Closed Form PHD Filtering for Linear Jump Markov Models

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Abstract - ***In recent years there has been much interest in the probability hypothesis density (PHD) filtering approach, an attractive alternative to tracking unknown numbers of targets and their states in the presence of data association uncertainty, clutter, noise, and miss-detection. In particular, it has been discovered that the PHD filter has a closed form solution under linear Gaussian assumptions on the target dynamics and birth. This finding opens up a new direction where the PHD filter can be practically implemented in an effective and reliable fashion. However, the previous work is not general enough to handle jump Markov systems (JMS), a popular approach to modeling maneuvering targets. In this paper, a closed form solution for the PHD filter with linear JMS is derived. Our simulations demonstrate that the proposed PHD filtering algorithm provides promising performance. In particular, the algorithm is capable of tracking multiple maneuvering targets that cross each other.***

Keywords: Multi-target tracking, optimal filtering, random sets, linear jump Markov models.

1 Introduction

Tracking a maneuvering target in clutter is a challenging problem and is the subject of numerous works [1, 2, 3, 4]. While a non-maneuvering target motion can be described by a fixed model, a combination of motion models that characterise different maneuvers may be needed to describe the motion of a maneuvering target. This problem is further compounded by detection uncertainty and clutter. Moreover, in a multi-target environment, the number of targets changes due to targets appearing, disappearing, and it is not known which target generated which measurement. Tracking multiple maneuvering targets involves jointly estimating the number of targets and their states at each time step, and is extremely difficult due to noise, clutter and uncertainties in target maneuvers, data association and detection.

The multiple models approach has been proven to be an effective tool for single maneuvering target tracking [5, 2]. In this approach the dynamics of a maneuvering target is modeled as a linear jump Markov system (LJMS), i.e. the target can switch between a set of linear models in a Markovian fashion. The multiple models approach can be combined with joint probabilistic data association (JPDA) [6, 7, 8, 9] or multiple hypothesis tracking (MHT) [10, 11] to track multiple maneuvering targets. However, the combinatorial nature of these data association-based approaches dictates an exponential increase in complexity [12, 1, 13, 14]. Although heuristic techniques can be used to reduce the computational load, the resulting algorithms are still computationally intensive in general.

In this paper we propose an efficient method for tracking multiple maneuvering targets in clutter using Mahler's Probability Hypothesis Density (PHD) filter [15]. The PHD filter is attractive in that it circumvents the combinatorial computations that arise from data association and accommodates detection uncertainty, Poisson false alarms, target motions, births, spawnings, and deaths. In [16], [17], the PHD filter was applied to track multiple maneuvering targets using sequential Monte Carlo (SMC) implementations [18, 19]. However, the main drawbacks of the SMC approach are the large number of particles and the unreliability of clustering for extracting multi-target state estimates [19, 20, 21]. Recently, a closed form solution to the PHD recursion has been found for linear Gaussian models that led to the development of the Gaussian mixture PHD filter [20, 21]. Although this approach is efficient and capable of handling nonlinear models, it is not general enough for addressing targets with multiple model dynamics. In this work, we derive a closed form solution to the PHD recursion for LJMS. Based on this, a multi-target filter capable of tracking targets that switch between multiple models is developed. Our result is a generalization of the Gaussian mixture PHD filter of [20, 21] to a broader class of practical multi-target models. Simulation results are presented to demonstrate the capability of the proposed approach.

The paper is structured as follows: Section 2 presents some background on JMS for modeling a maneuvering target and the PHD filter. In section 3 we describe the JMS multi-target model for the PHD filter and give the main result of this paper, a closed-form solution to the PHD recursion for LJMS. In Section 4 we demonstrate the capability of the proposed algorithm through simulations. This is followed by concluding remarks in Section 5.

2 Background

We begin with a review of JMS and introduce the class of LJMS for modeling maneuvering targets in Section 2.1. Using the RFS representations for multi-target states and sensor measurements, our problem is posed as a Bayesian filtering problem. The PHD filter is described in Section 2.2.

2.1 Jump Markov System (JMS)

A jump Markov system (JMS) refers to a system that can be described by a set of parameterised (state space) models whose underlying parameters evolve with time according to a finite state Markov chain. Let $\mathcal{M}$ denote the (discrete) set of all model labels. Let $\xi_k \in \mathbb{R}^n$ denote the kinematic state (e.g., the target coordinate and velocity of a maneuvering target), $z_k \in \mathbb{R}^{n_z}$ denote the observation, and $r_k \in \mathcal{M}$ denote the model in effect, at time k. In a JMS, the probability of a transition from model r_{k-1} at time $k-1$ to r_k at time k is given by

$$t_{k|k-1}(r_k|r_{k-1}). \tag{1}$$

Moreover, given a model r_k, the state transition density and the likelihood are given by

$$\tilde{f}_{k|k-1}(\xi_k|\xi_{k-1}, r_k), \tag{2}$$
$$g_k(z_k|\xi_k, r_k). \tag{3}$$

