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Magnetic Resonance Based Proof of Reality of Wavefunction

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Abstract

In this contribution, we use the formalism of Magnetic Resonance to present an argument for the reality of the solution of the Schrodinger Equation appropriate for the existence of Bloch Equation magnetization states. We take as our definition of Reality that the observable used can be measured in the laboratory such as the Cartesian x Component of Magnetization. We relate this real existing observable to the Density Matrix corresponding to the system and then argue the Density Matrix must have Physical Reality if the Magnetization has Physical Reality. Since the Density Matrix for a pure state is simply related to the wave function, we then argue that the corresponding wavefunction must also be Physically Real.

Keywords

Magnetic Resonance, Magnetization, Wavefunction, Density Matrix, Quantum Mechanics

1. Introduction

The issue of the Reality of the Quantum Mechanical Wavefunction as the solution of the appropriate non-relativistic Schrodinger Equation is a topic of ongoing debate in the Philosophy of Quantum Mechanics literature (Ney & Albert, 2013; Ney, 2023; Pusey et al., 2012; Chen, 2019; Lorenzetti, 2022; Gao, 2018). There are various schools of thought which are predominant in this arena. A succinct statement is that the wavefunction is viewed as an element of physical reality, or as just an instrumentalist tool for the prediction of physical events (Abragam, 1988; Slichter, 1996; Munowitz, 1988; Blum, 1981). In this communication, we proposed a succinct yet potentially definitive proof of the Reality of the Wavefunction based on an argument derived from commonly accepted experimental results in the Magnetic Resonance Community. A perusal of the literature by the author finds that this approach may be a new and unique methodology to address the problem of validating whether the wavefunction is an element of Reality (Ney & Albert, 2013; Gao, 2018; Blum, 1981; Albert, 2023; Popper, 1982; Lewis, 2016). The validity of the argument basically rests on accepting the x magnetization as a real physical quantity because it can be measured and quantitated by means validated for years in the Magnetic Resonance literature (Goldman, 1988; Abragam, 1988; Slichter, 1996; Munowitz, 1988; Blum, 1981). A simple relation is derived between the Quantum Mechanical expectation value of the x-magnetization for a Sine Cosine Radiofrequency Hamiltonian and the Trace of the product of the presented Density Matrix and the x-Spin Angular Momentum Operator. Since the elements of the Density Matrix are defined in terms of solutions to the Schrodinger equation wave function we assert that the wavefunctions must have the same element of Reality as the magnetization expectation value. The presented proposed method is currently limited to spin 1/2 particles but the general thrust of the proposed proof may be easily extended to more complicated spin systems.

2. Theory

We center our proof on the definition of the reduced density matrix (Blum, 1981) for a pure state of the general form:

$$\sigma[t] = |\psi(t)\rangle\langle\psi(t)| \tag{1}$$

The assumptions of the proof and the limitations of the model are carefully delineated and discussed. The formalism is presented and then the ramifications are discussed.

3. Formalism

We begin our theoretical formalism by first stating the Bloch Equations famous for the range of applications they have in the field of Magnetic Resonance (Goldman, 1988; Abragam, 1988; Slichter, 1996; Munowitz, 1988; Blum, 1981). The form of the Bloch equations specified is for the case without explicit relaxation included and also with time-dependent radio-frequency modulation functions. We choose the amplitude of the RF to be on the x component, but this does not detract from the generality of the argument.

We state the Bloch equations as follows [The Interested reader may consult the books by Abragam and Slichter (Abragam, 1988; Slichter, 1996) for example]

$$\frac{\mathrm{d}M_x[t]}{\mathrm{d}t} = -\Delta\omega[t]M_y[t] \tag{2}$$

$$\frac{\mathrm{d}M_y[t]}{\mathrm{d}t} = \Delta\omega[t]M_x[t] + \omega_1[t]M_z[t] \tag{3}$$

$$\frac{\mathrm{d}M_z[t]}{\mathrm{d}t} = -\omega_1[t]M_y[t] \tag{4}$$

Here we note that $\Delta\omega[t]$ is denoted as the Frequency Offset in the First Rotating Frame. While $\omega_1[t]$ is the Radio-Frequency Amplitude. The domain of validity for the Bloch equations is specified for spin 1/2 nuclei. This delineates a constraint that is imposed on the applicability of our proposed proof.

We note in passing that the Bloch equations as specified can be easily derived from a so-called Torque Relation between the Time Derivative of the Magnetization Vector and the vector cross product of the Magnetization Vector and the Vector of the applied radio-frequency pulse (Munowitz, 1988).

