jiang *et al.* [17] provided the corresponding asymptotic estimation theory and proved its effectiveness. There have been numerous studies on Lp quantile regression in recent years, such as those by Usseglio-Carleve [18], Girard *et al.* [19], and Tang and Chen [20], among others (for more research on Lp quantile, see [21]-[24]). These studies have explored various estimators combined with Lp quantile regression methods, empirically demonstrating that estimators using Lp quantile regression are effective, highlighting the superiority of the Lp quantile regression method. Inspired by this, this article aims to use the Lp quantile regression estimate of VaR to replace the standard deviation in the denominator of the traditional Sharpe index, thereby correcting the traditional Sharpe index as a measure of fund performance.

The remainder of the article is organized as follows. In Section 2, a brief introduction is provided on the Lp quantile regression estimation method used in this article, as well as its application in conjunction with nonlinear models for measuring financial risk, VaR. Section 3 primarily conducts an empirical study on the performance of 22 sample funds based on the Sharpe ratio. The Lp quantile regression estimation method introduced in Section 2 is utilized to calculate the VaR values. Subsequently, the Sharpe ratio is adjusted based on these VaR values. The performance of the funds is then evaluated using both the traditional Sharpe ratio and the Sharpe ratio adjusted with VaR. Finally, Section 4 concludes and looks ahead to future directions of the article.

2. *Lp* Quantile Nonlinear Regression Model

Lp quantile regression is a relatively popular estimation method in recent years. Following the introduction of quantile regression and expectile regression by Koenker and Bassett [25] and Newey and Powell [8], Chen [16] proposed the *Lp* quantile regression method with *p*-values greater than 1. Jiang *et al.* [17] further studied *Lp* quantile regression, proposing a *k*-th power expectile regression method that lies between quantile and expectile regression, demonstrating that *Lp* quantile regression with *p*-values between 1 and 2 can better balance robustness and efficiency. Lin *et al.* [26] provided the asymptotic theoretical properties of the estimators for *Lp* quantile regression with *p* greater than 1 and less than or equal to 2, and their simulation results showed that in certain data scenarios, this method has higher asymptotic efficiency than ordinary quantile regression and expectile regression, providing new avenues for the study of financial risk measurement. In the latest research, Sun *et al.* [27] focused on the application of *Lp* quantile regression in the accurate estimation of VaR, proposed the conditional *Lp* quantile nonlinear autoregressive regression model (CAR-LP-quantile model), and showed that this method has strong effectiveness and advantages through simulation and empirical study. It can be seen that *Lp* quantile, as a risk measurement tool in financial risk management, provides greater flexibility and adaptability for risk assessment.

Let Y be a random variable, $\tau \in (0,1)$, $p \in [1,2]$, then the *Lp* quantile $L_{p,\tau}(Y)$ may be defined as the solution that minimizes the asymmetric *p*-th power loss function:

$$L_{p,\tau}(Y) = \arg\min_{x \in R} E\left[|\tau - I(Y < x)| \cdot |Y - x|^p \right].$$

The τ denotes the asymmetry level in the loss function. For $p = 1$ and $p = 2$, quantile and expectile can be incorporated into *Lp* quantiles. Additionally, Kim and Lee [11] introduced a non-linear expectile regression method that can be applied to the estimation of VaR and Expected Shortfall (ES). This paper combines the *Lp* quantile regression method with non-linear models for fund performance evaluation. Referencing model (2.3) from the research by Kim and Lee [11], the following *Lp* quantile non-linear regression model is considered:

$$Y_t = f\left(Y_{t-1}, Y_{t-2}, \cdots; \beta^0\right) + \varepsilon_{t\tau} \equiv f_t\left(\beta^0\right) + \varepsilon_{t\tau}, 0 < \tau < 1, t \in \mathbb{Z},$$

