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$M^{[X]}/G/1$ MULTISTAGE QUEUE WITH RENEGING DURING VACATION AND BREAKDOWN PERIODS AND SECOND OPTIONAL REPAIR

C. Yuvarani[1], C. Vijayalakshmi[2]

[1]Department of Mathematics
Velammal Institute of Technology
Panchetti, INDIA

[2]Department of Mathematics
Vellore Institute of Technology
Chennai, INDIA

Abstract: This paper investigate a queueing model where customers arrive in batches according to Poisson process and a single server provides 'N' stages of heterogeneous service one after another in succession with general service time distribution having different vacation policies. Once the system gets breakdown, it enters into a two phase of repair process. The first phase is essential whereas the second phase is optional. In addition, customers may renege (leave the queue after joining) during server breakdown or during server vacation due to impatience. The steady state solution is derived using supplementary variable technique and numerical illustration is presented.

1. Introduction

Numerous authors have studied queueing models by importing many aspects on them such as server vacation, breakdown, repair, reneging, balking etc.

A batch arrival queueing model with different vacation policies is studied by many authors including R. Vimaladevi [1], G. Ayyappan & S. Shyamala [3] and Ke. J.C, Huang. H.I and Chu. Y.K [5]. G. Ayyappan ans S. Shyamala [4] obtained transient state and steady state of batch arrival queue subject to random breakdowns and Bernoulli schedule server vacation with second optional repair. Madhu Jain et al. [6] has discussed multi optional repair. The two stage $M[x]/G/1$ queue with reneging during vacation and breakdown period was investigated by, Monita Baruah et al. [2].

In this paper, We consider $M^{[x]}/G/1$ queue with 'N' stages of services under different vacation policy subject to system breakdown. After service completion of a customer, the server may remain in the system to serve the next customer with probability β_0 or he may proceed on j^{th} vacation scheme with probability β_j $(1 \leq j \leq M)$ and $\sum_{j=0}^{M} \beta_j = 1$. Once the server breakdown, it is immediately sent for first essential repair (FER). After the completion of FER, the server may opt for the second optional repair (SOR) with probability r or may join the system with probability $1-r$ to render the service to the customers. In addition we assume that the customers may renege during breakdown or vacation period due to impatience.

2. Model Assumptions

We make the following assumptions to describe the queueing model of our study.

a) Customers arrive in batches according to compound Poisson process with rate of arrival λ and they are provided one by one service on a first come - first served basis. Let $\lambda c_i dt$ $(i \geq 1)$ be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t; t + dt]$, where $0 \leq c_i \leq 1$ and $\sum_{i=1}^{\infty} c_i = 1$.

b) A single server provides 'N' stages of services for each customer, with the service times having general distribution. Let $B_i(v)$ and $b_i(v)$ $(i = 1, 2, 3, \ldots, N)$ be the distribution and the density function of i stage service respectively. Let $\mu_i(x)dx$ be the conditional probability density of service completion during the interval $(x; x + dx]$, given that the elapsed time is x.

c) The server's vacation time follows a general (arbitrary) distribution with distribution function $V_j(t)$ and density function $v_j(t)$. Let $\gamma_j(x)dx$ be the

conditional probability of a completion of a vacation during the interval $(x; x + dx]$ given that the elapsed vacation time is x.

d) Reneging is assumed to follow exponential distribution with parameter η. Thus ηdt is the probability that a customer can renege during a short interval of time $(t; t + dt]$.

e) The server breakdown is assumed to occur according to a poisson stream with mean breakdown rate $\alpha > 0$.

f) The repair process provides two types of repair in which the first type is essential whereas the second type is optional. Both are exponentially distributed with mean $1/p$ and $1/q$.

3. Equations Governing the System

We define
(i) $P_n^{(i)}(x, t)$ = Probability that at time t, there are 'n' ($n \geq 0$) customers in the queue excluding the one being served with elapsed service time x and the server is providing i^{th} stage ($i = 1, 2, 3, \ldots, N$) of service. Consequently $P_n^{(j)}(t)$ denotes the probability irrespective of the value of x.
(ii) $V_n^{(j)}(x, t)$ = Probability that at time t, there are 'n' ($n \geq 1$) customers waiting in the queue for service and the server is on j^{th} vacation ($j = 1, 2, 3, \ldots, M$) and consequently $V_n^{(j)}(t)$ denotes the probability irrespective of the value of x.
(iii) $R_n^{(1)}(t)$ = Probability that at time t, the server is inactive due to breakdown and the system is under FER while there are 'n' ($n \geq 0$) customers in the queue.
(iv) $R_n^{(2)}(t)$ = Probability that at time t, the server is inactive due to breakdown and the system is under SOR while there are 'n' ($n \geq 0$) customers in the queue.
(iii) $Q(t)$ = Probability that there are no customers in the queue at time t, and the server is idle but available in the system.

