DOI: 10.15439/2017F80
ISSN 2300-5963 ACSIS, Vol. 11

Modeling and Optimization of Multi-echelon Transportation systems - a hybrid approach

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***Abstract*—The efficient and timely distribution of freight goods is critical for supporting the demands of modern urban areas. Optimum freight ensures the survival and development of urban areas. In the contemporary logistic there are two main distribution strategies: direct distribution and multi-echelon distribution. In the direct distribution, means of transport, starting from the main distribution center, bring their freight directly to the delivery points , while in the multi-echelon systems, freight is delivered from the main distribution center to the delivery points through intermediate points (local warehouses, satellites).**

This study presents a concept and implementation of a integrated approach to modeling and optimization the Multi-Echelon Systems. In the proposed approach, two methods of constraint logic programming (CLP) and mathematical programming (MP) were integrated and hybridized. The proposed hybrid approach will be compared with classical mathematical programming on the same data sets (known benchmarks) for illustrative multi-echelon model - Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP).

I. INTRODUCTION

THE transportation of goods and services expresses one of the main activities that influences trade, market, economy, and society as it assures a vital link between suppliers and customers. Today, one of the most important aspects which takes place in freight transportation is the definition of different shipping strategies. In the current freight transportation there are two main distribution strategies: direct distribution and multi-echelon distribution. In the direct distribution, means of transport, starting from a source (the main distribution center, depot), bring their freight directly to the delivery points while in the multi-echelon systems, freight is delivered from the source to the delivery points through intermediate points (warehouses, satellites etc.).

Nowadays, multi-echelon systems have been introduced in different areas and issues:

- Urban and city logistics.
- Multimodal freight distribution.
- Different types of supply chains.
- E-commerce and home delivery distribution.
- Postal and courier services.

The overwhelming majority of formal models of optimization in distribution goods and city logistics have been formulated as the integer programming (IP), integer linear programming (ILP), or mixed integer linear programming (MILP) problems and solved using the OR-based methods. Most often used mathematical programming (MP) [1].

MP-based approach has some weaknesses. First of all, for the real size discrete optimization problems, it is time consuming and requires a lot of system resources (memory, processors, etc.). Secondly, it only allows modeling integer, binary and linear constraints [2].

This paper proposes the concept of a hybrid approach (where two approaches of constraint logic programming (CLP) and mathematical programming (MP) were integrated) to modeling and optimization of multi-echelon systems. The illustrative example shows the potential, efficiency and flexibility of this approach.

II. MULTI-ECHELONS TRANSPORTATION SYSTEMS

The hierarchical level in terms of distribution strategies is the way the freight goes to the delivery point. When the freight arrives to delivery point without changing means of transport unit, a direct shipping or single-echelon strategy is applied, whereas when freight is derived from its source (depot) to its final destination passing through intermediate points (satellites, warehouses), where the freight is unloaded, then loaded into the same or into a different means of transport unit, we can speak of a multi-echelon system. Especially in transportation, it is not always possible or comfortable to deliver the goods directly to the delivery point. In fact, some transportation systems use intermediary points (warehouses, distribution centers) where some operations (packing, palletizing, etc.) take place. The different means of transport unit that belong to these systems stop at some of these intermediate points, and in some cases the freight changes means of transport unit or even mode of transport. Moreover, some additional services, like palletizing, packaging, labeling, re-packing etc., can be realized at these intermediary points. One of the basic problems in such systems (multi-echelon) is Vehicle Routing Problem (VRP). The VRP is used to design an optimal route for a fleet of vehicles/means of transport units to service a set of customers' orders (known in advance), given a set of different type of constraints. The VRP is the NP-hard type. There are several variants of VRP like VRP with Time Windows (VRPTW), the capacitated VRP (CVRP), and Dynamic Vehicle Routing Problems (DVRP), Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) is a multi-echelon variant of CVRP etc. [2,3,4].