A linear JMS (LJMS) is a JMS with linear models, i.e. conditioned on a model with index r_k the state dynamics and observations are given by linear Gaussians of the form

$$\tilde{f}_{k|k-1}(\xi_k|\xi_{k-1}, r_k) = \mathcal{N}(\xi_k; F_{k-1}(r_k)\xi_{k-1}, Q_k(r_k)) \tag{4}$$
$$g_k(z_k|\xi_k, r_k) = \mathcal{N}(z_k; H_k(r_k)\xi_k, R_k(r_k)). \tag{5}$$

where $\mathcal{N}(\cdot; m, Q)$ denotes a Gaussian density with mean m and covariance Q, $F_{k-1}(r_k)$, and $H_k(r_k)$ denote the transition and observation matrices of model r_k, $Q_k(r_k)$ and $R_k(r_k)$ denote covariance matrices of the process noise and measurement noise. Such a system finds a range of applications in signal processing and provides a natural means to model a maneuvering target whose behavior cannot be characterised at all times by a single model [5, 3, 4].

The problem of tracking a single maneuvering target is to estimate the kinematic state ξ_k at time k, from a sequence of observations $z_{1:k}$ up to time k. Interested readers are referred to [3, 4] for a comprehensive survey of techniques for tracking maneuvering targets. The JMS (or multiple models) approach has been shown to be highly effective for maneuvering target tracking [5, 2].

2.2 The PHD Filter

In a multi-target scenario, each target maneuvers and generates observation according to (1)-(3). In addition, new targets can appear and existing targets can disappear in a random fashion. At the sensor, some target generated measurements may be undetected occasionally. Moreover these detected measurements are indistinguishable from the spurious measurements that the sensor receives. Multi-target tracking involves jointly estimating the time-varying number of states and the values of the states, given all observations up to the current time. This is a fundamentally difficult problem because in addition to the target maneuvers, the number of targets and the number of measurements both vary randomly in time and it is not known which target generated which measurement.

By concatenating the kinematic vector and the model label to form the augmented state vector $x_k = [\xi_k^T, r_k]^T \in \mathcal{X} \subseteq \mathbb{R}^n \times \mathcal{M}$, we can formulate the multi-target tracking problem in the augmented state. Let $x_{k,1}, \ldots, x_{k,N(k)}$ be the augmented states and $z_{k,1}, \ldots, z_{k,M(k)}$ be the measurements at time k, where $N(k)$ and $M(k)$ are the number of targets and measurements, respectively. Then the *multi-target state* X_k and *multi-target observation* Z_k, at time k, are

$$X_k = \{x_{k,1}, \ldots, x_{k,N(k)}\} \subset \mathcal{X}, \tag{6}$$
$$Z_k = \{z_{k,1}, \ldots, z_{k,M(k)}\} \subset \mathbb{R}^{n_z}. \tag{7}$$

Mahler's finite set statistics (FISST) approach provides an elegant Bayesian formulation of the multi-target tracking problem by using random finite sets to model the multi-target states and multi-target observations [15]. However, the Bayes multi-target filter is computationally intractable and intelligent approximations are necessary. The PHD filter [15] is a first order approximation to the multi-target Bayes filter, which propagates the first moment of the RFS representing the multi-target state.

Before proceeding to describe the PHD filter, let us recapitulate the notion of a RFS. A random finite set RFS X on a state space $\mathcal{X}$ is a finite set-valued random variable whose probability law can be specified by a discrete distribution and a family of joint distributions [22]. The discrete distribution characterises the cardinality of X while each of the joint distributions characterises the elements of X for a given cardinality. The first moment (also called the PHD or intensity function) of X, is a non-negative function v on $\mathcal{X}$ with the property that for any closed subset $\mathcal{S} \subseteq \mathcal{X}$

$$\mathbb{E}\left[|X \cap \mathcal{S}|\right] = \int_{\mathcal{S}} v(x)dx$$

where $|X|$ denotes the number of elements of X. In other words, for a given point x, the intensity $v(x)$ is the instantaneous expected number of targets per unit volume at x. The local maxima of v correspond to points in $\mathcal{X}$ with the highest local concentration of expected number of elements, which can be used to generate multi-target state estimates. An RFS X is *Poisson* if the cardinality of X is Poisson with mean $N = \int v(x)dx$ and given a cardinality the elements of X are i.i.d according to v/N. Thus, a Poisson RFS is completely charactertised by its intensity.

The PHD filter is based on the following assumptions:

A. 1 *Each target evolves and generates measurements independently of one another.*

A. 2 *The clutter RFS is Poisson and independent of the measurements.*

A. 3 *The predicted multi-target RFS is Poisson.*

Assumptions A.1 and A.2 are quite common in most multi-target tracking applications [1, 13]. Assumption A.3 is a simplification needed to derive the PHD update.

