

# Fractal Analysis of Currency Market: Hurst Index as an Indicator of Abnormal Events

Olena Liashenko<sup>1</sup>, Tetyana Kravets<sup>1</sup>

<sup>1</sup> Taras Shevchenko National University of Kyiv

lyashenko@univ.kiev.ua, tankravets@univ.kiev.ua

**Abstract.** The article is devoted to analysis of currency quotes behavior on the currency market by defining dynamic changes over time. The main tool of fractal analysis is the Hurst under the hypothesis of fractal market. For 17 major currency pairs on closing prices and the prices maximum-minimum the Hurst index is calculated by formula for the adjusted R/S analysis. Values at the currency markets of different countries in different economic conditions are compared during 2008-2015. For currency pairs Hurst index tends to maintain its average value in stable economic situation, while it is an indicator of events affecting directly or indirectly on the state's economy and the rate of its national currency. Application of sliding window method allows to simulate the dynamics of Hurst index for the currency pairs USD/JPY, GBP/JPY, EUR/USD, GBP/USD and establish certain patterns of conduct series of quotes due to appropriate reaction to economic, political and natural disturbances.

**Keywords.** Fractal analysis, exchange rates, crisis, Hurst index

**Key Terms.** Model, Research, Management

## 1 Introduction

The transition of Ukraine to the market economy, creation of modern infrastructure, evolution of ownership and increasing independence of participants of market relations inevitably leads to the need for constant monitoring of trends and identification of features of financial and monetary system functioning. Foreign exchange market as one of the main elements of the system in the last decade characterized by increasing globalization and transformation processes.

Due to the fact that international economic relations generate the corresponding cash requirements and obligations of the parties, a prerequisite for their settlement is national currency using, as the only universal global means of payment does not exist yet. It leads to the need to exchange one currency for another in the form of purchase of foreign currency by payer or recipient of funds in international operations. The international payment transactions related to payment of receivables and liabilities of

businesses and individuals around the world are serviced by foreign exchange market, defining its objective necessity.

The features of contemporary currency markets are internationalization, globalization, standardization and automation of communication facilities in the implementation of foreign exchange transactions. Thus there is the instability of exchange rates. Predicting future behavior of exchange rates is important because it allows to reduce currency risks and ensure the efficiency of various solutions in international financial management.

The aim is to study the behavior of the currency pairs using Hurst index monitoring as one of the tools of fractal analysis, under the hypothesis of fractal market.

## **2 Analysis of Recent Research**

During the last three decades an efficient market theory was the most famous theory of financial markets. The statement of this theory is that changes in asset prices reflect the important new information release fully and immediately. In addition, through a flow of information that can be provided between the current and the next trading period, changes in asset prices are independent. In other words, the unpredictable release of information drives asset prices in a random order, and price fluctuations comply the normal distribution [1-6].

Efficient market hypothesis (EMH), like all other economic concepts is based on linear paradigm, whereby each economic action (event) causes linearly proportional reaction that produced some cause and effect relationships. However, economic theory, based on the principles of balance, couldn't explain many complex financial phenomena. Revolution was needed and put into nonlinearity analysis.

Based on the nonlinear paradigm the fractal market hypothesis (FMH) emerged and was developed, whereby a certain action (or event) causes a nonlinear response that is exponential, unexpected, extremely strong and no one expected reaction. In contrast to the efficient market hypothesis the fractal market hypothesis states that the information is evaluated depending on the investment horizon of the investor. As different investment horizons evaluate information differently, dissemination of information is uneven also. At any certain time moment the price may not reflect all existing information, it can display only the part that is important for this investment horizon. FMH admits that chaotic mode occurs when investors lose confidence in the long-term fundamental information [1-5].

One of the results of exchange and stock markets research made by Mandelbrot, Peters [3, 1-2] is that the distribution of price changes are fractal Pareto distribution. This distribution has the property of statistical self-similarity in time (the presence of long memory). In addition, it was shown that the financial markets are nonlinear dynamic systems, which opened up opportunities for the study of financial markets by means of the theory of dynamical systems and deterministic chaos [5].