III. HYBRID APPROACH

Based on literature [5,6,7] and previous studies [8,9,10] was observed some advantages and disadvantages of both CLP-based and MP-based approaches. An integrated approach of constraint logic programming (CLP) and mixed linear integer programming/integer linear programming (MILP/ILP) can help to solve optimization problems which was impossible to solve with either of the two methods alone [11,12]. Although Constraint Logic Programming (CLP) and Operations Research (OR) methods like MP have different roots, the links between the two environments have grown stronger in recent years [11]. CLP and MP environments involve decision variables and constraints imposed on them. However, the types of the decision variables and different types of constraints that are used, and the way the constraints are represented, modeled and solved, are quite different in the two environments [10]. MP-based environments are based entirely on linear equations and inequalities, i.e., there are only two types of constraints: linear (linear inequalities or equations) and integrity (stating that the decision variables have to take their values in the binary and integer numbers). In CLP-based environments in addition to linear inequalities and equations, there are various other constraints: disequalities, nonlinear, logic and symbolic such as *cumulative(), ordered(), alldifferent(), sequence(), disjunctive(),* etc. In both MP-based and CLP-based environments, there is a group of constraints that can be solved with ease and a group of constraints that are difficult to solve. The easily solved constraints by MP methods are linear inequalities and equations over rational numbers. Integrity constraints are difficult to solve using MP algorithms such as branch-and-bound, branch-and-cost and cutting plane if the size of the problem is large. In CLP, domain constraints with integers and equations between two variables are easy to solve. The inequalities and general linear constraints (more than two variables), and symbolic constraints are difficult to solve. Taking all the above features of both approaches (MP and CLP), which in many areas complement each other, conducted research on how to integrate them. Several scenarios of their integration have been studied and reported in the literature [11].

Taking into account these studies and experiences with both environments, a hybrid approach has been proposed for modeling and solving multi-echelons problems.

- Main assumptions of the proposed hybrid approach were as follows:
- Integration of CLP and MP environments following the schedule proposed by the author;
- Use of strong points and compensation of weak points in terms of problem modeling and optimization revealed in both environments;
- Problem data representation in the form of sets of facts with a suitable structure based on the relational model [13];
- Introduction of model transformation as a presolving method;
- Substantial reduction of the feasible solution space for the post-transformation models;
- Automatic generation of implementing models and their translation into the MILP/ILP form.

Figure 1 presents the general concept of the hybrid approach implementation as an implementation platform. The hybrid approach comprises several phases: modeling, presolving, generating and solving. It has two inputs and uses the set of facts. Inputs are the set of constraints and the objectives to the reference model of a given problem. Based on them, the primary model of the problem is generated as a CLP model, which is then presolved. The built-in CLP method (constraint propagation [5,7]) and the method of problem transformation designed by the authors [8,9] (Section III.A) are used for this purpose. Presolving procedure results on the transformed model CLP^T. This model is the basis for the automatic generation of the MILP (Mixed Integer Linear Programming) model, which is solved in MP (with the use of an external solver or as a library of CLP).