Let $v_{k|k-1}$ and v_k denote the predicted intensity and posterior intensity at time k, respectively. The PHD recursion consists of a prediction step and an update step. The *PHD prediction* is given by

$$v_{k|k-1}(x) = \int \big[p_{S,k}(x')f_{k|k-1}(x|x') + \beta_{k|k-1}(x|x')\big]v_{k-1}(x')dx' + \gamma_k(x), \quad (8)$$

where $f_{k|k-1}(x|x')$ is the density of a transition from x' at time $k-1$ to x at time k, $p_{S,k}(x')$ is the probability that a target continues to exist at time k given that its previous state is x', $\beta_{k|k-1}(\cdot|x')$ is the intensity of the RFS of target spawned at time k, by a target with previous state x', and $\gamma_k(\cdot)$ is the intensity of the birth RFS. It is understood that an integral with respect to a discrete variable means a sum. Let Z_k denote the multi-target measurement received at time k. The *PHD update* is given by

$$v_k(x) = \big[1 - p_{D,k}(x)\big]v_{k|k-1}(x) + \sum_{z\in Z_k} \frac{p_{D,k}(x)g_k(z|x)v_{k|k-1}(x)}{\kappa_k(z) + \int p_{D,k}(x)g_k(z|x)v_{k|k-1}(x)dx}, \quad (9)$$

where $g_k(z|x)$ is the likelihood of a measurement z given a state x at time k, $p_{D,k}(x)$ is the probability of detecting a target with state x at time k, and $\kappa_k(\cdot)$ is the intensity of the (Poisson) clutter RFS.

Notice that the PHD filter avoids any data association computations and operates exclusively on the single target state space $\mathcal{X}$. The PHD recursion is still generally intractable due to the 'curse of dimensionality' in numerical integration.

A generic sequential Monte Carlo (SMC) implementation was proposed in [18, 19], and relevant convergence results were established in [19, 23, 24]. This so-called particle-PHD filter has been used to track multiple maneuvering targets in [16]. In [17], the authors argued that the particle-PHD filter does not provide a mechanism for handing changes in the target motion model, and went on to implement a multiple model particle-PHD filter. It turns out that the multiple model particle-PHD filter is a special case of the particle-PHD filter where the state vector is a hybrid of continuous and discrete components. The recently proposed Gaussian mixture PHD filter [20, 21] is much more efficient than the particle-PHD filter. However, it is not general enough to handle LJMS models. In the following section, we derive a closed form solution to the PHD recursion for LJMS and develop an efficient and reliable multi-target filter for tracking maneuvering targets.

3 The PHD Filter for LJMS

In this section we show that the closed-form PHD solution proposed in [20, 21] can be extended to the class of LJMS. We describe the JMS multi-target model and specify the assumptions on the multi-target submodels needed to derive the closed-form PHD solution in Section 3.1 and then go on to show the closed-form solution to the PHD recursion (8)-(9) in Section 3.2.

3.1 LJMS multi-target model

For a JMS (not necessarily linear) we have the following stochastic state space model

$$f_{k|k-1}(\xi, r|\xi', r') = \tilde{f}_{k|k-1}(\xi|\xi', r)t_{k|k-1}(r|r'), \quad (10)$$
$$g_k(z|x) = g_k(z|\xi, r). \quad (11)$$

where ξ' and r' denote the kinematic state and model label respectively at time $k-1$. Moreover, the birth and spawn processes are modeled by Poisson RFSs, the intensities of which are given by

$$\gamma_k(\xi, r) = \pi_k(r|\xi)\tilde{\gamma}_k(\xi), \quad (12)$$
$$\beta_{k|k-1}(\xi, r|\xi', r') = \pi_{k|k-1}(r|\xi, \xi', r')\tilde{\beta}_{k|k-1}(\xi|\xi', r'), \quad (13)$$

where $\tilde{\gamma}_k$ is the intensity of the kinematic state births at time k, $\pi_k(\cdot|\xi)$ is the probability distribution of the models for a given birth with kinematic state ξ at time k, $\tilde{\beta}_{k|k-1}(\cdot|\xi', r')$ is the intensity of the kinematic states spawned at time k from $[\xi'^T, r']^T$ and $\pi_{k|k-1}(\cdot|\xi, \xi', r')$ is the probability distribution of the models for a given kinematic state ξ, spawned at time k from $[\xi'^T, r']^T$. It can be shown from Campbell's theorem that (12) and (13) are indeed intensities of RFSs in the hybrid space $\mathbb{R}^n \times \mathcal{M}$.

In line with the LJMS assumption of state independent model transition probability $t_{k|k-1}$, the LJMS multi-target model assumes that,

$$\gamma_k(\xi, r) = \pi_k(r)\tilde{\gamma}_k(\xi), \quad (14)$$
$$\beta_{k|k-1}(\xi, r|\xi', r') = \pi_{k|k-1}(r|r')\tilde{\beta}_{k|k-1}(\xi|\xi', r'), \quad (15)$$

In our closed-form PHD recursion to be presented, we assume linear JMS.

A. 4 *Conditioned on the model label, the target dynamic are linear and Gaussian:*

$$f_{k|k-1}(x|x') = \mathcal{N}(\xi; F_{k-1}(r)\xi', Q_k(r))t_{k|k-1}(r|r'), \quad (16)$$
$$g_k(z|x) = \mathcal{N}(z; H_k(r)\xi, R_k(r)). \quad (17)$$

In addition some assumptions on the target birth, death and detection are made:

A. 5 *The probabilities of target survival and target detection are independent of the kinematic state:*

$$p_{S,k}(\xi', r') = p_{S,k}(r'), \tag{18}$$
$$p_{D,k}(\xi, r) = p_{D,k}(r). \tag{19}$$