There are several alternative approaches to assessing the fractal structure of the time series: R/S-analysis; method based on the determination of cell dimensions; standard fluctuation analysis; detrend fluctuation analysis (DFA); multifractal DFA. The description and practical application of these methods can be found in [3, 5-16]. For example, using the R/S-analysis in [6] the Hurst index was estimated and it was

proved that hypothesis of FMH could be "reasonable" generalization of the efficient market hypothesis. In [12], the authors conducted an empirical study of scaling and multifractal properties of currency pair USD/DEM.

Multifractal spectra singularities for different currency pairs were studied in [13-16]. The use of different approaches to assessing the performance of fractal time series supports the hypothesis of FMH and allows to generalize it to multifractal market hypothesis (MFMH). In [8] it is demonstrated that a change of fractal properties of returns in exchange rates are an indicator of a currency crisis.

### 3 Research method

R/S-analysis method of study of fractal time series was proposed by Mandelbrot [3] and it is based on research conducted by the British explorer Hurst. It is based on the analysis of accumulated magnitude deviation of observations series and standard deviation. Hurst offered new statistics - Hurst index, which is widely used in the analysis of time series due to its stability. Its calculation requires minimum assumptions about studied system and time series can be classified by the type and the depth of memory on its basis. It can distinguish a random series of non-random one, even if the random series has non-Gaussian distribution [18].

Calculation of Hurst index can be carried out as follows:  $R/S = (aN)^H$  or  $H = \frac{\log(R/S)}{\log(aN)}$ , where  $H$  - Hurst index;  $S$  - standard deviation of observations number;  $R$  - variation of accumulated deviation;  $N$  - number of observation periods;  $a$  - positive constant [9, 17].

The scale of the accumulated deviation  $R$  is the most important element of Hurst index formula:  $R = \max_{1 \leq u \leq N} (Z_u) - \min_{1 \leq u \leq N} (Z_u)$ , where  $Z_u$  - the accumulated deviations number of values  $x$  from the mean  $\bar{x}$ , i.e.  $Z_u = \sum_{i=1}^u (x_i - \bar{x})$ . Formula for Hurst index shows that the increasing scale, reducing standard deviation and reducing number of observations influence on its growth.

For further calculations we use  $a = \frac{\pi}{2} \approx 1,5708$  as the choice of another constant for calculating. Hurst index inflates its value significantly. It will lead to erroneous conclusions about persistence of random series [9].

While a small number of observations, actual calculations of normalized variation R/S for random series give much too low results compared to theoretical ones

$R/S = \sqrt{\frac{N\pi}{2}}$ . This contradiction leads to lower values of Hurst index when the number of observations  $N < 250$ . To avoid this contradiction, it is necessary to transform actually calculated values of normalized variation using the formula [9]:

$$R / S_T = R / S \times 0,998752 + 1,051037. \tag{1}$$

However, due to feature of logarithmic calculations of Hurst index, the adjusted value of normalized variation will contain a minor error also. On the basis of the correlation between the number of observations and the ratio of the standard and actual Hurst index it is necessary to adjust the formula for Hurst index calculation so that its value was close at most to a standard one for random series ( $H = 0,5$ ) for all  $N$ . The final formula is [9]:

$$H_T = \frac{\log(R / S_T)}{\log(\pi \times N / 2)} \times (-0,0011 \times \ln N + 1,0136) \tag{1}$$

So Hurst index above 0.5 confirms the presence of long-term memory of the market: current depends on the past and the future depends on the present.

The economic literature is usually gives recommendation to calculate the accumulated variation on closing prices. However, for practical market trade minimum and maximum prices set in the middle of interval are also important. To calculate the Hurst index on the maximum-minimum prices we use the formula [9]:

$$H_M = \frac{\log(R / S_T)}{\log(\pi \times (N - 1) / 2)} \times (-0,0011 \times \ln(N - 1) + 1,0136) \tag{3}$$

Testing hypotheses about market on the basis of Hurst index can be done in case of data mixing. If the result of calculations on randomly mixed data is Hurst index close to 0.5, and it is different from the actual calculations, it may indicate that some data is not Brownian motion.

## 4 Results

The studies calculated the Hurst index by adjusted formulas for 17 major currency pairs on annual data within 2008 - 2014. Figure 1 presents the dynamics of Hurst index change, which is calculated on closing prices for different currency pairs.

