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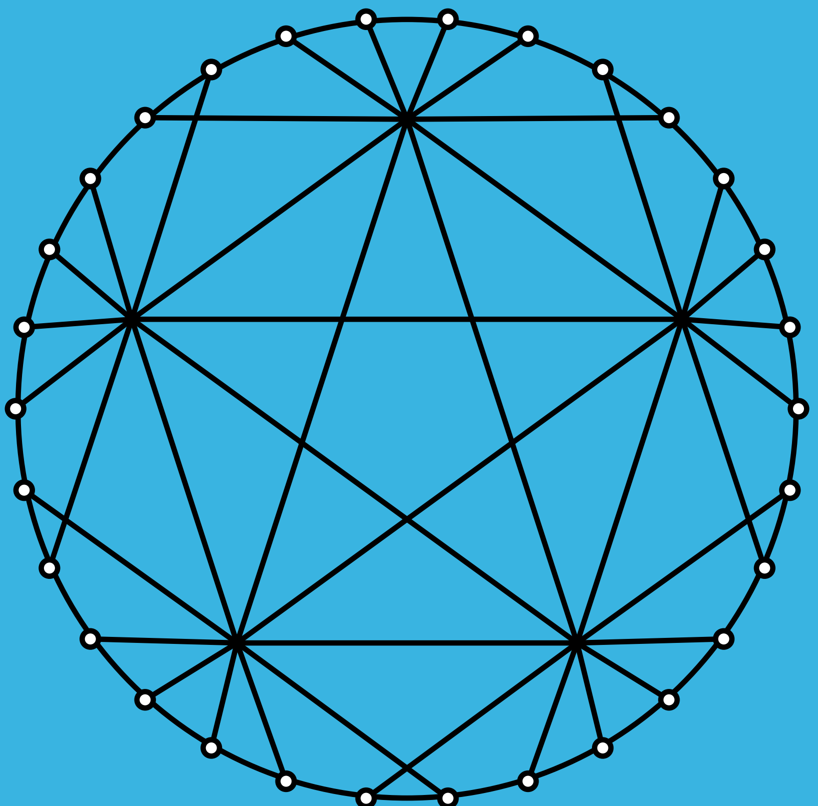

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Two related (?) 2-edge-Hamiltonian bigraph conjectures addendum

Gustavus J. Simmons
gsimmons30@comcast.net

One way of looking at the entries in Table 1 in [1] is that an N represents a $\mathrm{P}_{m,k}$ whose 2-edge-Hamiltonicity remains in doubt after Theorem 2.4 has eliminated those cases known to be 2-edge-Hamiltonian because $gcd(m+1, k+1) \leq 2$. Similarly, although not reflected in the table, Theorem 2.6 eliminates the cases in which $gcd(m+1, k+1) = k+1$ and Theorem 2.9 those for which $k = \lfloor (m-1)/2 \rfloor$, i.e. the entries on the diagonal in Table 1. The entries that remain in doubt after Theorems 2.4, 2.6 and 2.9 have all been applied are those for which $2 < gcd(m+1, k+1) < k+1$. This was noted in [1], but full advantage wasn't taken there of the isomorphisms of the $\mathrm{P}_{m,k}$ [2] to eliminate as many of the remaining uncertain cases as possible. The table below shows the rows and columns from Table 1 which contain one or more of the $2 < gcd(m+1, k+1) < k+1$ entries, represented by an X to indicate their 2-edge-Hamiltonicity is as yet unproven. The status of most of these can be resolved by two simple appeals to isomorphism. Obviously if k is isomorphic to a k' for which $\mathrm{P}'_{m,k}$ is 2-edge-Hamiltonian, then so is $\mathrm{P}_{m,k}$. Less obvious is that if k is a member of an isomorphism triple $\mathrm{P}_{m,k}$ is 2-edge-Hamilton since that means both the Plus and Minus mappings describe Hamilton cycles which guarantees every edge pair is in a Hamilton cycle. It is worth noting that if m is a prime all isomorphic groupings are triples, but triples can occur for m that are not primes; for example $(4, 7, 10)$ when $m = 27$.

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$m \backslash k$	5	7	8	9	11	13	14
14	X						
19		Ⓧ					
20	Ⓧ		Ⓧ				
23			X				
24				Ⓧ			
26	X				X		
27		X			X		
29			X		X		
32	X				X		
34				Ⓧ		X	Ⓧ

That leaves only five $P_{m,k}$, indicated by the circled X, whose 2-edge-Hamiltonicity remains in doubt after the isomorphic reductions have been made. The three cases for $m = 20$ and 24 were shown to be 2-edge-Hamiltonian by direct computation in [1]. Neither member of the isomorphic pair $P_{34,9}$ and $P_{34,14}$ is known to be 2-edge-Hamiltonian and m is much too large for direct computation. So, in the absence of some other way of establishing 2-edge-Hamiltonicity, the most that can be said is that all $P_{m,k}$ on 66 or fewer vertices are 2-edge-Hamiltonian, lending further credence to Conjecture 1.

References

[1] G.J. Simmons, Two related (?) 2-edge-Hamiltonian bigraph conjectures, *Bull. Inst. Combin. Appl.*, **88** (2020), 98–117.

[2] G.J. Simmons, Hamilton-laceable properties of polygonal bigraphs, *Congr. Numer.*, **226** (2016), 121–138.