A. 6 *The intensities of birth and spawn RFS can be expressed as Gaussian mixtures of the form:*

$$\gamma_k(\xi, r) = \pi_k(r) \sum_{i=1}^{J_{\gamma,k}(r)} w_{\gamma,k}^{(i)}(r)\mathcal{N}\big(\xi; m_{\gamma,k}^{(i)}(r), P_{\gamma,k}^{(i)}(r)\big), \tag{20}$$

$$\beta_{k|k-1}(\xi,r|\xi', r') = \pi_{k|k-1}(r|r') \sum_{j=1}^{J_{\beta,k|k-1}(r,r')} w_{\beta,k|k-1}^{(j)}(r,r') \cdot \mathcal{N}\big(\xi; F_{\beta,k-1}^{(j)}(r, r')\xi' + d_{\beta,k-1}^{(j)}(r, r'), Q_{\beta,k-1}^{(j)}(r, r')\big), \tag{21}$$

where $J_{\gamma,k}(r), w_{\gamma,k}^{(i)}(r), m_{\gamma,k}^{(i)}(r)$, and $Q_{\gamma,k}^{(i)}(r)$, $\forall\ i = 1, 2, \ldots, J_{\gamma,k}(r)$ are given model parameters that characterise the birth intensity $\tilde{\gamma}_k(\xi|r)$. Similarly, $J_{\beta,k|k-1}(r, r'), w_{\beta,k|k-1}^{(j)}(r, r'), F_{\beta,k-1}^{(j)}(r, r'), d_{\beta,k-1}^{(j)}(r, r')$, and $Q_{\beta,k-1}^{(j)}(r, r')$, $\forall\ j = 1, 2, \ldots, J_{\beta,k|k-1}(r, r')$ characterise the spawning intensity $\tilde{\beta}_{k|k-1}(\xi|r, \xi', r')$.

Assumptions A.4 and A.5 follow from standard assumptions of target tracking algorithms for computational tractability (see for example [1, 13, 12]). Some remarks regarding assumption A.6 can be found in [20, 21].

3.2 Closed-form PHD Recursion

Proposition 1 *Suppose that Assumptions A.4-A.6 holds, and that the posterior intensity $v_{k-1}(\xi', r')$ at time $k-1$ is a Gaussian mixture for each r', i.e.*

$$v_{k-1}(\xi', r') = \sum_{i=1}^{J_{k-1}(r')} w_{k-1}^{(i)}(r')\mathcal{N}(\xi'; m_{k-1}^{(i)}(r'), P_{k-1}^{(i)}(r')). \tag{22}$$

Then the predicted intensity $v_{k|k-1}(\xi, r)$ is also a Gaussian mixture for each r given by

$$v_{k|k-1}(\xi,r) = \sum_{r'} \sum_{i=1}^{J_{k-1}(r')} w_{k-1}^{(i)}(r')\Big[p_{S,k}(r')t_{k|k-1}(r|r') \cdot \mathcal{N}\big(\xi; m_{k|k-1}^{(i)}(r,r'), P_{k|k-1}^{(i)}(r,r')\big) + \sum_{j=1}^{J_{\beta,k|k-1}(r,r')} w_{\beta,k|k-1}^{(j)}(r,r') \cdot \pi_{k|k-1}(r|r')\mathcal{N}\big(\xi; m_{k|k-1}^{(i,j)}(r,r'), P_{k|k-1}^{(i,j)}(r,r')\big)\Big] + \gamma_k(\xi, r) \tag{23}$$

where $\gamma_k(\xi, r)$ is given in (20),

$$m_{k|k-1}^{(i)}(r,r') = F_{k-1}(r)m_{k-1}^{(i)}(r'), \tag{24}$$
$$P_{k|k-1}^{(i)}(r,r') = Q_{k-1}(r) + F_{k-1}(r)P_{k-1}^{(i)}(r')F_{k-1}^T(r), \tag{25}$$

and

$$m_{k|k-1}^{(i,j)}(r, r') = F_{\beta,k-1}^{(j)}(r, r')m_{k-1}^{(i)}(r') + d_{\beta,k-1}^{(j)}(r,r'), \tag{26}$$
$$P_{k|k-1}^{(i,j)}(r, r') = Q_{\beta,k-1}^{(j)}(r, r') + F_{\beta,k-1}^{(j)}(r, r')P_{k-1}^{(i)}(r')\Big(F_{\beta,k-1}^{(j)}(r, r')\Big)^T. \tag{27}$$

Proposition 2 *Suppose that Assumptions A.4-A.6 holds, and that the predicted intensity $v_{k|k-1}(\xi, r)$ is a Gaussian mixture for each r, i.e.*

$$v_{k|k-1}(\xi, r) = \sum_{i=1}^{J_{k|k-1}(r)} w_{k|k-1}^{(i)}(r)\mathcal{N}(\xi; m_{k|k-1}^{(i)}(r), P_{k|k-1}^{(i)}(r)). \tag{28}$$