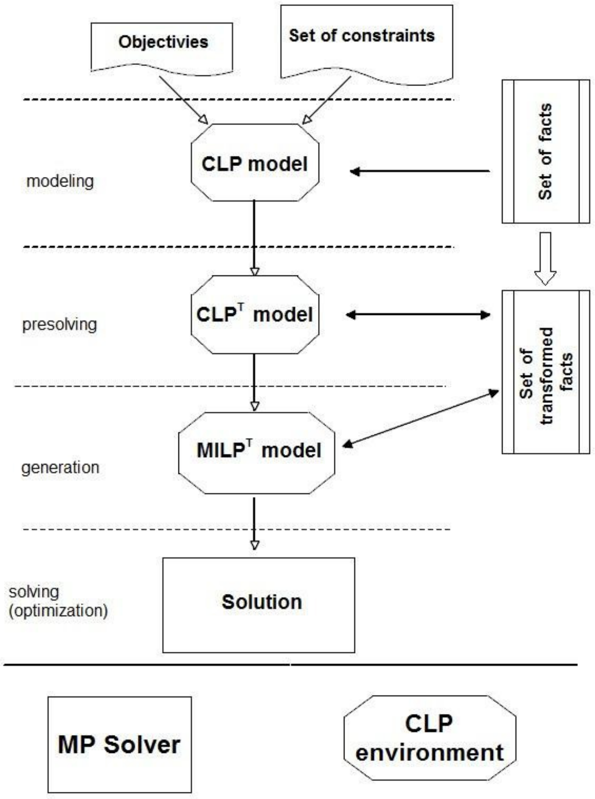

Fig. 1 A concept of a hybrid approach as an implementation platform

The general concept of hybrid approach as an implementation platform consists in modeling and presolving of a problem in the CLP environment with the final solution (optimization) found in the MP environment. In all its phases, the platform uses the set of facts having the structure appropriate for the problem being modeled and solved (see Figure 2 for illustrative problem). The set of facts is the informational layer of the implementation platform, which can be implemented as database, XML files, etc. Description of the facts has been shown in Appendix A

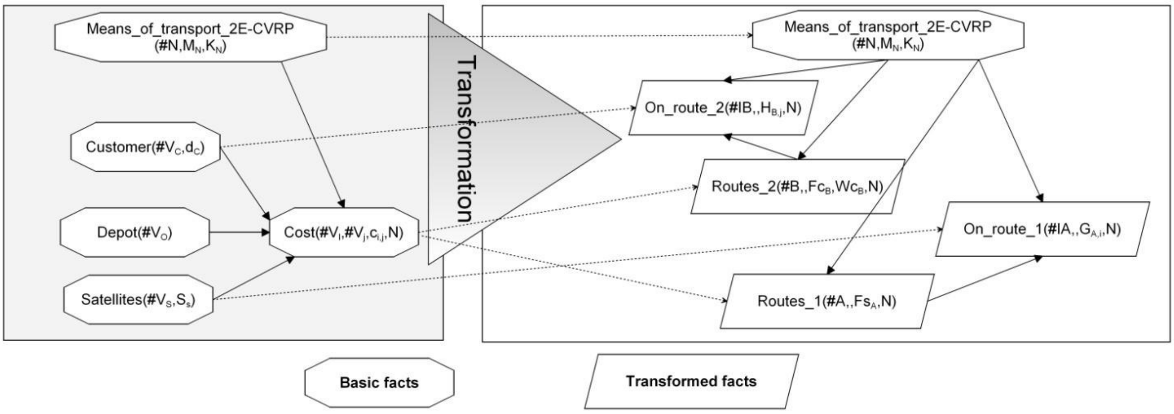

Fig. 2 A structure of facts for illustrative problem (2E-CVRP)

A. *Transformation of the problem-presolving phase*

The presolving phase is an important element of this approach as it makes it possible to simplify the model for the problem being solved and to reduce the problem search space. For the presolving phase to be effective, unfeasible combinations of model dimensions (indicies) have to occur. In practice, unfeasible combinations of the index of decision variables and/or facts occur. The proposed platform uses constraint propagation and transformation for the presolving procedure. Constraint propagation is a concept and method that appears in constrained-based environments. Constraint propagation embeds any reasoning which consists in explicity forbiding values from some varable domain of a problem, because all constraints can not be satisfied otherwise [5,7].

In the case of the ilustrated problem presented, the transformation consisted in changing the problem representation from graph to routing. Instead of analyzing all possible trnasport connections from the source to the intermidate points and then from the intermidate points to the delivery points, only the feasible connections (source-intermidiate point-delivery point) were generated and named routes. This resulted in the removal of certain indices and in the aggregation of other indices for decision variables, parameters, etc., which eventually led to the reduction in the number of decision variables and constraints [8,9,14]. The new set of decision variables, constraints and facts was the basis for creating the CLP^{T}model.

IV. ILLUSTRATIVE EXAMPLE –TWO-ECHELON VEHICLE ROUTING PROBLEM (2E-CVRP)

Possibility of using hybrid approach to modeling and optimization of multi-echelon systems is shown for the illustrative example. A good illustrative example of a multi-echelon system is 2E-CVRP. The Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP) is an extension of the classical Capacitated Vehicle Routing Problem (CVRP) where the delivery source-delivery points pass through intermediate points (called satellites). As in CVRP, the goal is to deliver goods to delivery points (retailers, customers, etc.) with known ordered demands, minimizing the total delivery cost in the fulfillment of vehicle capacity constraints. Multi-echelon systems presented in the literature such as 2E-CVRP usually explicitly consider the routing problem at the last level of the transportation system, while a simplified routing problem is considered at higher levels [4,15].

