Descriptive Models of System Dynamics

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Abstract. Nowadays in the course of investigation researchers are facing large arrays and datasets required fast processing, analysis and drawing adequate conclusions. Data mining, statistical methods and big data analytics provide an impressive arsenal of tools allowing scientists to solve these tasks. However, investigators often require techniques that enable with use of relatively simple and cheap measurements of easily accessible parameters to build useful and meaningful concepts.
In our paper two classes of dynamical models aimed at revealing between-component relationships in natural systems with feedback are presented. The idea of both models follows from the frameworks of theoretical biology and ecology regarding pairwise interactions between the parts of the system as background of system behavior. Both deterministic and stochastic cases are considered, that allow us to determine the direction of pairwise relationships in the deterministic case and the direction and strength of relationships in the stochastic one.

Keywords: dynamical systems, deterministic models, stochastic models, Markov chains
Keyterms: Model, MathematicalModel, MathematicalModeling, ComputerSimulation

1 Introduction

The world of scientific research has been immersing into an extraordinary information explosion over past decades, accompanied by the rapid growth in the use of Internet and the number of connected computers worldwide. We see a rate of increase in data growth that is faster than at any period throughout history. Enterprise application and machine-generated data continue to grow exponentially, challenging experts and researchers to develop new innovative techniques to evaluate hardware and software technologies and to develop new methods of big data investigation [1].

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Heterogeneity, scale, timeliness, complexity, and privacy problems with big data impede at all phases of obtaining value from data. The problems begin during data acquisition, when the amount of data requires us to make decisions, currently in an *ad hoc* manner, about importance and interpretability of data. Besides, much data today are not natively in structured format, have gaps and incomplete. Hence, data analysis, organization, retrieval, and modeling are foundational challenges. Finally, presentation of the results and its interpretation by non-technical domain experts is crucial to extracting actionable knowledge.

Our study is devoted to the well-known problem of revealing conditions of stability in natural systems providing long and steady development and existence of systems. Today there is a large amount of online big data collections comprising datasets taken from different branches of biology, health sciences, ecology etc. As examples we can mention Data Centre of International Council for the Exploration of the Sea (includes hundreds of thousands marine biology related datasets), which were used in the current investigation.

The problem of homeostasis and stability in the living organisms community or natural systems (biological or ecological) is closely related to the problem of dynamic stability. The practical aspect of this problem is connected to the disturbance in stability of systems, that is often accompanied, for example, by outbreaks in number or biomass of species.

The study of stability in communities or natural systems is closely connected to investigation of relationships that determine the dynamic characters of a system, i. e. relationships between systems parameters having influence on the system dynamics.

For decades systemic methods, for example, based on the Shannon index of diversity, have been used for studying the relationships between the structure and stability of a system. Generalizing that and other approaches, Margalef [2] states that “the ecologist sees in any measure of diversity an expression of the possibilities of constructing feedback systems, or any sort of links, in a given assemblage of species”. Similar ideas were therefore presented in studying the structure of correlation pleiads, in using cluster analysis and other statistical techniques to establish such relationships for investigating similar problems.

Despite different approaches to revealing between-component relationships, in biology and ecology there is a general approach in studying such relationships, based on the following pairwise relationships: $(+,+)$, $(-,-)$, $(-,+)$, $(-,0)$, $(+,0)$, $(0,0)$. For the multi-component systems, this set of relationships exhausts all possible pairwise inter-component relationships categorized by the type of effect and have been studied at length, particularly, in biology and ecology [3–5]. Therefor, in the current paper the analysis of the relationships structure is based on the idea of regarding the objects (i. e., living organisms in a community, species etc.) as components of a system between which the pairwise relationships mentioned above are possible. This allows us to present the structure of relationships in an explicit form of relationships between the components of a natural system.

It should be noted, that mentioned relationships not always can be revealed with the help of statistical methods. For example, correlation analysis is initially used for estimation of a relationship between two or more variables, but it covers only statistical relation and cannot reveal a cause-effect relationship [6].

There are statistical methods (structural relation modeling, analysis of path and adjacent techniques), which are devoted to revealing between-component relationships (and other tasks as latent variables' analysis) and can be used for casuality analysis [7, 8]. But these methods express the relationships in a system in the terms of regression coefficients and not in the form of paired relationships. Besides, interpretation of results of this analysis is occasionally difficult (e.g. studying relationship between a feedback system and homeostasis in a community).