The queueing model is then, governed by the following set of differential-difference equations:

$$\frac{\partial}{\partial x} P_n^{(i)}(x, t) + \frac{\partial}{\partial t} P_n^{(i)}(x, t) + [\lambda + \mu_i(x) + \alpha] P_n^{(i)}(x, t)$$

$$= \lambda \sum_{k=1}^{n} c_k P_{n-k}^{(i)}(x, t) \quad i = 1, 2, 3, \ldots, N, n \geq 1$$

$$\frac{\partial}{\partial x}P_0^{(i)}(x,t) + \frac{\partial}{\partial t}P_0^{(i)}(x,t) + [\lambda + \mu_i(x) + \alpha]P_0^{(i)}(x,t) = 0$$

$$\frac{\partial}{\partial x}V_n^{(j)}(x,t) + \frac{\partial}{\partial t}V_n^{(j)}(x,t) + [\lambda + \gamma_j(x) + \eta]V_n^{(j)}(x,t)$$

$$= \eta V_{n+1}^{(j)}(x,t) + \lambda \sum_{k=1}^{n} c_k V_{n-k}^{(j)}(x,t) \quad j = 1,2,3,\ldots,M, n \geq 1$$

$$\frac{\partial}{\partial x}V_0^{(j)}(x,t) + \frac{\partial}{\partial t}V_0^{(j)}(x,t) + [\lambda + \gamma_j(x) + \eta]V_0^{(j)}(x,t) = \eta V_1^{(j)}(x,t)$$

$$\frac{d}{dt}R_n^{(1)}(t) + (\lambda + p + \eta)R_n^{(1)}(t) = \lambda \sum_{k=1}^{n} c_k R_{n-k}^{(1)}(t) + \eta R_{n+1}^{(1)}(t)$$

$$+ \alpha \sum_{i=1}^{N} \int_0^\infty P_{n-1}^{(i)}(x,t)dx$$

$$\frac{d}{dt}R_0^{(1)}(t) + (\lambda + p)R_0^{(1)}(t) = \eta R_1^{(1)}(t)$$

$$\frac{d}{dt}R_n^{(2)}(t) + (\lambda + q + \eta)R_n^{(2)}(t) = \lambda \sum_{k=1}^{n} c_k R_{n-k}^{(2)}(t) + \eta R_{n+1}^{(2)}(t) + rpR_n^{(1)}(t)$$

$$\frac{d}{dt}R_0^{(2)}(t) + (\lambda + q)R_0^{(2)}(t) = \eta R_1^{(2)}(t) + rpR_0^{(1)}(t)$$

$$\frac{d}{dt}Q(t) = \beta_0 \int_0^\infty P_0^{(N)}(x,t)\mu_N(x)dx + \sum_{j=1}^{M} \int_0^\infty V_0^{(j)}(x,t)\gamma_j(x)dx$$

$$- \lambda Q(t) + (1-r)pR_0^{(1)}(t) + qR_0^{(2)}(t)$$

The above equations are to be solved subject to the following boundary conditions:

$$P_n^{(1)}(0,t) = \beta_0 \int_0^\infty P_{n+1}^{(N)}(x,t)\mu_N(x)dx + \sum_{j=1}^{M} \int_0^\infty V_{n+1}^{(j)}(x,t)\gamma_j(x)dx$$

$$+ \lambda c_{n+1}Q(t) + (1-r)pR^{(1)}_{n+1}(t) + qR^{(2)}_{n+1}(t)$$

$$P^{(i)}_n(0,t) = \int_0^\infty \mu_{i-1}(x)P^{(i-1)}_n(x,t)dx \ \ i = 2,3,\ldots,N$$

$$V^{(j)}_n(0,t) = \beta_j \int_0^\infty \mu_N(x)P^{(N)}_{n+1}(x,t)dx \ \ j = 2,3,\ldots,M$$

The initial conditions are

$$V^{(j)}_0(0) = V^{(j)}_n(0) = 0, \ \ j = 1,2,3,\ldots,M \text{ and } Q(0) = 1 \text{ and}$$
$$P^{(i)}_n(0) = 0 \text{ for } n = 0,1,2,\ldots, \ \ i = 1,2,3,\ldots,N$$