In 2E-CVRP, the freight delivery from the source (depot) to the delivery points is managed by shipping the freight through intermediate points (satellites). Thus, the transportation network (Figure 3) is decomposed into two levels: the 1st level connecting the source point/depot (d) to intermediate points/satellites (s) and the 2nd one connecting the intermediate points/satellites (s) to the delivery points/customers (c). The objective is to minimize the total transportation cost of the vehicles involved in both levels. Constraints on the maximum capacity of the vehicles and the intermediate points are considered, while the timing of the deliveries is ignored.

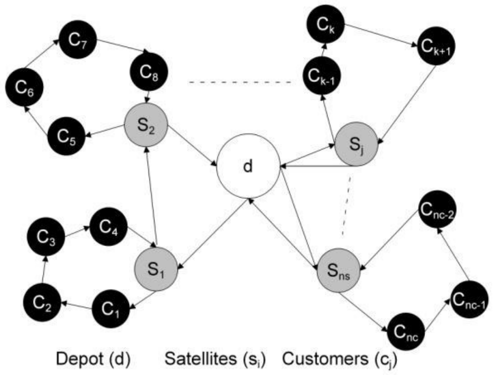

Fig. 3 Sample transportation network for 2E-CVRP

A. *Mathematical model for 2E-CVRP*

The formal mathematical model for 2E-CVRP in the form of MILP was taken from [4]. Table I shows the parameters and decision variables of the model. Figure 3 shows sample transportation network for 2E-CVRP.

TABLE 1

SUMMARY INDICES, PARAMETERS AND DECISION VARIABLES

Symbol	**Description**
Indices	
n_s	Number of intermediate points (warehouses, distribution centers, etc.)
n_c	Number of delivery points (retailers, shops, etc.)
V_0-$[v_0]$	Source (main distribution center)t
$V_s=\{v_{s1}, v_{s2}, \ldots, v_{sn}\}$	Set of intermediate points
$V_c=\{v_{c1}, v_{c2}, \ldots, v_{cn}\}$	Set of delivery points
Parameters	
M_1	Number of the means of transport unit (i.e. vehicles, trucks, etc.) (1st-level)
M_2	Number of the means of transport unit (i.e. vehicles, pick-ups) (2nd-level)
K_1	Capacity of the means of transport unit for the 1st level
K_2	Capacity of the means of transport unit for the 2nd level
d_i	Order quantity by customer *i*
$c_{i,j}$	Time /Cost of the *arc (i,j)*
s_k	Cost of unloading/loading procedure of the means of transport unit in intermediate point *k*
Decision variables	
$X_{i,j}$	An integer decision variable (the 1st-level) routing is equal to the number of means of transport units (1st-level) using *arc (i,j)*
$Y_{k,i,j}$	A binary decision variable (the 2nd-level) routing is equal to 1 if a (2nd-level) means of transport unit makes a route starting from intermediate point *k* and goes from node i to node j and 0 otherwise
$Q^1_{i,j}$	The freight flow *arc (i,j)* for the 1st-level
$Q^2_{k,i,j}$	The freight *arc (i,j)* where *k* represents the intermediate point where the freight is passing through.
$Z_{k,j}$	A binary decision variable that is equal to 1 if the freight to be delivered to delivery point *j* is consolidated in intermediate point *k* and 0 otherwise

$$\min \sum_{i,j\in V_0\cup V_s} (c_{i,j}\cdot X_{i,j}) + \sum_{k\in V_s}\sum_{i,j\in V_s\cup V_c} (c_{i,j}\cdot Y_{k,i,j}) + \sum_{k\in V_s}(s_k\cdot Ds_k) \tag{1}$$

