An Indirect Method for Closed-loop Identification of Sparsely Controlled Networks

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1 Introduction

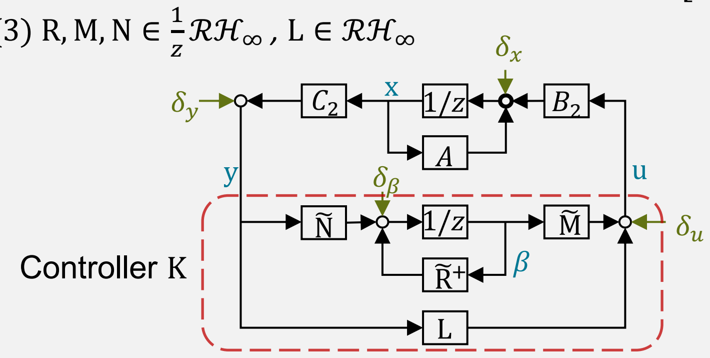
- Modern Cyber-Physical Systems (CPS) are large-scale, physically distributed with decentralized controllers.
- Exploit the apriori knowledge of the controller's sparsity to improve closed-loop identification of CPS.

2 System Level Parameterization[a,b]

- Consider the discrete time linear time invariant (LTI) system P as $\mathrm{P} = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$
- Let *system response* $\{\mathrm{R},\mathrm{M},\mathrm{N},\mathrm{L}\}$ be *defined* as the mapping $\begin{bmatrix} \mathrm{x} \\ \mathrm{u} \end{bmatrix} = \begin{bmatrix} \mathrm{R} & \mathrm{N} \\ \mathrm{M} & \mathrm{L} \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}$, where δ_x and δ_y are sensor and process disturbances.
- ***Theorem:*** *For the above output feedback system, the controller* $\mathrm{K} = \mathrm{L} - \mathrm{M}\mathrm{R}^{-1}\mathrm{N}$ *is internally stabilizing if*

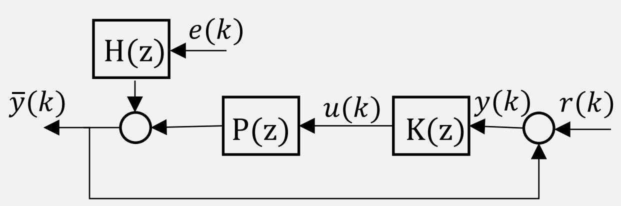
$$(1)\ \begin{bmatrix} zI - A & -B_2 \end{bmatrix} \begin{bmatrix} \mathrm{R} & \mathrm{N} \\ \mathrm{M} & \mathrm{L} \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix} \quad (2)\ \begin{bmatrix} \mathrm{R} & \mathrm{N} \\ \mathrm{M} & \mathrm{L} \end{bmatrix} \begin{bmatrix} zI - A \\ -C_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

$$(3)\ \mathrm{R},\mathrm{M},\mathrm{N} \in \frac{1}{z}\mathcal{RH}_\infty,\ \mathrm{L} \in \mathcal{RH}_\infty$$



- Thus, the sparsity of K *translates to sparsity* of $\mathrm{L},\mathrm{M},\mathrm{R}$, and N

3 Closed Loop Identification



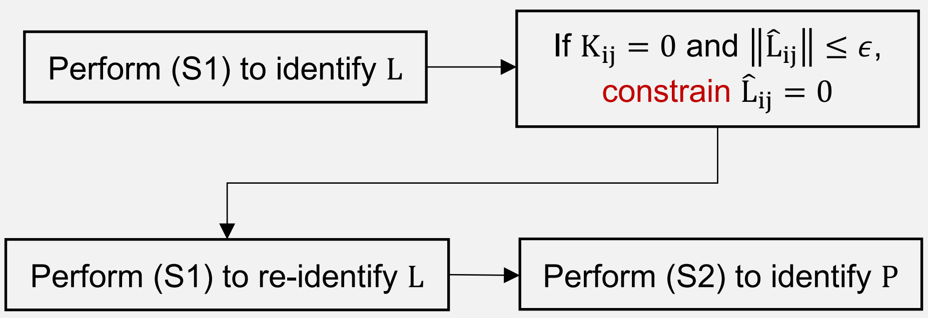
Lemma: Let $\mathrm{K} := \mathrm{L} - \mathrm{M}\mathrm{R}^{-1}\mathrm{N}$ be a stabilizing controller for the above plant P, then