2 Theory

Here we present two dynamical models developed for revealing between-component relationships on the base of observation data obtained from a real natural system.

First model has deterministic dynamic, finite number of states and discrete time. As it is described in [9] at length, here we describe the model in brief. The second model is stochastic and will be describe in more detailed. Both models have a common background, so we begin with its description and later will go to specific properties of each models.

We assume that a natural system to be modelled comprises N components, which can be denoted by $A_1, A_2, \ldots, A_N$. It is assumed that the component take integer values 1, 2, ..., K, i. e. K value for each component. The value 1 means a minimum amount of a component, the value K means maximum, i. e. the component value varies from 1 to K.

The system develops in discrete time and the moments of time are to be denoted $t = 0, 1, \ldots$. So, the value of the component A_i at the moment of time $t = 0, 1, \ldots$ are numbers $A_i(0)$, $A_i(1)$, ...

The next properties of a system are different for deterministic and stochastic cases, so we shall describe them separately.

2.1 Deterministic model revealing the direction of between-component relationships

We begin with deterministic case discussed, as mentioned, in [9] and was named the Discrete model of dynamical systems with feedback. For the deterministic system its state at the moment $t+1$ is fully and definitively determined by the state at the moment t.

If the system at the moment t is in the state $(A_1(0), A_2(0), \ldots, A_N(0))$, all the following states can be written as the trajectory, where each column is a state at corresponding moment:

$$\begin{pmatrix} A_1(0) & A_1(1) & A_1(2) & \ldots \\ A_2(0) & A_2(1) & A_2(2) & \ldots \\ \vdots & \vdots & \vdots & \ldots \\ A_N(0) & A_N(1) & A_N(2) & \ldots \end{pmatrix}. \tag{1}$$

In the theory of dynamical systems [10], such a system is called a free dynamical system with discrete time. The system has only finite number of states, so there exists a positive integer $\mathcal{T}$, which can be called a period of the trajectory, for which the conditions of periodicity hold $A_i(s) = A_i(s+\mathcal{T})$ for enough large s.

Taking into account the periodicity, it is possible to extract the following minor form (1)

$$\begin{pmatrix} A_1(s) & A_1(s+1) & \ldots & A_1(s+\mathcal{T}-1) \\ A_2(s) & A_2(s+1) & \ldots & A_2(s+\mathcal{T}-1) \\ \vdots & \vdots & \ddots & \vdots \\ A_N(s) & A_N(s+1) & \ldots & A_N(s+\mathcal{T}-1) \end{pmatrix} \tag{2}$$

presenting full description of the dynamics of the system.

Now we introduce the concept of relationships between components. Let $\Omega = \{-, 0, +\}$. A relationship between specified components A_i and A_j is determined as an entry from the set $\Omega\times\Omega$ and denoted by $\Lambda(A_i, A_j) = (\omega_1, \omega_2)$, where $\omega_1 \in \Omega$, $\omega_2 \in \Omega$. If $\Lambda(A_i, A_j) = (\omega_1, \omega_2)$, this means that:

- if $\omega_1 = \{-\}$, then large values of the A_j will lower the value of the A_i.

- if $\omega_1 = \{0\}$, then the A_j doesn't influence the value of the component A_i.

- if $\omega_1 = \{+\}$, then large values of the A_j will raise the value of the A_i.

The relationship Λ is antisymmetric in the following sense: $\Lambda(A_i, A_j) = (\omega_1, \omega_2)$ implies $\Lambda(A_j, A_i) = (\omega_2, \omega_1)$.