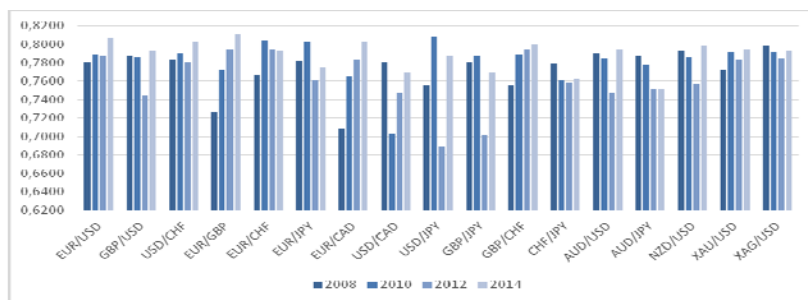


Fig. 1. Dynamics of Hurst index, calculating on closing prices

Note that the values of the Hurst index are higher than 0.68, i.e. series are persistent. Herewith there is the tendency to change the depth of long-term memory for different periods. Most pairs characterized by increase in the degree of persistence an average over the period 2008-2014, which means the stabilization of the economic situation in the world, overcoming of the global economic crisis.

However, for some currency pairs, especially USD/JPY, USD/CAD, GBP/JPY, Hurst index dynamics is characterized by significant fluctuations throughout the study period.

Currency pairs EUR/USD, GBP/USD, USD/CHF, USD/JPY are highly liquid financial instruments that are characterized by significant volatility and therefore have great potential for profit. The most popular is EUR/USD. The EUR is highly dependent on interest rates, economic conditions in the euro area, policies of central banks of the US and EU, political stability in the world [19].

Currency pair GBP/USD is one of the most moving and aggressive currency pairs. The dynamics of the currency pair largely follows the trend of the currency pair EUR/USD.

The feature of the currency pair USD/CHF is more dependence of its change of information on the economic situation in the United States than in Switzerland. The growth trend of US dollar against the Swiss franc, the tendency of weakening of other currencies against the US dollar is observed. This is especially true for currency pairs USD/CAD, AUD/USD [18].

Japanese yen - one of the world reserve currency, a tool for international settlements of countries with the lowest short-term interest rates. There is a significant correlation between currency pairs USD/JPY, EUR/JPY and CHF/JPY. Since the bulk of Japan's funds invested in European assets, changing the course of the currency pair EUR/JPY to a large extent depends on the level of interest rates of Eurozone and Japan.

Figure 2 presents the dynamics of Hurst index change, which is calculated by the maximum-minimum prices for various currency pairs.

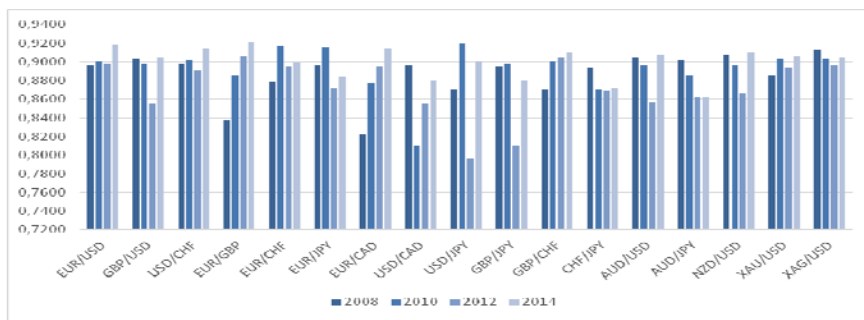
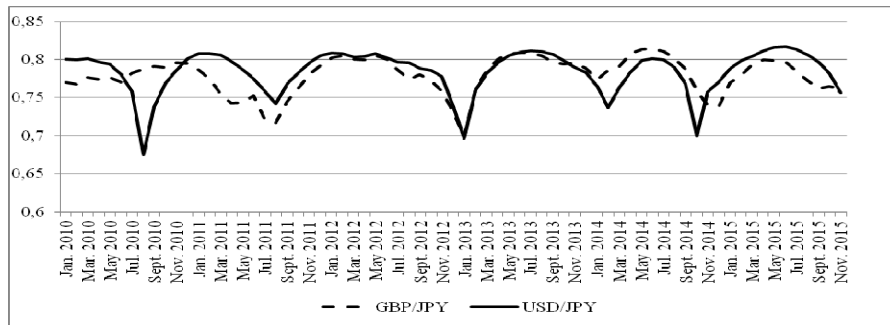