where $f_t(\cdot)$ is a stochastic process parameterized by $\beta \in R^p$, $\beta^o = \beta^o(\tau, p)$ is a true parameter, $\varepsilon_{t\tau}$ are error terms satisfying $\mu_{\varepsilon_{t\tau}|\Omega_t}(\tau) = 0$, and Ω_t denotes the information set available up to time t, say, $\sigma(Y_{t-1}, Y_{t-2}, \cdots)$. To estimate the parameter β^o, one needs to approximate $f_t(\beta)$ using the observation $\tilde{f}_t(\beta)$ from Y_t. Therefore, consider:

$$\tilde{f}_t(\beta) = f\left(Y_{t-1}, Y_{t-2}, \cdots, Y_1, 0, \cdots; \beta\right),$$

and estimate β^o by the following equation:

$$\hat{\beta}_n(\tau,p)=\arg\min_{\beta} n^{-1}\sum_{p=1.1}^{2}\sum_{t=1}^{n}\left|\tau-I\left(Y_t<\tilde{f}_t(\beta)\right)\right|\left|Y_t-\tilde{f}_t(\beta)\right|^p, \tag{1}$$

where $\tau\in(0,1)$, p is taken from 1.1 to 2 at intevals of 0.1.

In empirical studies of fund performance, the observation Y_t is the daily return data of the fund. We consider using the following linear GARCH(1, 1) model to evaluate the performance of the Lp quantile nonlinear regression estimate:

$$Y_t=\sqrt{h_t}\eta_t,$$
$$h_t=1+\alpha Y_{t-1}^2+\gamma h_{t-1},$$

where η_t is the error distribution, α and γ are parameters, therefore the τ-th condition Lp quantile regression model is $Y_t=f_t(\tau)+\varepsilon_{t\tau}=\sqrt{h_t}\xi_\tau+\varepsilon_{t\tau}$, ξ_τ is the Lp quantile of error distribution η_t. The parameters ξ_τ, α and γ of the model are estimated by the optimization function $\hat{\beta}_n(\tau,p)$ using the parameters estimated by the quasi-likelihood as initial values in *R*.

3. An Empirical Study on Fund Performance in China

3.1. Sample Selection and Data Sources

This article selects 22 open-end funds with complete data from January 1, 2017 to May 15, 2024 as the research object, each fund has 1785 samples. In empirical research, the entire sample is divided into two parts: one part is the in-sample data, which is used as an estimation sample, using the first 1000 data to estimate the model parameters; the other part is the out-of-sample data, which is used as a prediction sample or test sample, and the last 785 data are used for the prediction of out-of-sample VaR. All data in this article are derived from Eastmoney Choice Data.

3.2. Empirical Results and Analysis

3.2.1. Description of Basic Statistics

The paper initially presents the names and corresponding codes (see **Table 1**) of the 22 open-end funds, followed by providing basic statistics on their returns. Based on the analysis in **Table 2**, it is evident that, with regards to mean returns, all funds except Castrol Research Selected Mix and China Merchants CSI 300 Real Estate Equal-weighted Index A fund exhibit positive mean returns. Furthermore, the standard deviation of returns for all funds is greater than zero. In terms of skewness, Wanxia Xinli Flexible Allocation Hybrid and China Merchants CSI 300 real estate eq ual weight index A fund demonstrate positive skewness indicating right-skewness, whereas other funds display negative skewness implying left-skewness. Additionally, all fund return series exhibit high peaks and kurtosis values exceeding 3, suggesting that most sample funds have less dispersed return distributions with a more convex shape compared to a normal distribution. These findings indicate biased distributions with sharp peaks and fat tails for all fund returns.

Finally, the Jarque-Bera statistic is employed to test the normality assumption of the sample fund returns. The statistical values significantly surpassing the critical value at a significance level of 5% strongly reject the null hypothesis that daily return time series of sample funds follow a normal distribution.

Table 1. Names and codes of 22 funds.