Also, Equations (2)-(4) can be succinctly written in the following notation:

$$\frac{\mathrm{d}\tilde{M}[t]}{\mathrm{d}t} = \tilde{\tilde{W}}[t]\tilde{M}[t] \tag{5}$$

where $\tilde{M}[t]$ is a Cartesian vector with Components of the magnetization in the First Rotating Frame (FRF) specified as:

$$\begin{bmatrix} M_x[t] \\ M_y[t] \\ M_z[t] \end{bmatrix} \tag{6}$$

And $\tilde{\tilde{W}}[t]$ is a three by three square matrix of the form:

$$\begin{bmatrix} 0 & -\Delta\omega[t] & 0 \\ \Delta\omega[t] & 0 & \omega_1[t] \\ 0 & -\omega_1[t] & 0 \end{bmatrix} \tag{7}$$

Central to our argument is that the solution to the Bloch equations for the Magnetization Components can be obtained from the Reduced Density Matrix, meaning a density matrix with only the time-dependence of the system of interest included neglecting any so-called environmental contributions. [For a discussion consider the Books by Blum (1981) and Munowitz (1988)]

The argument we use is centered on elementary Quantum Mechanics as applied to Magnetic Resonance. [See The Book by Maurice Goldman (1988) for example]

Here we need to define the so-called Pauli matrices as:

$$\begin{aligned} s_x &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ s_y &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \\ s_z &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{aligned} \tag{8}$$

Then we enlist the so-called Trace relation in the Schrodinger Representation (Goldman, 1988) as

$$\langle S_\alpha \rangle[t] = Tr\left[\tilde{\tilde{\sigma}}[t] \cdot s_\alpha\right], \alpha = x, y, z$$

Here we note that the left-hand side of the previous equation is the Quantum Mechanical Expectation value of the operator s_α (Goldman, 1988; Slichter, 1996).

Another important assumption in our argument is that the Expectation Value of the Cartesian Spin Angular Momentum of our considered spin 1/2 model is proportional to the Magnetization. [See for Example Chapter 1 of (Slichter, 1996)]

Explicitly, we write:

$$\langle M_\alpha \rangle[t] = \mu \langle S_\alpha \rangle[t] \quad \alpha = x, y, z$$

where μ is the magnetic moment of the spin 1/2.

Our proof then unfolds immediately by using Equation (8) and Equation (9)

We can write:

$$\begin{aligned}\langle S_\alpha \rangle[t] &= Tr\left[\tilde{\tilde{\sigma}}[t] \cdot s_\alpha\right] = Tr\left[\left|\psi[t]\right\rangle\left\langle\psi[t] \cdot s_\alpha\right|\right] \\ &= Tr\left[\left\langle\psi[t] s_\alpha \middle| \psi[t]\right\rangle\right] = \left\langle\psi[t] s_\alpha \middle| \psi[t]\right\rangle = \langle S_\alpha \rangle[t]\end{aligned} \tag{10}$$

For concreteness let us consider the case $\alpha = x$.

Then we obtain in Equation (10) the result using the representation given of the Pauli Matrices and standard matrix multiplication and the definition of the trace relationship:

that

$$\langle M_x[t] \rangle = \mu\left(\sigma_{12}[t] + \sigma_{21}[t]\right) \tag{11}$$

where:

$$\sigma_{ij}[t] = \left|\psi^i[t]\right\rangle\left\langle\psi^j[t]\right| \tag{12}$$

where:

$\left|\psi[t]\right\rangle = \begin{bmatrix} \left|\psi^1[t]\right\rangle \\ \left|\psi^2[t]\right\rangle \end{bmatrix}$ is the defined spinor representation of the wavefunction that is a solution of the non-relativistic Schrodinger equation of the form:

$$i\left|\dot{\psi}[t]\right\rangle = \tilde{\tilde{H}}[t]\left|\psi[t]\right\rangle \tag{13}$$

Where the Hamiltonian utilized is of the form:

$$\tilde{\tilde{H}}[t] = \begin{bmatrix} \Delta\omega[t] & \omega_1[t] \\ \omega_1[t] & -\Delta\omega[t] \end{bmatrix} \tag{14}$$

Therefore, we can now write:

$$\begin{aligned}\langle M_x \rangle[t] &= \mu \langle S_x \rangle[t] = \mu\left(\sigma_{12}[t] + \sigma_{21}[t]\right) \\ &= \mu\left(\left|\psi^1[t]\right\rangle\left\langle\psi^2[t]\right| + \left|\psi^2[t]\right\rangle\left\langle\psi^1[t]\right|\right)\end{aligned} \tag{15}$$

We now assert that if we accept the Reality of the Expectation value of the x-Magnetization component, it forces the assertion that the wave function components of the spinor utilized in the equation must therefore be Real using the same criterion of Reality.

4. Discussion

The novelty of the proof presented of the Reality of the wavefunction is that it relies on the use of experimental methodologies commonplace in the Magnetic Resonance Discipline. The literature is replete with experimental attempts to prove the Reality of the wave function. (Brown, 2017; Ringbauer, 2015; Lundeeni, 2011)

The methods used are usually comparatively involved to implement. In our contribution, the proposed method utilizes means that are standard procedures in the Magnetic Resonance Community. (Goldman, 1988; Abragam, 1988; Slichter, 1996; Munowitz, 1988; Blum, 1981)

The result is so straightforward it might be regarded as self-evident and not in need of any experimental verification.

The assertions are straightforward in the Proof presented and then can be easily supported or debunked in the critical literature. The limitations of the Proof in that it considers spin 1/2 nuclei and a particular physical system can be easily addressed and be subject to subsequent development and generalization.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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