Assume that all the relationships $\Lambda(A_j, A_i)$ between all pairs (A_j, A_i) of components $A_1, A_2, \ldots, A_N$ are given. For each A_j and each $(s, u) \in \Omega \times \Omega$ it is possible to find the set of components, with which A_j has the relationship (s, u)

$$L_j(s, u) = \{A_i | \Lambda(A_j, A_i) = (s, u)\}.$$

Let $\varkappa = \{1, 2, \ldots, K\}$ is to be the set of the states of an individual component and $N_j(s, u)$ is the number of components in the set $L_j(s, u)$, $j = 1, 2, \ldots, N$, $(s, u) \in \Omega \times \Omega$. A transition from the state at t to the state at $t + 1$ is described by N transition functions F_j, each of which defines the mapping

$$\varkappa^{N_j(+,+)+N_j(+,0)+N_j(+,-)+N_j(-,+)+N_j(-,0)+N_j(-,-)} \mapsto \varkappa.$$

Two types of relationships, intrinsic to natural systems. For more detailed description of the dynamics of a natural system, one needs to specify the explicit form of the transitional mapping. mino We introduced two approaches based on the concepts of biological interactions: weight functions' approach and approach based on principles of Justus von Liebich's law.

Define the following functions on the set $\varkappa$: $\mathrm{Inc}(A) = \min\{K, A + 1\}$, $\mathrm{Dec}(A) = \max\{1, A - 1\}$.

The system dynamics based on weight functions' approach. First we define the type of dynamics, which takes into account the weighted sum of all $A_j(t)$ (inclusive $A_i(t)$) for calculating the value of the component A_i at the moment $t + 1$.

As we defined above, for each j ($j = 1, 2, \ldots, N$) and each pair $(s, u) \in \Omega \times \Omega$ there exists the set $L_j(s, u)$ with $N_j(s, u)$ entries. Assume that the function $\varphi_{j,1}^{\langle s,u \rangle}(\cdot), \varphi_{j,2}^{\langle s,u \rangle}(\cdot), \ldots, \varphi_{j,N_j(s,u)}^{\langle s,u \rangle}(\cdot)$ are to be the functions of interactions of those components, with which the A_j has relationships (s, u).

The functions are defined on the discrete set $\varkappa$ and have the following properties: (i) $\varphi_{j,k}^{\langle +,+ \rangle}(\cdot)$, $\varphi_{j,k}^{\langle +,0 \rangle}(\cdot)$, $\varphi_{j,k}^{\langle +,- \rangle}(\cdot)$ are increasing functions; (ii) $\varphi_{j,k}^{\langle -,+ \rangle}(\cdot)$, $\varphi_{j,k}^{\langle -,0 \rangle}(\cdot)$, $\varphi_{j,k}^{\langle -,- \rangle}(\cdot)$ are decreasing functions; (iii) $\varphi_{j,k}^{\langle s,u \rangle}(1) = 0$ for any $(s, u) \in \Omega \times \Omega$.

We also introduce the numbers $\delta_j > 0$ $(j = 1, 2, \ldots, N)$ which can be called thresholds of sensitivity.

For the system's state at the moment of time t the following value is calculated

$$d_j = \sum_{A_k \in L_j(+,+)} \varphi_{j,k}^{\langle +,+\rangle}(A_k(t)) + \sum_{A_k \in L_j(+,0)} \varphi_{j,k}^{\langle +,0\rangle}(A_k(t)) + \sum_{A_k \in L_j(+,-)} \varphi_{j,k}^{\langle +,-\rangle}(A_k(t)) + \sum_{A_k \in L_j(-,+)} \varphi_{j,k}^{\langle -,+\rangle}(A_k(t)) + \sum_{A_k \in L_j(-,0)} \varphi_{j,k}^{\langle -,0\rangle}(A_k(t)) + \sum_{A_k \in L_j(-,-)} \varphi_{j,k}^{\langle -,-\rangle}(A_k(t)). \quad (3)$$

The value of the component A_j is being changed according to the value d_j by the following rules

1. if $d_j \geq \delta_j$, then $A_j(t+1) = \mathrm{Inc}(A_j(t))$;
2. if $d_j \leq -\delta_j$, then $A_j(t+1) = \mathrm{Dec}(A_j(t))$;
3. if $-\delta_j < d_j < \delta_j$, then $A_j(t+1) = A_j(t)$.

Now, the meaning of introduced transition functions can be explained in clear way. For example, the functions $\varphi_{j,k}^{\langle -,+\rangle}(\cdot)$ $(k = 1, 2, \ldots, N_j(-,+))$ reflects the influence upon the component A_j of components in the set $L_j(-,+)$, which are related with A_j by relationship $(-,+)$. The greater the influence (i. e. the greater values of $A_i(t)$ from the set $L_j(-,+)$), the lower the values of d_j.