Fund name	Fund code
Jingshun Small and Medium Enterprise Board and Growth Enterprise Market Selected Stocks	000586
Investment Industry Select Stock Fund	000746
Frontier Open Source Dividend Rate Top 100 Stocks	000916
Castrol New Consumption Stock Fund	001044
Golden Eagle Technology Innovation Stock A	001167
Nuoan Low Carbon Economy Stock A	001208
China New Silk Source Hybrid A	002871
Southern National Policy Power	001692
Dacheng Industrial Upgrade Stock	160919
HSBC Jinxin Small and Medium Cap Stocks	540007
Yi Fangda Consumer Industry Stocks	110022
Fuguo Tianrui Strong Mix	100022
Guangfa Steady Growth Hybrid A	270002
China Medicine ETF	510660
Castrol Research Selected Mix	070013
Castrol CSI 500ETF-A	000008
Rongtong Industry Boom Mixed A	161606
Wanxia Xinli Flexible Allocation Hybrid	519191
Xingquan Trend Investment Hybrid (LOF)	163402
Dacheng CSI Dividend Index A	090010
Fu Guo Zheng Coal Index A	161032
China Merchants CSI 300 Real Estate Equal-weighted Index A	161721

Table 2. Descriptive statistics of open-end funds.

Fund code	Mean	Standard deviation	Skewness	Kurtosis	JB statistics
000586	0.000437	0.0151	−0.243	4.533	192.47
000746	0.000734	0.0140	−0.101	4.657	207.40
000916	0.000455	0.0098	−0.476	6.740	1109.52
001044	0.000681	0.0113	−0.041	4.303	127.02
001167	0.000727	0.0163	−0.160	4.511	177.20
001208	0.000677	0.0102	−0.130	5.127	341.88
002871	0.000330	0.0113	−0.116	10.148	3810.98
001692	0.000550	0.0135	−0.083	4.688	214.28
160919	0.000674	0.0132	−0.079	4.477	164.56

Continued

540007	0.000684	0.0155	−0.172	4.363	126.87
110022	0.000686	0.0149	−0.163	4.769	241.21
100022	0.000494	0.0140	−0.186	4.630	207.82
270002	0.000300	0.0064	−0.180	4.548	188.04
510660	0.000319	0.0150	−0.035	4.541	177.46
070013	−0.000049	0.0126	−0.221	5.004	313.71
000008	0.000061	0.0124	−0.504	7.578	1637.48
161606	0.000388	0.0156	−0.091	4.768	235.27
519191	0.000615	0.0160	0.046	5.297	394.05
163402	0.000259	0.0110	−0.076	4.851	257.01
090010	0.000415	0.0099	−0.674	8.297	2224.66
161032	0.000592	0.0177	−0.125	4.845	257.87
161721	−0.000161	0.0173	0.389	5.771	615.62

3.2.2. Computation of the Sharpe Ratio

Fund performance evaluation involves using historical data of fund operations to comprehensively assess the actual investment outcomes of the fund. The Sharpe ratio, in particular, is based on the Capital Market Line (CML) and assumes that returns follow a normal distribution. It adjusts total risk using standard deviation. The Sharpe ratio is a standardized metric used to evaluate fund performance, also known as the Sharpe Index. The traditional formula for calculating the Sharpe ratio is:

$$SharpeRatio = \frac{E\left(R_P\right) - R_f}{\sigma_P}$$

where $E\left(R_P\right)$ is the expected annualized return rate of the investment portfolio, R_f is the annualized risk-free rate, and σ_P is the standard deviation of the annualized return rate of the investment portfolio. According to the common practice of using the yield of national short-term government bonds as the risk-free rate, this article considers the one-year maturity yield of the China Bond Government Bond as of May 15, 2024, as the risk-free rate, which is 1.6134. To ensure that the data results of average returns, standard deviations, and risk-free rates for each fund are consistent over time, the annualized Sharpe ratio is calculated for each fund, excluding weekends and holidays, with the assumption that there are 252 trading days in a year. The following formula is used to convert the daily average returns and daily return standard deviations of each fund into annual average returns and annual return standard deviations. The basic steps for calculating the Sharpe ratio are as follows:

Firstly, obtain the latest net asset value per unit of the fund within the research period, and calculate the daily return rate:

$$Rt = \frac{P_t - P_{t-1}}{P_{t-1}}.$$

According to the traditional Sharpe ratio calculation formula, the resulting daily yield needs to be annualized and denoted as R_{annual}, then the annualized yield calculation formula is:

$$R_{annual} = \left(1+\bar{R}\right)^{252} - 1,$$

where $\bar{R} = \frac{1}{N}\sum_{i=1}^{N} Rt_i$, $\bar{R}$ denotes the mean of daily returns.