Then the posterior intensity $v_k(\xi, r)$ is also a Gaussian mixture for each r given by

$$v_k(\xi, r) = \big[1 - p_{D,k}(r)\big] v_{k|k-1}(\xi, r) + \sum_{z\in Z_k} v_{D,k}(x, z) \tag{29}$$

where

$$v_{D,k}(x, z) = \sum_{j=1}^{J_{k|k-1}(r)} w_k^{(j)}(z)\mathcal{N}\big(\xi; m_{k|k}^{(j)}(r, z), P_{k|k}^{(j)}(r)\big), \tag{30}$$

with

$$w_k^{(j)}(z) = \frac{p_{D,k}(r)w_{k|k-1}^{(j)}(r)q_k^{(j)}(z, r)}{\kappa_k(z) + \sum_{r'} p_{D,k}(r')\sum_{i=1}^{J_{k|k-1}(r')} \psi_k^{(i)}(z)}, \quad \psi_k^{(i)}(z) = w_{k|k-1}^{(i)}(r')q_k^{(i)}(z, r'), \tag{31}$$

and

$$q_k^{(j)}(z, r) = \mathcal{N}(z; H_k(r)m_{k|k-1}^{(j)}(r), R_k(r) + H_k(r)P_{k|k-1}^{(j)}(r)H_k^T(r)), \tag{32}$$
$$m_{k|k}^{(i)}(z, r) = m_{k|k-1}^{(i)}(r) + K_k^{(i)}(r)(z - H_k(r)m_{k|k-1}^{(i)}(r)), \tag{33}$$
$$P_{k|k}^{(i)}(r) = [\ I - K_k^{(i)}(r)H_k(r)\ ]P_{k|k-1}^{(i)}(r), \tag{34}$$
$$K_k^{(i)}(r) = P_{k|k-1}^{(i)}(r)H_k^T(r) \cdot (H_k(r)P_{k|k-1}^{(i)}(r)H_k^T(r) + R_k(r))^{-1}. \tag{35}$$

Propositions 1 and 2 are established by applying standard results for Gaussian functions (e.g., Lemma 1 and Lemma 2 in [20, 21]). Substituting (16), (18), (20), (21) and (22) into PHD prediction (8) and applying Lemma 1 [20, 21], we obtain (23). Substituting (17), (19) and (28) into PHD update (9) and applying Lemma 1 and Lemma 2 [20, 21], (29) is obtained.

Propositions 1 and 2 show closed-form expressions for recursive computation of means, variances and weights of $v_{k|k-1}$ and v_k. Assuming the initial prior intensity v_0 is a Gaussian mixture it can be inferred by induction that the subsequent predicted and posterior intensities are also Gaussian mixtures. Propositions 1 and 2 also indicate that the number of Gaussian components of the predicted and posterior intensity increases with time. This poses a problem in implementation, but it has been found [20, 21] that this issue can be effectively handled by applying some simple pruning procedure.

4 Simulation Results

In this section we test the performance of the proposed PHD filter for LJMS in tracking an unknown and time-varying number of maneuvering targets in clutter. In particular, Example 2 investigates the robustness of the proposed filter in a scenario where the paths of two targets cross repeatedly in time. For illustration purposes we consider only two-dimensional scenarios.

Example 1

Consider the case where targets can appear in a surveillance region at different locations and times. The targets are observed in a square region $[-10000, 10000] \times [-10000, 10000]\, m^2$ by a sensor located at $(0, 0)\, m$. The kinematic state $\xi = [\ p_x,\ \dot{p}_x,\ p_y,\ \dot{p}_y\ ]^T$ of each target consists of position (p_x, p_y) and velocity $(\dot{p}_x, \dot{p}_y)$. The measurements, sampled at a period of $T = 60\, s$, comprise of position only and follow the observation model with observation matrix and measurement noise covariance matrix given by

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad R_k = \sigma_\epsilon^2 I_2 \tag{36}$$

where I_n denotes a $n \times n$ identity matrix, and $\sigma_\epsilon = 10\, m$ denotes the standard deviation of measurement noise. Targets are detected with a probability $p_{D,k}(r) = 0.98$ and have a probability of survival $p_{S,k}(r') = 0.99$. Clutter is modeled as a Poisson RFS with intensity

$$\kappa_k(z) = \lambda_c V \mathcal{U}(z) \tag{37}$$

where $\mathcal{U}(\cdot)$ denotes a uniform density over the surveillance region, $V = 4 \times 10^8\, m^2$ is the *volume* of the surveillance region and $\lambda_c = 1.25 \times 10^{-7}\, m^{-2}$ denotes the average number of clutter returns per unit volume.

The motion models are described as follows. Model $r = 1$ is the constant velocity (CV) model. The single target state transition and process noise covariance matrices that characterise the target dynamics are given by

$$F_{k-1}(r=1) = \begin{bmatrix} A_1 & 0_2 \\ 0_2 & A_1 \end{bmatrix}, \ A_1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{38}$$

$$Q_k(r=1) = \sigma_{1,v}^2 \begin{bmatrix} \Sigma_1 & 0_2 \\ 0_2 & \Sigma_1 \end{bmatrix}, \Sigma_1 = \begin{bmatrix} \frac{T^4}{4} & \frac{T^3}{2} \\ \frac{T^3}{2} & T^2 \end{bmatrix} \tag{39}$$

where 0_n denotes a $n \times n$ zero matrix, and $\sigma_{1,v} = 3 \times 10^{-3}\, m\, s^{-2}$ denotes the standard deviation of process noise.