$$\sum_{i\in V_s} X_{0,i} \le M_1 \tag{2}$$

$$\sum_{j\in V_s\cup V_0, j\ne k} X_{j,k} = \sum_{i\in V_s\cup V_0, i\ne k} X_{k,i} \text{ for } k\in V_s\cup V_0 \tag{3}$$

$$\sum_{k\in V_s}\sum_{j\in V_c} Y_{k,k,j} \le M_2 \tag{4}$$

$$\sum_{i\in V_c, j\in V_c} Y_{k,i,j} = \sum_{i\in V_c, j\in V_c} Y_{k,j,i} \text{ for } k\in V_s \tag{5}$$

$$\sum_{i\in V_0\cup V_s, i\ne j} Q^1_{i,j} - \sum_{i\in V_s, i\ne j} Q^1_{j,i} = \begin{cases} Ds_j & j \text{ is not thedeport} \\ \sum_{i\in V_c} -d_i & \text{otherwise} \end{cases} \text{ for } j\in V_s\cup V_0 \tag{6}$$

$$Q^1_{i,j} \le k_1\cdot X_{i,j} \text{ for } i,j\in V_s\cup V_0, i\ne j \tag{7}$$

$$\sum_{i\in V_s\cup V_c, i\ne j} Q^2_{k,i,j} - \sum_{i\in V_c, i\ne j} Q^2_{k,j,i} = \begin{cases} Z_{k,j} d_j & j \text{ is not a satelite} \\ -D_j & \text{otherwise} \end{cases} \text{ for } j\in V_c\cup V_S, k\in V_s \tag{8}$$

$$Q^2_{k,i,j} \le k_2\cdot Y_{k,i,j} \text{ for } i,j\in V_s\cup V_c, i\ne j, k\in V_s \tag{9}$$

$$\sum_{i\in V_s} Q^1_{i,V_0} = 0 \tag{10}$$

$$\sum_{j\in V_c} Q^2_{k,j,k} = 0 \text{ for } k\in V_s \tag{11}$$

$$Y_{k,i,j} \le Z_{k,j} \text{ for } i\in V_s\cup V_c, j\in V_c, k\in V_s \tag{12}$$

$$Y_{k,j,i} \le Z_{k,j} \text{ for } i\in V_s, j\in V_c, k\in V_s \tag{13}$$

$$\sum_{i\in V_s\cup V_c} Y_{k,i,j} = Z_{k,j} \text{ for } k\in V_s, j\in V_c, i\ne k \tag{14}$$

$$\sum_{i\in V_s} Y_{k,j,k} = Z_{k,j} \text{ for } k\in V_s, j\in V_c, i\ne k \tag{15}$$

$$\sum_{i\in V_s} Z_{i,j} = 1 \text{ for } j\in V_c \tag{16}$$

$$Y_{k,i,j} \le \sum_{l\in V_s\cup V_0} X_{k,l} \text{ for } k\in V_s, i,j\in V_c \tag{17}$$

$$Y_{k,i,j}\in\{0,1\}, Z_{k,l}\in\{0,1\} \text{ for } k\in V_s, i,j\in V_s\cup V_c, l\in V_c \tag{18}$$

$$X_{k,j}\in Z^+ \text{ for } k,j\in V_s\cup V_0 \tag{19}$$

$$Q^1_{i,j}\ge 0 \text{ for } i,j\in V_s\cup V_0; Q^2_{k,i,j}\ge 0 \text{ for } i,j\in V_s\cup V_c, k\in V_s \tag{20}$$