Secondly, calculate the corresponding annualized standard deviation, denoted as σ_{annual}, with the formula:

$$\sigma_{annual} = \sqrt{252} * \sigma,$$

where $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(Rt_i - \bar{R}\right)^2}$, σ represents the standard deviation of daily returns.

Finally, the traditional Sharpe ratio formula is obtained as:

$$Sharpe = \frac{R_{annual} - R_f}{\sigma_{annual}}. \tag{2}$$

Due to the traditional Sharpe ratio's assumption of normally distributed returns, which overlooks downside risk, it fails to fully measure the true risk of a fund. Therefore, we consider using VaR to adjust the traditional Sharpe ratio. For the calculation of the Sharpe ratio adjusted with VaR, using VaR values estimated through *Lp* quantile regression instead of the standard deviation in the traditional Sharpe ratio formula, we can derive the formula for the Sharpe ratio adjusted with VaR as follows:

$$Sharpe_{VaR} = \frac{R_{annual} - R_f}{VaR}. \tag{3}$$

There are several main implications for the Sharpe ratio modified by VaR: Firstly, more attention should be paid to tail wind, because the standard deviation mainly reflects the overall volatility, while VaR focuses more on extreme losses. By introducing VaR, investors can better understand the potential significant loss risk; Secondly, it is suitable for non-normal distribution. The traditional Sharpe ratio assumes that the return distribution is normal, while VaR can be better applied to non-normal distribution data, especially in the face of peak and fat-tailed distribution. In addition, the revised Sharpe ratio can be used as a tool for investment decision-making. It helps investors to consider not only profitability but also potential extreme risks when evaluating investment performance, which is especially important for risk-averse investors; Finally, the revised Sharpe ratio can be applied in practice. In actual risk management, the use of VaR as a risk measure can enable financial institutions and investors to carry out more effective capital allocation and risk hedging in the face of market crisis or potential loss.

In addition to this, the VaR values used to replace the standard deviation in the adjusted Sharpe ratio reflect different risk preferences and data characteristics when different regression methods are employed to estimate VaR. Compared to the expectile, the use of Lp quantile regression methods for estimating VaR in this paper is more advantageous. Firstly, the square loss function used in the expectile is highly sensitive to outliers, leading to estimation bias when extreme values or data noise are present. In contrast, the p loss function used in Lp quantile regression, where p is between 1 and 2, is more robust to outliers, making the estimated VaR values more reliable and thus improving the accuracy of the adjusted Sharpe ratio. Secondly, financial market data often exhibit asymmetric distribution characteristics, especially during extreme market fluctuations. Lp quantile regression allows for different degrees of punishment on the tails of the distribution, better capturing the asymmetric characteristics of the data. Moreover, financial data typically show a heavy-tailed phenomenon, with a higher probability of extreme returns. Lp quantile regression is more sensitive to data with heavier tails, providing a more conservative VaR estimate, which enhances the reliability of its application in risk management. Lastly, different p-values can be chosen based on varying market conditions and data characteristics to obtain the most optimal risk assessment model. While the square loss function is computationally simpler, its applicability is limited, especially in non-normal and heavy-tailed distributions, making Lp quantile regression more widely applicable due to its flexibility in practical applications.

Overall, using VaR as an alternative to standard deviation in Sharpe ratio calculations, VaR not only focuses on tail risk, but also applies to non-normal distributions, adding a stronger risk management perspective to the analysis of risk-adjusted returns, allowing investors to consider risk factors more fully when evaluating portfolio performance. Lp quantile regression is more robust, able to capture asymmetry and heavy-tailed distribution characteristics, and more flexible than expectile regression in estimating fund VaR. Therefore, using VaR based on Lp quantile estimation to modify Sharpe ratio can provide more accurate and reliable results, thus improving the quality of risk management and decision-making.