Model $r = 2$ is a constant turn (CT) model [2, 25] with a counterclockwise turn at 9 degree per minute. The single target state transition and process noise covariance matrices that characterize the target dynamics are given by

$$F_{k-1}(r=2) = \begin{bmatrix} A_2 & -\tilde{A}_2 \\ \tilde{A}_2 & A_2 \end{bmatrix} \tag{40}$$

with

$$A_2 = \begin{bmatrix} 1 & \frac{\sin \omega T}{\omega} \\ 0 & \cos \omega T \end{bmatrix}, \ \tilde{A}_2 = \begin{bmatrix} 0 & \frac{1-\cos \omega T}{\omega} \\ 0 & \sin \omega T \end{bmatrix}$$

and

$$Q_k(r=2) = \sigma_{2,v}^2 \begin{bmatrix} \Sigma_2 & -\tilde{\Sigma}_2 \\ \tilde{\Sigma}_2 & \Sigma_2 \end{bmatrix} \tag{41}$$

with

$$\Sigma_2 = \begin{bmatrix} \frac{2(\omega T - \sin \omega T)}{\omega^3} & \frac{1-\cos \omega T}{\omega^2} \\ \frac{1-\cos \omega T}{\omega^2} & T \end{bmatrix},$$

$$\tilde{\Sigma}_2 = \begin{bmatrix} 0 & -\frac{\omega T - \sin \omega T}{\omega^2} \\ \frac{\omega T - \sin \omega T}{\omega^2} & 0 \end{bmatrix}$$

where ω denotes the turn rate and $\sigma_{2,v} = 2 \times 10^{-2}\, m\, s^{-2}$ denotes standard deviation of the process noise.

Model $r = 3$ is also a constant turn (CT) model but with a clockwise turn at 9 degree per minute. The Markovian transition probability matrix is taken as

$$[t_{k|k-1}(r|r')] = \begin{bmatrix} 0.95 & 0.025 & 0.025 \\ 0.15 & 0.85 & 0 \\ 0.15 & 0 & 0.85 \end{bmatrix} \tag{42}$$

The spontaneous birth RFS is Poisson with intensity

$$\gamma_k(\xi, r) = 0.1 \pi_k(r) [\mathcal{N}(\xi; m_\gamma^{(1)}, P_\gamma) + \mathcal{N}(\xi; m_\gamma^{(2)}, P_\gamma) + \mathcal{N}(\xi; m_\gamma^{(3)}, P_\gamma)] \tag{43}$$

with

$$m_\gamma^{(1)} = [\ -7000,\ \ 3,\ \ 4000,\ \ 2\ ]^T \tag{44}$$
$$m_\gamma^{(2)} = [\ 6200,\ \ -1,\ \ 6000,\ \ -4\ ]^T \tag{45}$$
$$m_\gamma^{(3)} = [\ -6000,\ \ 1.5,\ \ -6000,\ \ 4\ ]^T \tag{46}$$
$$P_\gamma = diag\Big([\ 100,\ \ 25,\ \ 100,\ \ 25\ ] \Big) \tag{47}$$

and $\pi_k(r) = 0.8$ for $r = 1$ and $\pi_k(r) = 0.1$ for $r = 2$ and $r = 3$.

The RFS of targets spawned from a target with a previous state $[\xi'^T, r']^T$ is Poisson with intensity

$$\beta_{k|k-1}(\xi, r|\xi', r') = 0.05 \pi_{k|k-1}(r|r') \mathcal{N}(\xi; \xi', Q_\beta) \tag{48}$$

$$Q_\beta = diag\Big([100,\ \ 250,\ \ 100,\ \ 250] \Big) \tag{49}$$

and

$$[\pi_{k|k-1}(r|r')] = \begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.2 & 0.8 & 0 \\ 0.2 & 0 & 0.8 \end{bmatrix} \tag{50}$$

Fig. 1 shows the true target trajectories in the x-y plane. A 1-D view of these trajectories along each axis with cluttered measurements plotted against time is shown in Fig. 2. As indicated in Figs. 3 and 4 which show the position and velocity estimates respectively of the PHD filter against time, the filter provides accurate tracking performance in clutter. Note that we apply the same pruning procedures and parameters as in [20, 21] in our implementation.

The mean absolute error in the number of targets and the average probability of track lost (see [21] for a definition of these measures), estimated from 1×10^3 Monte Carlo runs, are shown in Fig. 5. The mean absolute error function peaks at the times of target births. The sharp peaks indicate that a new target appeared, and that the filter immediately detects the new targets. In the interval $[67, 75]$ the increased uncertainty due to the proximity of the targets as Target 4 approaches Target 1 on the turn and then crosses Target 5 causes the error to exceeds 0.2.

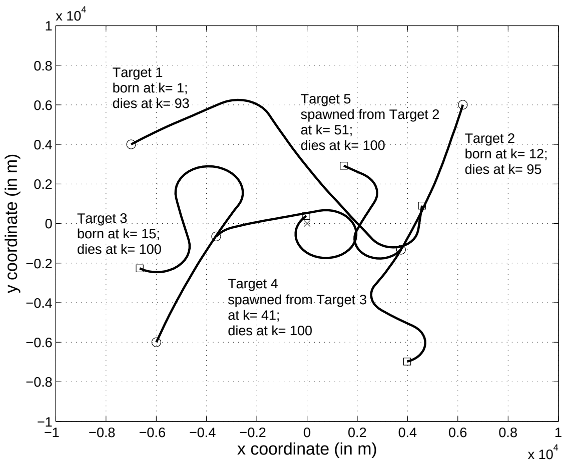

Figure 1: Target trajectories. '○'– locations of target births; '□'– locations of target deaths ('×'– location of sensor).