Fig. 2. Hurst index dynamics, calculated by the maximum-minimum prices

Comparison of the results (Fig. 1, 2) suggests a similarity diagrams and increasing values of Hurst index, designed for maximum-minimum prices compared with the Hurst index for closing prices. Calculation of average values of years gives identical pattern shift 0.1.

For further study consider a pair of the most pronounced drop Hurst index: USD/JPY and GBP/JPY. Using the method of sliding windows simulate of dynamic change of Hurst index for these pairs in the time period 2010-2015 (Fig. 3). On the horizontal axis the right end of the time window is marked. There is a consistency of behavior expected of these pairs, due to the presence of the yen and the interconnect- edness of economies of the US and Britain as the developed countries, members of the Group of Seven.



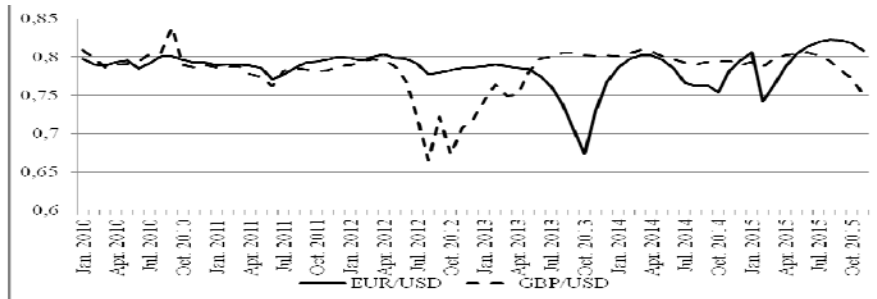
**Fig. 3.** The dynamics of Hurst index for closing prices of USD/JPY and GBP/JPY

The graphs in Fig. 3 are typical manifestations of cyclical and periodic drop of Hurst index in relatively insignificant period of time. Consider in more detail the dynamics of Hurst index for pair USD/JPY, because it is one of the most influential in the foreign exchange market.

The first decline of Hurst index and thus persistence weakening can be seen in the summer of 2010. It is a consequence of the global currency crisis 2008. The event which caused the decrease in the degree of predictability of the market currencies, was active company fight for the preservation of the national economies. It was in 2010, the government of Japan has allocated 1 trillion yen for forming a reserve to combat the crisis and restore regions, and in summer 2010 "new growth strategy" was adopted and a significant reform of the economy of rising sun was conducted. Large inflows of funds, coupled with the instability of the economic situation led to a drop in Hurst index, reducing the persistence [20].

Thus, the events were a shock for the foreign exchange market and caused a drop in Hurst index. During these time periods it was difficult to predict further developments for the currency pair USD/JPY, as its behavior has not been dictated by the internal events in the market, but external, such as monetary policy in Japan and the US.

Figure 4 presents the dynamics of Hurst index change for relatively stable and very influential pairs in the foreign exchange: EUR/USD and GBP/USD.



**Fig. 4.** The dynamics of Hurst index, calculated at closing prices for EUR/USD and GBP/USD

For the currency pair EUR/USD we could see only one, but significant "failure" of Hurst index, which happened in the fall of 2013. The cause of this could be the announcement by governments of the US and EU about the start of negotiations for the establishment of so-called Transatlantic trade and investment partnership (TTIP). However, the very mention of this agreement has caused some instability in the economy, many meetings and public criticism from the media.

## 5 Conclusions

Hurst index as a tool of fractal analysis allows to determine the degree of persistency of financial series, the presence of the long-term memory at foreign exchange market. In stable economic situation the Hurst index for currency pairs tends to maintain their average. However, this index is very responsive to events that directly or indirectly affect the state's economy and the rate of its national currency. The biggest jump of index can be seen when the country holds planned intervention to improve their economic situation, thus reducing its currency.