The dynamics based on the Liebig's law of the minimum. Next approach is based on principles of Justus von Liebich's law (Liebig's law of the minimum) and essentially differs from the first approach, which is basically additive.

Omitting the details, enough to say, that according to this approach, transition from the state $(A_1(t), A_2(t), \ldots, A_n(t))$ is defined by relations of the state with two matrices C and C^* playing the role of a threshold.

The system identification with use of the observation data. While dealing with real data, we often don't observe the data in dynamics. Often real data come unordered in time in contrast to data used for time series modeling. So we don't observe any dynamism described by the trajectory (1) or the minor (2).

Usually, the result of observation is represented by a table of cases:

$$\tilde{M} = \begin{pmatrix} C_{11} & C_{12} & \ldots & C_{1B} \\ C_{21} & C_{22} & \ldots & C_{2B} \\ \vdots & \vdots & \ddots & \vdots \\ C_{N1} & C_{N2} & \ldots & C_{NB} \end{pmatrix}, \quad (4)$$

where columns correspond to cases and rows correspond to components (N components and B cases). We emphasize unordered character of the data above, i. e. these is no time order between the cases in the table $\tilde{M}$.

Here we describe a principle allowing to reveal the system relationships of above mentioned type on the basis of the observation table $\tilde{M}$.

This algorithm determines inter- and intra-component relationships, which are as close as possible to relationships, which form matrix (2) in some sense.

Assume that relationships structure is given. In that case for initial state $(A_1(0), A_2(0), \ldots, A_N(0))$ and for given sets $L_1(u,s)$, $L_2(u,s)$, …, $L_N(u,s)$, $u \in \Omega$, $s \in \Omega$ the minor (2) can be calculated. Let P is to be the correlation matrix (Pearson or Spearman) between the rows of the minor (2) with entries $r_{i,j}$. Also, for the table $\tilde{M}$, the correlation matrix $\tilde{P}$ (with entries $\rho_{i,j}$) of its rows can be calculated.

Introduce the measure of distance between the matrices P and $\tilde{P}$

$$D(P, \tilde{P}) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (r_{ij} - \rho_{ij})^2. \tag{5}$$

We state the task of minimization $D(P, \tilde{P})$ by all possible vectors of initial states $(A_1(0),\ A_2(0),\ \ldots,\ A_N(0))$ and all allowable sets $L_j(s,u)$, $s \in \Omega$, $u \in \Omega$ for all j

$$D(P, \tilde{P}) \mapsto \min \begin{array}{l}(\text{by all initial states } \&\\ \text{ by all allowable sets } L_j(s,u)).\end{array} \tag{6}$$

The stated task means the search for such relationships between components, that the minor (2) is to be as close as possible to the table of observations regarding the measure (5).

The following theorem proved in [9] shows that this task is well-grounded in probabilistic sense.

Theorem 1. *If the table of observations $\tilde{M}$ is obtained from the minor (2) by equiprobable choice of columns, then the Pearson correlation matrix of the observations table $\tilde{P}$ converges to the correlation matrix of minor P (in probability)*

$$\lim_{B \to \infty} \rho_{ij} = r_{ij}, \quad i = 1, 2, \ldots, N,\ j = 1, 2, \ldots, N.$$

The same result takes place for the Spearman correlation matrix as well.

2.2 Additive stochastic model of between-component relationships

Our second model is also described by a set of components $A_1, A_2, \ldots, A_N$ taking discrete values 1, 2, …, K.

But, in contrast to the first one, the second model introduces into consideration not only direction of relationships (in fact, for the first model we considered three direction — negative, neutral, and positive), but also a strength of relationships. Hence, relationships in this case can be recognized besides directions by the strength.

The structure of relationships between components $A_1, A_2, \ldots, A_N$ is described by the following relationships matrix

$$\mathcal{M} = \begin{pmatrix} m_{1,1} & m_{1,2} & \ldots & m_{1,N} \\ m_{2,1} & m_{2,2} & \ldots & m_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ m_{N,1} & m_{N,2} & \ldots & m_{N,N} \end{pmatrix}.$$

Any entry $m_{i,j}$ reflects the strength and direction of influence of the component A_j upon the component A_i. The direction of influence is expressed by the sign of the value $m_{i,j}$ (may be $-, 0, +$) and the strength — by modulus of $m_{i,j}$ and varies from 0 to 1. So, $-1 \leq m_{i,j} \leq 1$ for each i, j. The influence of the component A_i on A_j is expressed by $m_{j,i}$. It is easy to see, that the relationship between the components A_i on A_j is described by the pair $(m_{i,j}, m_{j,i})$, which is close by implication to the relationship (ω_1, ω_2) introduced for the first model.