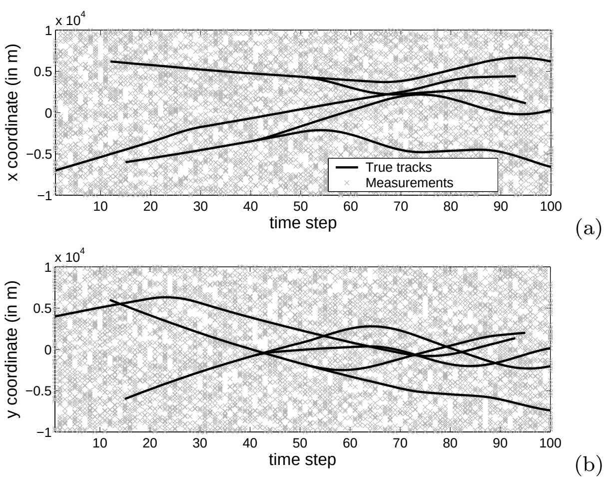

Figure 2: Measurement data and true target positions.

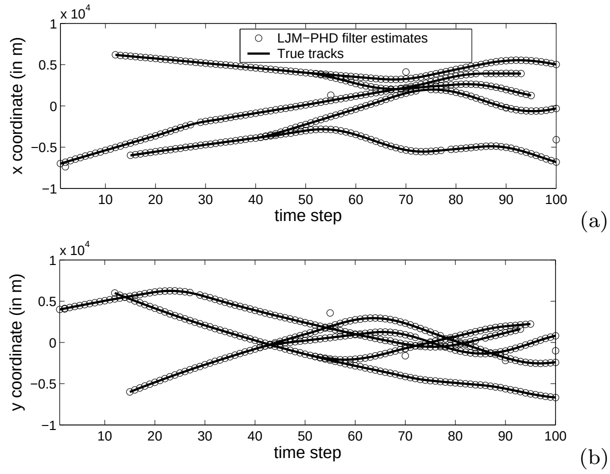

Figure 3: Position estimates of the Gaussian mixture LJM-PHD filter.

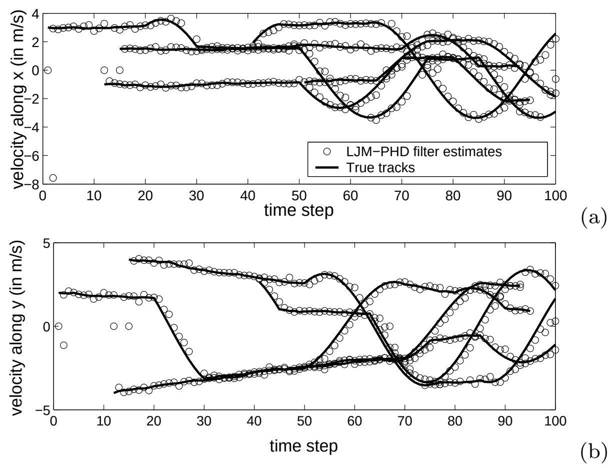

Figure 4: Velocity estimates of the Gaussian mixture LJM-PHD filter.

Example 2

In this example we examine a situation where two targets cross paths repeatedly. The simulation settings are the same as those of the previous example, except that the birth intensity is given by

$$\gamma_k(\xi, r) = 0.1\pi_k(r)\big[\mathcal{N}(\xi; m_\gamma^{(1)}, P_\gamma) + \mathcal{N}(\xi; m_\gamma^{(2)}, P_\gamma) + \mathcal{N}(\xi; m_\gamma^{(3)}, P_\gamma)\big]$$

with

$$m_\gamma^{(1)} = \left[\ 0,\quad 1,\quad 8000,\quad -3\ \right]^T \tag{51}$$

$$m_\gamma^{(2)} = \left[\ 5000,\quad -3,\quad 0,\quad 0\ \right]^T \tag{52}$$

$$m_\gamma^{(3)} = \left[\ -4000,\quad 2.5,\quad -6000,\quad 3\ \right]^T \tag{53}$$

$$P_\gamma = diag\Big(\left[\ 100,\quad 25,\quad 100,\quad 25\ \right]\Big) \tag{54}$$

Fig. 6 shows the true target trajectories. The position and velocity estimates of the PHD filter are shown in Figs. 7 and 8. Although as shown in Fig. 9 the absolute error in the number of targets is slightly higher in this situation, the PHD filter provides accurate position estimates at most times without losing track.