3.2.3. VaR Calculation

Before parameter estimation, we use Ljung-Box test method to test the correlation of the return series of 22 selected funds. The lag order is 10, and the p-value is obviously larger than the commonly used significance level of 0.05 or 0.01. This means that we cannot reject the null hypothesis that the autocorrelation of time series data is zero. Since there is no significant autocorrelation, consider using GARCH models to analyze this data set to demonstrate the rationality of volatility models.

Take the Jingshun Small and Medium Enterprise Board and Growth Enterprise Market Selected Stocks board as an example, use linear GARCH(1, 1) model to calculate VaR value, and calculate VaR value of other funds similarly. First, for a given quantile θ of 5%, when p takes different values from 1.1 to 2, we use the sample data of GEM selected stocks in Inveshun to obtain the corresponding τ

estimates in the linear GARCH(1, 1) model. As shown in **Table 3**, under the given 5% quantile, the estimated values of 22 funds selected in this paper, such as Jingshun Small and Medium Enterprise Board and Growth Enterprise Market Selected Stocks, are given at different values τ of *p*. Then, the estimated values of parameters ξ_τ, α, γ in linear GARCH(1, 1) model are −0.5383803, 0.5809303 and 0.8927805 by *Lp* quantile regression method of Equation (1). Finally, since the τ-th condition *Lp* quantile of Y_t is given by:

$$f_t(\tau) = \sqrt{h_t}\,\xi_\tau,$$

where ξ_τ is the τ-th *Lp* quantile of the error distribution η_t thus for a given 5% quantile the formula for calculating the VaR value corresponding to the Jingshun Small and Medium Enterprise Board and Growth Enterprise Market Selected Stocks data is:

$$VaR_t(0.05) = \sqrt{h_t}\,\xi_{\tau_{0.05}}.$$

After obtaining the corresponding VaR value estimated by the *Lp* quantile regression of the selected equity funds of the Jingshun Small and Medium Enterprise Board and Growth Enterprise Market Selected Stocks, bring it into Equation (3) and calculate that the Sharpe ratio value based on the VaR correction is −1.6314, and the Sharpe ratio value of the remaining funds based on the VaR correction can be obtained similarly.

3.2.4. Fund Ranking Based on the Sharpe Ratio

Calculated the traditional Sharpe ratio results for 22 sample funds based on Equation (2). Then, using the example of the Jingshun Small and Medium Enterprise Board and Growth Enterprise Market Selected Stocks, obtained the Sharpe ratio values for the remaining 21 sample funds based on VaR correction at the optimal *p*, and recorded the VaR-corrected Sharpe ratio results for the expectile regression estimation at $p = 2$. As shown in **Table 3**, we compared the rankings of the three types of Sharpe ratios: traditional Sharpe ratio, expectile, and *Lp* expectile regression estimation based on VaR correction for the performance of open-end funds.

From the results in **Table 4**, it is evident that the ranking of the 22 funds based on the Sharpe ratio obtained from three methods shows significant differences. The number of funds ranked higher based on the Sharpe ratio adjusted for expectile and *Lp* quantile is 9 and 10, respectively, compared to the traditional Sharpe ratio calculation. This indicates that these funds have a better ability to control downside risk, and it also reflects the deficiency in the traditional Sharpe ratio where the standard deviation fails to capture tail risk.

Simultaneously, under the expectile and *Lp* quantile regression adjustments to the Sharpe ratio, the ranking of the adjusted values is largely similar. Six funds experienced changes in their rankings, with three funds, China New Silk Source Hybrid A, China Medicine ETF, and Dacheng CSI Dividend Index A, showing improved performance rankings after the *Lp* quantile method adjustment. This is due to the *Lp* quantile method's more robust and effective approach to measuring VaR.

Therefore, incorporating the Lp quantile method to measure VaR values for adjusting the Sharpe ratio holds significant value for investors and is both feasible and operable for fund managers.