Now describe the dynamics of transition from the state of the system at the moment t to the state at the next moment $t+1$. As for the weight functions' approach, we assume, that a set of functions $\psi_{i,j}(\cdot)$, ($i, j = 1, 2, \ldots, N$) reflecting relationships between all pairs of components, including inner relationships, are given. The functions $\psi_{i,j}(\cdot)$ have the following properties: (i) $\psi_{i,j}(\cdot)$ are defined on the set $\varkappa$; (ii) $\psi_{i,j}(1) > 0$; (iii) $\psi_{i,j}(\cdot)$ are increasing functions on $\varkappa$.

Also assume that a positive number δ playing the role of threshold, is given. Let the system is to be in the state $(A_1(t), A_2(t), \ldots, A_N(t))$. For each pair of indices define the random variable $\xi_{i,j}$ as follows

$$\xi_{i,j} = \begin{cases} \operatorname{sign}(m_{i,j}) \cdot \psi_{i,j}(A_j(t)) & \text{with probability } |m_{i,j}|, \\ 0 & \text{with probability } 1 - |m_{i,j}|. \end{cases}$$

Then we calculate the set of N random variables $d_i = \sum_{j=1}^{N} \xi_{i,j}, \ i = 1, 2, \ldots, N$.

Using the set $(d_1, d_2, \ldots, d_N)$, it's possible to calculate the set of probabilities (p_i^-, p_i^0, p_i^+) for each i according to the rule

$$p_i^- = P(d_i \geq \delta), p_i^0 = P(-\delta < d_i < \delta), p_i^+ = P(d_i \leq -\delta)$$

for each i from 1 to N.

This definition implies the equality $p_i^- + p_i^0 + p_i^+ = 1$. The transition from the state at the moment t to the next state at $t+1$ is defied by following rule

$$A_i(t+1) = \begin{cases} \mathrm{Dec}(A_i(t)) & \text{with probability } p_i^-, \\ A_i(t) & \text{with probability } p_i^0, \\ \mathrm{Inc}(A_i(t)) & \text{with probability } p_i^+. \end{cases}$$

That is, at the moment $t+1$ the value of A_i can increase by 1, remain the same or decrease by 1 with probabilities p_i^-, p_i^0, p_i^+ correspondingly. Applying this rule for each i, the probabilities of transition from any appropriate state $(A_1(t), A_2(t), \ldots, A_N(t))$ can be calculated.

It can be shown, that if each row of the matrix $\mathcal{M}$ include both negative and positive entries, we obtain the Markov chain with K^N states $A_1(t), A_2(t), \ldots, A_N(t)$ $(A_i \in \varkappa)$. Besides, this chain is regular, so there a unique steady-state stochastic vector $\mathbf{w}$.

Now the reasons that state behind this model, can be explained. We assume, that a natural system is described by this model, and the probability of staying the system in states converges to the entries of the vector $\mathbf{w}$. Using the states $A_1 A_2, \ldots, A_N$ and the components of the steady-state vector $\mathbf{w}$ we can calculate a weighted Pearson correlation matrix [11] between the components. Denote such the matrix by $R_{\mathbf{w}}$.

We suppose, that the true dynamics of our natural system is not visible, i. e. we cannot observe time series of states, but can record a state of the system at random moments of time. These observations are collected in the observation table $\tilde{M}$ having N variables and B cases (after B observations) in similar way as for analogous table (4) of the first model. Let the Pearson correlation matrix between rows of (4) is denoted by $\tilde{R}$.

Theorem 2. If the observation table $\tilde{M}$ is obtained according to the way described above, we have

$$\tilde{R} \to R_{\mathbf{w}} \text{ in probability when } B \to \infty.$$

This means component-wise convergence.

Proof. Omitted for short.