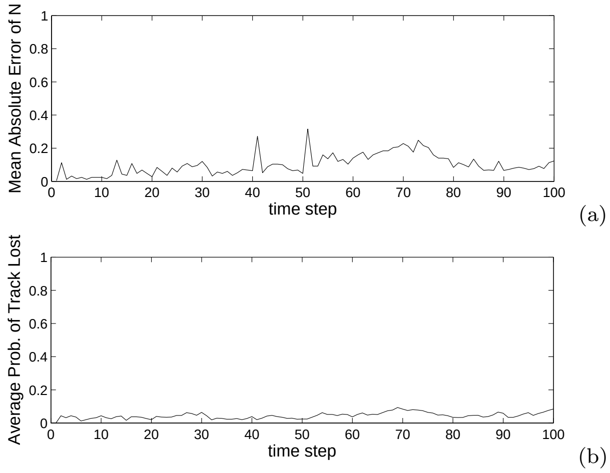

Figure 5: Mean absolute error of N and average probability of track lost.

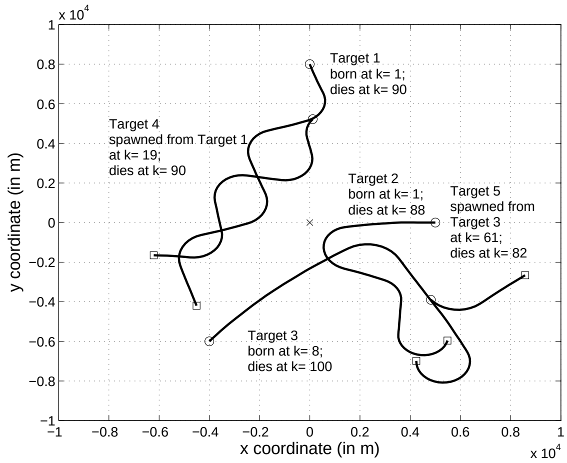

Figure 6: Target trajectories. '○'– locations of target births; '□'– locations of target deaths ('×'– location of sensor).

5 Conclusions

This paper presents a closed-form solution to the PHD recursion for LJMS. We have shown that for an initial prior intensity of Gaussian mixture form, the posterior intensity propagates in time with a similar form. Closed-form recursions for the weights, means and covariances of the Gaussian components modeling the posterior intensity are derived. We show that by incorporating the motion model index in the PHD filtering framework the filter can account for the maneuvers executed by each target. Simulation results demonstrate that the PHD filter is a promising candidate for multi-target tracking with an unknown number of maneuvering targets.

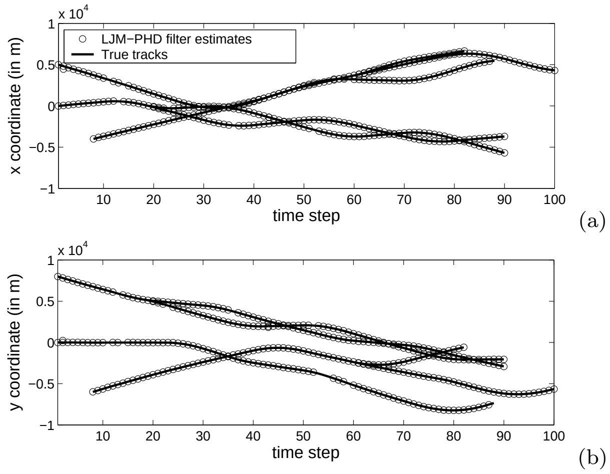

Figure 7: Position estimates of the Gaussian mixture LJM-PHD filter.

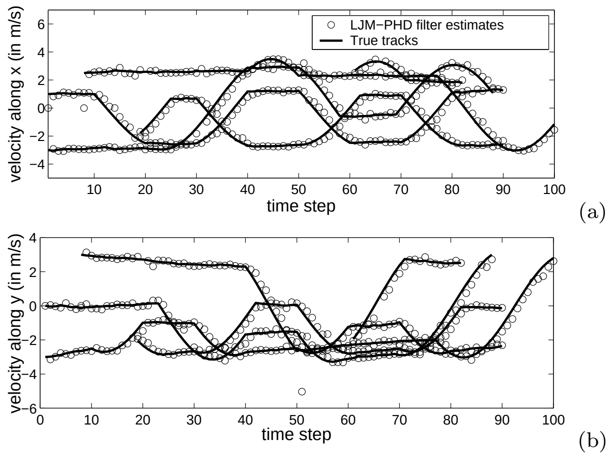

Figure 8: Velocity estimates of the Gaussian mixture LJM-PHD filter.

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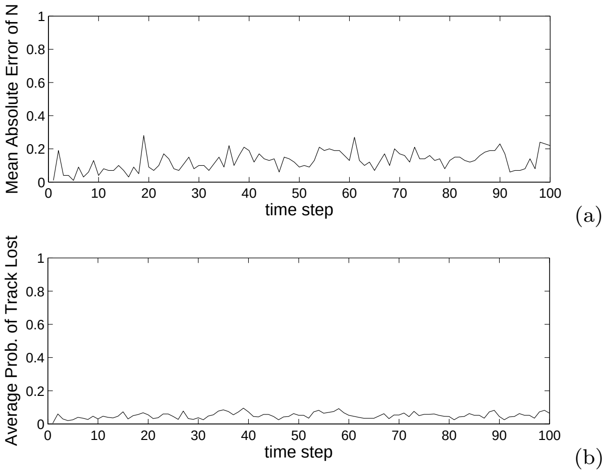

Figure 9: Mean absolute error of N and average probability of track lost.

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