4. The Steady State Results

We obtain the following steady state probability distribution functions by using supplementary variable technique

$$\bar{P}^{(i)}(z,s) = \frac{\left[\begin{array}{c} f_2(z)f_3(z)\lambda(C(z)-1)\bar{B}_1(f(z)) \\ \bar{B}_2(f(z))\ldots\bar{B}_{i-1}(f(z))[1-\bar{B}_i(f_1(z))]Q \end{array}\right]}{DR}$$

$$i = 1,2,\ldots,N$$

$$\bar{V}^{(j)}(z) = \frac{\left[\begin{array}{c} \beta_j f_1(z)f_2(z)f_3(z)\lambda(C(z)-1)\bar{B}_1(f(z)) \\ \bar{B}_2(f(z))\ldots\bar{B}_N(f(z))Q \end{array}\right]}{DR}\frac{1-\bar{V}_i(T)}{T}$$

$$\bar{R}^{(1)}(z) = \frac{\alpha z f_3(z)\lambda(C(z)-1)[1-\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\ldots\bar{B}_N(f_1(z))]Q}{DR}$$

$$\bar{R}^{(2)}(z) = \frac{rp\alpha z\lambda(C(z)-1)[1-\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\ldots\bar{B}_N(f_1(z))]Q}{DR}$$

where
$$DR = f_1(z)f_2(z)f_3(z)\left\{z - \bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\ldots\bar{B}_N(f_1(z))\right.$$
$$\left.\left[\beta_0 + \sum_{j=1}^M \beta_j\bar{V}_j(T)\right]\right\} - \alpha pz[(1-r)f_3(z) + rq]$$

$$[\bar{B}_1(f_1(z))\bar{B}_2(f_1(z))\ldots\bar{B}_N(f_1(z))]$$

$f_1(z) = s + \lambda(1 - C(z)) + \alpha, \quad f_2(z) = s + \lambda(1 - C(z)) + \eta - \frac{\eta}{z} + p$

$f_3(z) = s + \lambda(1 - C(z)) + \eta - \frac{\eta}{z} + q$ and $T = s + \lambda(1 - C(z)) + \eta - \frac{\eta}{z}$

$\bar{B}_i(f_1(z))$, $\bar{V}_j(f_2(z))$ are the Laplace-Stieltjes transform of Service time $B_i(x)$ and vacation time $V_j(x)$ respectively.

Let $W_q(z)$ be the PGF of queue size irrespective of the state of the system. Then we have,

$$W_q(z) = \sum_{i=1}^{N} \bar{P}^{(i)}(z) + \sum_{j=1}^{M} \bar{V}^{(j)}(z) + \bar{R}^{(1)}(z) + \bar{R}^{(2)}(z)$$

In order to obtain Q, using the normalization condition

$W_q(1) + Q = 1$

Let L_q denote the mean number of customers in the queue under the steady state, then

$$L_q = \frac{d}{dz}[W_q(z)]_{z=1}$$

5. Numerical Results

In order to examine the validity of results of queueing system, numerical approaches is an useful way. For the purpose of a numerical result, we choose the following arbitrary values: $N = 3$, $M = 1$, $E(I) = 1$; $E(I(I - 1)) = 0$; $\lambda = 2$; $\mu_1 = 2$; $\mu_2 = 4$; $\mu_3 = 4$; $\gamma = 5$; $\beta = 0.5$; $r = 1$; $p = 0.5$; $q = 0.5$

Figure 1: Effect of reneging η and breakdown at $\alpha = 1$ on the mean queue size L_q and mean waiting time W_q

Figure 2: Effect of reneging η and breakdown at $\alpha = 2$ on the proportion of idle time Q and the utilization factor ρ

Thus Figure 1 shows that as breakdown occurs, it increases the average

Table 1: Computed values of various
queue characteristics

η	α	Q	ρ	L_q	W_q
5	1	0.5353	0.4647	11.3921	5.6961
6		0.6078	0.3922	8.7275	4.3638
7		0.6607	0.3393	5.9181	2.9591
8		0.7011	0.2989	5.1961	2.5981
5	2	0.5648	0.4352	5.1737	2.5869
6		0.6352	0.3648	4.0386	2.0193
7		0.6861	0.3139	3.3789	1.6895
8		0.7245	0.2755	2.9487	1.4744
5	3	0.5753	0.4247	4.1627	2.0814
6		0.6449	0.3551	3.2279	1.6140
7		0.6949	0.3051	2.6794	1.3397
8		0.7325	0.2675	2.3190	1.1595

length of the queue and due to reneging of the customers from the queue, the average waiting time also decreases. Again from Figure 2, it is clear that the proportion of the idle time of the server increases and utilization factor decreases due to breakdown of the system and reneging. The trends shown by the table are as expected.

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