$$Ds_k = \sum_{l\in V_c} (d_j\cdot Z_{k,j}) \text{ for } k\in V_s \tag{21}$$

The objective function (1) minimizes the sum of the handling operations and transport costs (according to the individual arcs of the route). Constraint (3) ensures, for $k=V_0$, that each 1st-level route begins and ends at the source point, while when k is a intermediate point, impose the balance of means of transport units entering and leaving that point. Constraint (5) specifies that each 2nd-level route to begin and end to one intermediate point and the balance of means of transport units entering and leaving each delivery point. The number of the routes in the 1-st and 2-nd levels must not exceed the number of mode of transport units for that level, as forced by constraints (2) and (4). The flows balance on each network node is equal to order quantity of this node, except for the source point, where the exit flow is equal to the total order quantity of the delivery points, and for the intermediate points at the 2nd-level, where the flow is equal to the order quantity (unknown) assigned to the intermediate points which ensure constraints (6) and (8). Moreover, constraints (6) and (8) forbid the presence of sub-routes not containing the source or a intermediate point, respectively. In fact, each node receives an amount of flow equal to its order quantity, preventing the presence of sub-routes. The capacity constraints are formulated in (7) and (9), for both levels. Constraints (10) and (11) do not allow residual flows in the routes, making the returning flow of each route to the source (1st-level) and to each intermediate point (2nd-level) equal to 0. Constraints (12) and (13) indicate that delivery point *j* is served by a intermediate point *k* ($Z_{k,j}=1$) only if it receives freight from that intermediate point ($Y_{k,i,j}=1$). Constraint (16) assigns each delivery point to one and only one intermediate point, while constraints (14) and (15) indicate that there is only one 2nd-level route passing through each delivery point and connect the both levels. Constraints (17) allow to start a 2nd-level route from a intermediate point *k* only if a 1st-level route has served it. Constraints from (18) to (20) result from the character of the MP-formulated problem. Constraint (21) determines transsphipment volume for satellite Vs.

B. Mathematical model for 2E-CVRP after transformation

The most important feature that characterize the hybrid approach is the presolving phase. The presolving is usually used to reduce the size of the problem (the number of decision variables and constraints), what results in an increase in the effectiveness of the search for a solution.

In hybrid approach, the main method of presolving is model transformation. In this case the transformation is based on the transition from arc to the route notation (Section III.A). During the transformation the TSP - traveling salesman problem is repeatedly solved and only the best routes in terms of costs are generated. In the process of transformation, the capacity vehicles constraints and those resulting from the set of orders are taken into account at both first and second level. Transformation is also subject to a set of facts describing the problem. The obtained model after the transformation (TC1)..(TC9) has different decision variables (Table II) and different constraints than those in the (1)..(24). Some of the decision variables are redundant; other variables are subject to aggregation. This results in a very large reduction in their number. The transformation also reduces or eliminates some of the constraints of the model.

$$\min\sum_{a=1}^{W}(Z_a\cdot Fs_a)+\sum_{b=1}^{F}U_b\cdot Fc_b \qquad \text{(TC1)}$$

$$\sum_{b=1}^{F}U_b\le M2 \qquad \text{(TC2)}$$

$$\sum_{b=1}^{F}U_b\cdot H_{b,j}=1\,\forall\, j=1..n_c \qquad \text{(TC3)}$$

$$\sum_{b=1}^{F}U_b\cdot H_{b,i}\cdot Wc_b=\sum_{a}^{W}X_a\cdot G_{a,i}\ \forall\, i=1..n_s \qquad \text{(TC4)}$$

$$\sum_{i=1}^{n_s}\sum_{b=1}^{F}U_b\cdot H_{b,i}\cdot Wc_b=\sum_{a}^{W}X_a \qquad \text{(TC5)}$$