Another group of events affecting the persistence of the currency market are important for the country's situations, such as changing the government, announcement of a new policy or natural disasters. All this violates the usual course of events in the currency markets, and changing of the Hurst index of exchange quotations is the indicator of this. The foreign exchange market disturbance passes over the index approaches to its average value inherent in each currency pair.

## References

1. Peters, E.E.: Chaos and Order in the Capital Markets: A New View of Cycles, Prices, and Market Volatility, John Wiley&Sons, Inc. (1996)
2. Peters, E.E.: Fractal Market Analysis: Applying Chaos Theory to Investment and Economics, John Wiley&Sons, Inc. (1994)
3. Mandelbrot, B.B., Hudson, R.L.: The (Mis)behavior of Markets: A Fractal Views of Financial Turbulence, Basic Books, (2005)
4. Soros, G.: The New Paradigm for Financial Markets: The Credit Crisis of 2008 and What It Means, NY: Public Affairs (2008)

5. Crownover, R.M.: Introduction to Fractals and Chaos, Jones and Bartlett (1995)
6. Corazza, M., Malliaris, G.: Multifractality in Foreign Currency Markets. *Multinational Finance Journal*, 6, 387–401 (2009)
7. Benbachir, S. & El Alaoui, M.: A Multifractal Detrended Fluctuation Analysis of the Moroccan Stock Exchange. *International Research Journal of Finance and Economics*, 78, 6–17 (2011)
8. Mansurov, A.K.: Forecasting Currency Crises by Fractal Analysis Techniques. *Studies on Russian Economic Development*, 19(1), 96–103 (2008)
9. Naiman, E.: The Hurst Index Calculation to Identify Persistence of the Financial Markets and Macroeconomic Indicators. *Ukrainian Journal Ekonomist*, 10, 18–28 (2009)
10. Nava, N., Di Matteo, T., Aste, T.: Anomalous Volatility Scaling in High Frequency Financial Data. *Physica A*, 447, 434–445 (2015)
11. Morales, R., Di Matteo, T., Gramatica, R., Aste, T.: Dynamical Generalized Hurst Exponent as a Tool to Monitor Unstable Periods in Financial Time Series. *Physica A*, 391(11), 3180–3189 (2011)
12. Xua, Z., Gencay, R.: Scaling, Self-similarity and Multifractality in FX Markets. *Physica A*, 323, 578–590 (2003)
13. Drozd, S., Kwapien, J., Oswiecimka, P., Rak, R.: Foreign Exchange Market: Return Distributions, Multifractality, Anomalous Multifractality and Epps Effect. *New Journal of Physics*, 12, online <http://iopscience.iop.org/article/10.1088/1367-2630/12/10/105003/pdf> (2010)
14. Oh, G., Eom, C., Havkin, S., Jung, W.-S., Wang, F., Stanley, H.R., Kim, S.: A Multifractal Analysis of Asian Foreign Exchange Markets. *Eur. Phys. J. B.* online <http://polymer.bu.edu/hes/articles/oehjwsk12.pdf> (2012)
15. Norouzzadeh, P. & Rahmani, B.: Multifractal Detrended Fluctuation Description of Iranian Rial – US Dollar Exchange rate. *Physica A*, 367, 328–336 (2006)
16. Gunay, S.: Source Of The Multifractality In Exchange Markets: Multifractal Detrended Fluctuations Analysis. *Journal of Business & Economics Research*, 12(4), 371–384 (2014)
17. Schroeder, M.: *Fractals, Chaos, Power Laws: Minute from an Infinite Paradise*, New York: W.H.Freeman and Co. (1991)
18. Cannon, M.J., Percival, D.B., Caccia, D.C., Raymond, G.M., Bassingthwaigte, J.B.: Evaluating Scaled Windowed Variance Methods for Estimating the Hurst Coefficient of Time Series. *Physica A*, 241(3-4), 606–626 (1997)
19. Feldstein, M.: The Failure of the Euro. *The Foreign Affairs*. online <https://www.foreignaffairs.com/articles/europe/2011-12-13/failure-euro> (2012)
20. Harari, Daniel. Japan's Economy: from the 'Lost Decade' to Abenomics. House of Commons Library, Standard Note SN06629. London: Oct 24, 2013