Table 3. For a given 5% quantile, the linear GARCH(1, 1) model estimates the value of τ when p is taken with different values.

Fund code	C1[a]	C2	C3	C4	C5	C6	C7	C8	C9	C10
000586	0.043	0.038	0.034	0.031	0.028	0.026	0.024	0.023	0.021	0.02
000746	0.042	0.036	0.031	0.027	0.025	0.022	0.019	0.018	0.017	0.016
000916	0.045	0.042	0.039	0.036	0.034	0.033	0.026	0.030	0.029	0.028
001044	0.043	0.038	0.033	0.029	0.027	0.024	0.022	0.020	0.019	0.017
001167	0.044	0.038	0.034	0.031	0.029	0.026	0.025	0.023	0.022	0.021
001208	0.045	0.042	0.039	0.036	0.034	0.032	0.031	0.029	0.028	0.027
002871	0.045	0.042	0.039	0.036	0.034	0.033	0.032	0.031	0.031	0.030
001692	0.044	0.039	0.036	0.032	0.030	0.027	0.026	0.023	0.022	0.021
160919	0.045	0.042	0.037	0.035	0.033	0.031	0.028	0.027	0.025	0.024
540007	0.043	0.039	0.035	0.031	0.029	0.027	0.025	0.024	0.023	0.022
110022	0.045	0.041	0.037	0.035	0.032	0.030	0.028	0.026	0.025	0.024
100022	0.044	0.039	0.0353	0.032	0.029	0.027	0.025	0.023	0.021	0.020
270002	0.0439	0.0388	0.0348	0.031	0.029	0.026	0.024	0.023	0.021	0.020
510660	0.0442	0.0393	0.0352	0.032	0.029	0.026	0.024	0.022	0.020	0.019
070013	0.0437	0.0392	0.0357	0.033	0.030	0.029	0.027	0.025	0.024	0.023
000008	0.0435	0.0385	0.0346	0.032	0.029	0.027	0.026	0.024	0.023	0.023
161606	0.0446	0.0402	0.0367	0.034	0.031	0.029	0.027	0.025	0.024	0.023
519191	0.0444	0.0399	0.0362	0.033	0.030	0.028	0.026	0.025	0.023	0.022
163402	0.0444	0.0399	0.0361	0.033	0.030	0.028	0.026	0.024	0.023	0.022
090010	0.0446	0.0403	0.0369	0.034	0.032	0.030	0.028	0.027	0.026	0.025
161032	0.0448	0.0405	0.0369	0.034	0.031	0.029	0.027	0.025	0.024	0.022
161721	0.0437	0.0386	0.0344	0.031	0.028	0.0258	0.024	0.022	0.021	0.019

[a]The values in columns C1 to C10 represent estimated values of τ when p takes the values 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9 and 2 for varying parameters in the linear GARCH model.

Table 4. Ranking of open-end funds based on 3 Sharpe ratios.

Fund code	Sharpe[a]	Rank	EVaR-Sharpe[a]	Rank	LVaR-Sharpe[a]	Rank
000586	0.4067	15	−1.6213	14	−1.6314	14
000746	0.8412	3	2.5414	9	2.4854	9
000916	0.6567	8	−4.0849	18	−3.4936	18
001044	0.9507	2	0.3067	12	0.2997	12
001167	0.7016	6	−3.7388	17	−3.8339	19
001208	1.0321	1	−4.4211	20	−3.9102	20
002871	0.3937	16	−2.1414	16	−1.7627	15
001692	0.6186	9	0.8716	11	0.87076	11

Continued

160919	0.8067	4	−1.6072	13	−1.5470	13
540007	0.6832	7	−2.1212	15	−2.1666	16
110022	0.7281	5	3.6048	7	3.5983	8
100022	0.5225	13	3.7748	6	4.0538	6
270002	0.6090	10	5.0744	4	5.1769	4
510660	0.2832	19	3.3358	8	3.7531	7
070013	−0.1416	21	7.5070	1	7.3670	1
000008	−0.0034	20	0.9004	10	0.8956	10
161606	0.3488	17	6.0303	3	5.8303	3
519191	0.5976	11	−7.2162	22	−7.4163	22
163402	0.2944	18	6.3134	2	5.8736	2
090010	0.5970	12	−4.2732	19	−3.2764	17
161032	0.5140	14	−6.3952	21	−6.1137	21
161721	−0.2033	22	4.3584	5	4.3050	5

[a]Sharpe, EVaR-Sharpe, and LVaR-Sharpe represent the traditional Sharpe ratio, Sharpe ratio modified based on expectile, and *Lp* quantile, respectively.