$$Z_a\cdot K1\ge Ps_a\ \ \forall\ a=1..W \qquad \text{(TC6)}$$

$$\sum_{a=1}^{W}Z_a\le M1 \qquad \text{(TC7)}$$

$$U_b\in\{0,1\}\ \ \forall\, b=1..F \qquad \text{(TC8)}$$

$$Z_a\in C\ \text{for}\ a=1..W \qquad \text{(TC9)}$$

TABLE II
SUMMARY INDICES, PARAMETERS AND DECISION VARIABLES FOR TRANSFORMED MODEL

Symbol	**Description**
	Indices
n_s	Number of intermediate points (warehouses, distribution centers, etc.)
n_c	Number of delivery points (retailers, shops, etc.)
W	Number of possible routes from source point to intermediate points (determined by CLP during transformation)
F	Number of possible routes from intermediate points to delivery points (determined by CLP during transformation)
i	Intermediate point index
a	Source point-intermediate point route index
j	Dleivery point index
b	Intermediate point –delivery point route index
M_1	Number of the 1st-level means of transport units
M_2	Number of the 2nd-level means of transport units
	Input parameters
Wc_b	Total demand for route *b* (determined by CLP during transformation)
Fs_a	Route *a* cost (determined by CLP during transformation)
Fc_b	Route *b* cost (determined by CLP during transformation)
$G_{a,i}$	If *i* is located on route *a* $G_{a,i}=1$, otherwise $G_{a,i}=0$
$H_{b,j}$	If intrmediate or delivery point *j* is located on route *b* $H_{b,j}=1$, otherwise $H_{b,j}=0$
K_1	Capacity of the means of transport unit for the 1st level
	Decision variables
Z_a	If the tour takes place along the route a from the route set generated for level 1, then Z_a=1, otherwise Z_z=0
U_b	If the tour takes place along the route *b* from the route set generated for level 2, then $U_b=1$, otherwise $U_b=0$
	Computed quantities
X_a	Total demand for route a

V. NUMERICAL EXPERIMENTS

For the validation of the proposed hybrid approach and the implementation platform, benchmark data for 2E-CVRP was selected. The instances for numerical experiments were built from the existing instances for CVRP [16] denoted as E-n13-k4. All the instance sets can be downloaded from the website [17]. The instance set was composed with 1 depot, 12 customers and 2 satellites. The full set of instances consisted of 66 instances because the two satellites were placed over twelve customers in all 66 possible ways (number of combinations: 2 out of 12). Twenty instances were selected for the numerical experiments.

Numerical experiments were conducted for the same data in two runs. The first run was a classical implementation of model (1)..(21) and its solution in the MP-based environment (MP). In the next run the model (1)..(21) was transformed to (TC1)..(TC9) and solved in the proposed hybrid implementation platform (HYBRID). The calculations were performed using a computer with the following specifications: Intel(R) Core(TM) I3-2100, 2x 3,106GHZ RAM 8 GB.

The results are presented in Table III. As seen above, application of the hybrid approach reduced the calculation time needed to find the optimal solution from 3 to more than 50 times, depending on data instance, in relation to mathematical programming. For some examples, mathematical programming did not find the optimal solution in acceptable time.

The final stage of the research was to optimize Two-Echelon Capacitated VRP with Time Windows (2E-CVRP-TW). In literature, this problem is the extension of 2E-CVRP where time windows on the arrival or departure time at the satellites and/or at the customers are considered.

In our case, the time window is interpreted as a non-transient time of transport at the first and second levels (independently). This interpretation of the time window is of great practical significance, i.e. it defines, for example, the maximum working time of the driver (legal regulations), the transport time of the product (freshness), etc. In this case, the hybrid approach not only accelerated the calculations but enabled time windows to be introduced without the need to change the model. During the transformation, only those routes that fulfilled the condition imposed by the time window were accepted. The results obtained for different time window values for the selected data instances are shown in Table IV. In addition, the obtained results are illustrated by diagrams showing selected routes for E-n13-k4-20 instances without time windows and TW = 50 and TW = 60 (Fig. 4, Fig. 5 and Fig.6).