$$\mathrm{LPK} = \mathrm{L} - \mathrm{K} \text{ and } \mathrm{KPL} = \mathrm{L} - \mathrm{K}$$

A two-stage open-loop identification strategy[c]

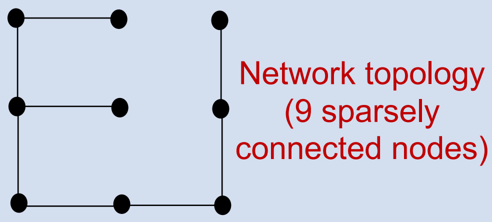
- (S1) Stage 1: $\mathrm{u} = \mathrm{L}(-\mathrm{r}) + \mathrm{LHe}$ (estimate $\hat{L} \approx L$)
- (S2) Stage 2: $\bar{y} = \mathrm{Pv} + (\mathrm{PL} + \mathrm{I})\mathrm{He}$ (estimate $\hat{P} \approx P$) where $\mathrm{v} = \hat{\mathrm{L}}(-\mathrm{r})$

4 Proposed Identification Algorithm



5 Simulation Results

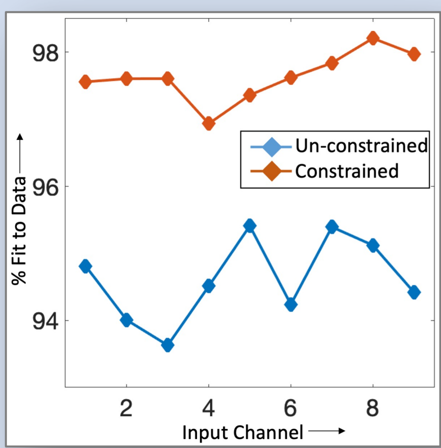
Identifying sparsely controlled Power System



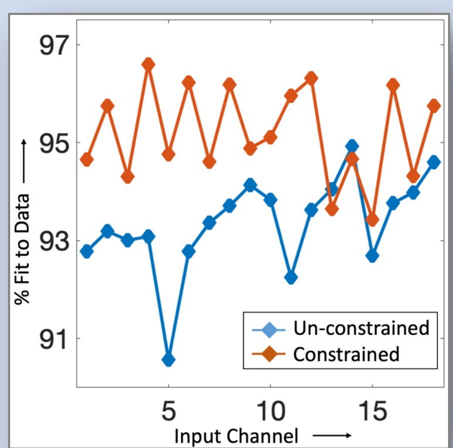
- Node i: state $x_i = [\theta_i\ \dot{\theta}_i]$

$$x_i[t+1] = A_{ii}x_i[t] + \sum_{j \in N_i} A_{ij}x_t[t] + B_{ii}u_i[t] + \delta_{x_i}[t]$$

- P: 9×18 MIMO plant
- K: 18×9 MIMO controller
- *Apriori information*: sparsity structure in K



(a) Identification of L



(b) Identification of P

References

a. Wang, Y. S., Matni, N., & Doyle, J. C. (2019). A system-level approach to controller synthesis. *IEEE Transactions on Automatic Control*, *64*(10), 4079-4093.

b. Anderson, J., Doyle, J. C., Low, S. H., & Matni, N. (2019). System level synthesis. *Annual Reviews in Control*, *47*, 364-393.

c. Van Den Hof, P. M., & Schrama, R. J. (1993). An indirect method for transfer function estimation from closed loop data. *Automatica*, *29*(6), 1523-1527.

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