4. Conclusion

This paper proposes a new method to calculate Sharpe ratio, which is based on *Lp* quantile regression to estimate VaR, and then uses estimated VaR instead of standard deviation to modify traditional Sharpe ratio as a new performance evaluation index. In order to measure VaR more accurately, we construct *Lp* quantile nonlinear regression model, which has better robustness, can capture asymmetry and heavy-tailed distribution characteristics, is more flexible, and can better describe the tail risk of fund return when estimating VaR value by *Lp* quantile. Therefore, using VaR based on *Lp* quantile estimation to modify Sharpe ratio can provide more accurate and reliable results, thus improving the quality of risk management and decision-making. The empirical results also show that our fund ranking method is effective and feasible. In addition, although this study provides a new idea for fund performance evaluation based on *Lp* quantile regression, the calculation is more complicated because this method selects the sum of multiple p-values as part of the loss function. In future research, we can choose the optimal p-value and different models to measure the risk in *Lp* quantile regression method, so as to analyze the performance of open-end funds in China more accurately and comprehensively.

Acknowledgements

This work is partly supported by the Graduate Textbook Construction Project of Sichuan University of Science and Engineering (Grant No. KA202011) and the Opening Project of Sichuan Province University Key Laboratory of Bridge Non-destruction Detecting and Engineering Computing (2024QYY02).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Sharpe, W.F. (1966) Mutual Fund Performance. *The Journal of Business*, **39**, 119-138. https://doi.org/10.1086/294846

[2] Jack, T. (1965) How to Rate Management of Investment Funds. *Harvard Business Review*, **43**, 63-75.

[3] Jensen, M.C. (1968) The Performance of Mutual Funds in the Period 1945-1964. *The Journal of Finance*, **23**, 389-416. https://doi.org/10.1111/j.1540-6261.1968.tb00815.x

[4] Peng, H. and Wu, Q. (2004) Application of the Value at Risk (VaR) Based Sharpe Ratio in Fund Performance Evaluation. *Management Review*, **2**, 17-23, 29-63. (In Chinese)

[5] Shi, M., Wang, S. and Xu, S. (2006) Modified Sharpe Ratio and Its Application in Fund Performance Evaluation. *Systems Engineering Theory and Practice*, **7**, 1-10. (In Chinese)

[6] Yu, T., Tian, J. and Wang, Y. (2010) Research on the Performance Evaluation of Open-End Funds Based on the Modified Sharpe Ratio. *China Soft Science*, **7**, 175-183. (In Chinese)

[7] Su, X. and Zhou, Y. (2015) Evaluation and Test of China's Open-End Fund Performance Based on Asymmetric Power Distribution. *Mathematical Statistics and Management*, **34**, 162-174. (In Chinese) https://doi.org/10.13860/j.cnki.sltj.20130426-005

[8] Newey, W.K. and Powell, J.L. (1987) Asymmetric Least Squares Estimation and Testing. *Econometrica*, **55**, 819-847. https://doi.org/10.2307/1911031

[9] Kuan, C., Yeh, J. and Hsu, Y. (2009) Assessing Value at Risk with CARE, the Conditional Autoregressive Expectile Models. *Journal of Econometrics*, **150**, 261-270. https://doi.org/10.1016/j.jeconom.2008.12.002

[10] Taylor, J.W. (2007) Estimating Value at Risk and Expected Shortfall Using Expectiles. *Journal of Financial Econometrics*, **6**, 231-252. https://doi.org/10.1093/jjfinec/nbn001