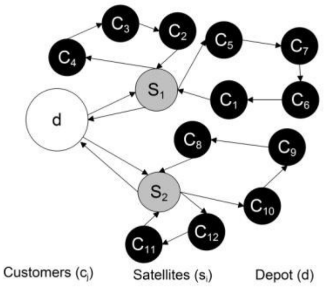

Fig. 4 Transportation routes for instance I-20, Fc=276

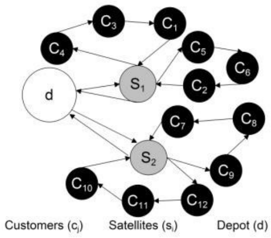

Fig. 5 Transportation routes for instance I -20 with time window TW=50, Fc=294

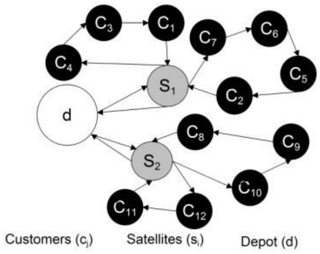

Fig. 6 Transportation routes for instance I -20 with time window TW=60, fc=280

TABLE III
THE RESULTS OF NUMERICAL EXPERIMENTS FOR 2E-CVRP

Instance	MP		HYBRID	
	T	Fc	T	Fc
I-01	600*	280	16	280
I-04	52	218	8	218
I-05	86	218	7	218
I-06	123	230	9	230
I-07	51	224	7	224
I-11	600*	276	11	276
I-13	600*	288	14	288
I-14	54	228	14	228
I-15	69	228	15	228
I-20	487	276	9	276
I-22	600*	312	8	312
I-23	40	242	12	242
I-24	74	242	11	242
I-25	97	252	8	252
I-26	55	248	7	248
I-32	600*	246	9	246
I-33	101	258	7	258
I-40	30	254	9	254
I-46	600*	280	9	280
I-53	120	300	10	300
I-54	600*	304	11	304
I-55	600*	310	11	310
I-56	132	310	15	310
I-57	600*	326	13	326
I-58	600*	326	7	326

*calculations stopped after 600s
instance I= E-n13-k4 (I-01 short for E-n13-k4-01)

As you can see, time windows affect both the optimal value of objective function (Table IV) and the way of distribution (different routes) (Fig.4,Fig.5,Fig.6.)

TABLE IV
THE RESULTS OF NUMERICAL EXAMPLES FOR 2E-CVRP-TW

Instance	40	50	60	70	80	90	100
I-01	-	-	-	-	-	-	280
I-07	-	224	224	224	224	224	224
I-11	-	-	304	276	276	276	276
I-20	-	294	280	276	276	276	276
I-26	-	248	248	248	248	248	248
I-32	-	-	262	246	246	246	246
I -33	-	258	258	258	258	258	258
I-40	-	284	284	254	254	254	254

VI. CONCLUSION

The effectiveness of the proposed hybrid approach results from the reduction of the problem space and using the best properties of both components – MP and CLP. The hybrid method (Table III) makes it possible to find optimal solutions in the shorter time. In addition to solving larger problems faster, the proposed approach provides virtually unlimited modeling options with many types of constraints.

Applying a hybrid approach to this type of problems also allows you to introduce a group of constraints such as different time windows, logic exclusion etc. without having to change the model itself.

Therefore, the proposed hybrid method is recommended for optimization multi-echelon distribution problems that have a structure similar to the illustrative model (Section IV). This structure is characterized by the constraints and objective function in which the decision variables are summed up.

Further work will focus on running the optimization models with non-linear and logical constraints, multi-objective, uncertainty etc. in the hybrid optimization platform. The planed experiments will employ proposed hybrid method for Two-Echelon Capacitated VRP with Satellites Synchronization, 2E-CVRP with Pickup and Deliveries and others VRP issues in logistic issues [18].

In addition, it is envisaged to include in future models the lead times [19,20]. In the course of further work on the hybrid approach, it is planned to use it for modeling and optimization of IoT processes [21].

APPENDIX A

TABLE A1
DESCRIPTION OF FACTS FOR 2E-CVRP

Name	Description
Means_of_transport_2E-CVRP (#N,M_N,K_N)	A fact that describes a particular type of transport with ID #N, including: Information on the number of means of transport on 1-level and 2-level and their capacities.
Customer(#V_C,d_C)	A fact that describes the recipients, including information about their orders.
Depot(#V_o)	A fact that describes the depot.
Satelites(#Vs,S_s)	A fact that describes the satellites.
Cost(#V_i, #V_j,$C_{i,j}$,,N)	A fact describing the distance between points (costs).
Routes_1(#A,Fs_A,N)	A fact describing routes from depot to satellites.
On_route_1(#A,G_A,N)	The fact states which points are on the route_1
Routes_2(#B,Fc_B, $Fc_{B,}$ N)	A fact describing routes from satellites to customers.
On_route_2(#B,H_b,N)	The fact states which points are on the route_2

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