[11] Kim, M. and Lee, S. (2016) Nonlinear Expectile Regression with Application to Value-at-Risk and Expected Shortfall Estimation. *Computational Statistics & Data Analysis*, **94**, 1-19. https://doi.org/10.1016/j.csda.2015.07.011

[12] Yao, H. (2011) Linear Heteroscedastic Models Based on Expectile. Ph.D. Thesis, University of Science and Technology of China. (In Chinese)

[13] Xie, S. (2011) Statistical Modeling of Financial Risk Measurement and Management and Its Applications. Ph.D. Thesis, Graduate School of the Chinese Academy of Sciences. (In Chinese)

[14] Hu, Z., Li, Y. and Wan, C. (2020) VaR and ES Risk Measurement Based on Bayesian GARCH-Expectile Model. *Mathematical Statistics and Management*, **39**, 467-477. (In Chinese) https://doi.org/10.13860/j.cnki.sltj.20191013-001

[15] Su, X. and Zhou, Y. (2013) Conditional Autoregressive Expectile Models and Their Application in Fund Performance Evaluation. *Chinese Management Science*, **21**, 22-29. (In Chinese) https://doi.org/10.16381/j.cnki.issn1003-207x.2013.06.007

[16] Chen, Z. (1996) Conditional L_p-Quantiles and Their Application to the Testing of Symmetry in Non-Parametric Regression. *Statistics & Probability Letters*, **29**, 107-115. https://doi.org/10.1016/0167-7152(95)00163-8

[17] Jiang, Y., Lin, F. and Zhou, Y. (2019) The Kth Power Expectile Regression. *Annals of the Institute of Statistical Mathematics*, **73**, 83-113. https://doi.org/10.1007/s10463-019-00738-y

[18] Usseglio-Carleve, A. (2018) Estimation of Conditional Extreme Risk Measures from Heavy-Tailed Elliptical Random Vectors. *Electronic Journal of Statistics*, **12**, 4057-4093. https://doi.org/10.1214/18-ejs1499

[19] Girard, S., Stupfler, G. and Usseglio-Carleve, A. (2020) An *Lp*-Quantile Methodology for Estimating Extreme Expectiles. HAL (Le Centre pour la Communication Scientifique Directe). https://inria.hal.science/hal-02311609v3

[20] Tang, L. and Chen, Y. (2020) L_p Quantile Regression with Realized Measure. *Journal of University of Science and Technology of China*, **50**, 1478-1487.

[21] Stupfler, G. and Usseglio-Carleve, A. (2022) Composite Bias-Reduced L^p-Quantile-Based Estimators of Extreme Quantiles and Expectiles. *Canadian Journal of Statistics*, **51**, 704-742. https://doi.org/10.1002/cjs.11703

[22] Arnroth, L. and Vegelius, J. (2023) Quantile Regression Based on the Skewed Exponential Power Distribution. *Communications in Statistics—Simulation and Computation*. https://doi.org/10.1080/03610918.2023.2224945

[23] Arnroth, L. (2024) Bayesian composite L^p-Quantile Regression. *Metrika*. https://doi.org/10.1007/s00184-024-00950-8

[24] Gao, J. and Wang, L. (2023) Communication-Efficient Low-Dimensional Parameter Estimation and Inference for High-Dimensional L^p-Quantile Regression. *Scandinavian Journal of Statistics*, **51**, 302-333. https://doi.org/10.1111/sjos.12683

[25] Koenker, R. and Bassett, G. (1978) Regression Quantiles. *Econometrica*, **46**, 33-50. https://doi.org/10.2307/1913643

[26] Lin, F., Jiang, Y. and Zhou, Y. (2022) The Kth Power Expectile Estimation and Testing. *Communications in Mathematics and Statistics*, **12**, 573-615. https://doi.org/10.1007/s40304-022-00302-w

[27] Sun, P., Lin, F., Xu, H. and Yu, K. (2024) Estimation of Value-At-Risk by L^p Quantile Regression. *Annals of the Institute of Statistical Mathematics*. https://doi.org/10.1007/s10463-024-00911-y