Introduce the measure of proximity for the matrices R and $\tilde{R}$

$$D(R_{\mathbf{w}}, \tilde{R}) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\tilde{R}_{i,j} - [R_{\mathbf{w}}]_{i,j})^2. \qquad (7)$$

The result proved in the theorem 2 means that the sample observation matrix consistently represents a true dynamics, not observed straightforwardly. This result works as a base for identifications of entries of the relationships matrix $\mathcal{M}$. Therefore we can try to calculate transition probabilities of the Markov chain, that provide the best approximation of a true correlation matrix by a sample matrix in the sense of the measure (7). So, $\mathcal{M}$ is obtained by resolving the following optimization task

$$D(R_{\mathbf{w}}, \tilde{R}) \mapsto \min \text{ by entries } m_{i,j}.$$

In fact, we find the relationships matrix $\mathcal{M}$, which makes the modelled correlation matrix as close as possible to the observe correlation matrix.

3 Case Studies

We present here three examples from different areas, where our models were applied.

First example concerns analysis of system factors affecting activity of social networks users, playing an important role in modern culture [12]. The structure of relationships between components of the system for two states of the Internet-forum on fantasy literature were calculated and compared. This comparison aimed to reveal system aspects of forum visiting in two periods. One state can be regarded as "low-performance", other as "high-performance" according to number of written fanfictions (also abbreviated as fan fics, fanfics) of visitors at the site dedicated to the cycle of novels of Joanne Rowling about Harry Potter (snapetales.com). The period of first half of December 2010 is regarded as "high-performance", the second period of the first half of December 2014 is called "low-performance". For these two periods a statistically significant difference according to Student t-test ($p < 0.05$) in average number of visits per day was also detected.

The fanfictions were divided into 4 categories according to their length: fanfictions of small, large, and medium size; the last, fourth category includes fanfictions not related to the novels about Harry Potter.

The following values were taken as the components of the system reflecting the authors activity

– the number of fanfictions *of small size* per day related to the cycle of novels about Harry Potter (denoted by **MIN**);
– the number of similar fanfictions *of large size* per day (**MAX**);
– the number of similar fanfictions *of medium size* denoted by (**MID**);
– the number of fanfictions denoted by not related to the cycle of novels about Harry Potter, based on another literary works (**OTHER**).

For the "high-performance" and "low-performance" periods, the structure of relationships were built. We identified the models using the Pearson correlation matrix and the approach on the base of von Liebig law, with $K = 3$ levels of components values. The structure of relationships for both period is presented in Figs. 1 and 2. The notation on the graphs corresponds to the models and is quiet understandable: the components are presented as rectangulars connected by ovals presenting considered relationships. For example, $\Lambda(\mathbf{MIN}, \mathbf{MID}) = (+, -)$, that is clearly shown on the graph.

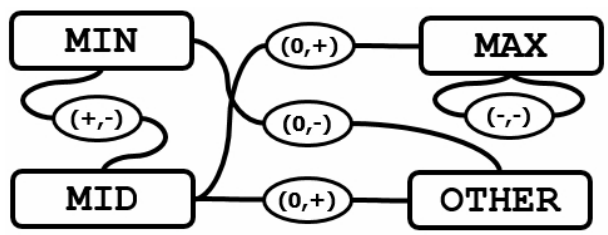

Fig. 1. The structure of relationships for "high-performance" period. Rounded rectangulars present the components of the system, the ovals include relationships between the components.

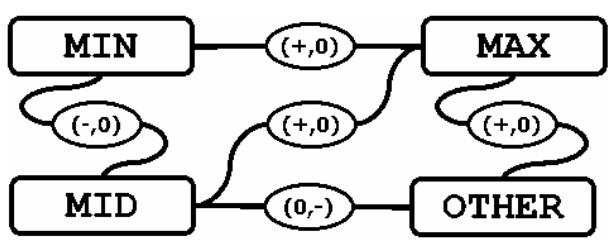

Fig. 2. The structure of relationships for "low-performance" period.

Comparing the graphs in Fig. 1 and Fig. 2 shows a system-forming role of the component **MID** for the "high-performance" period, in which **MID** positively affected other three components. This affect disappeared in the "low-performance' period together with loss a stabilizing mechanism through the relationship $(+, -)$ between **MID** and **MIN** supporting a dynamic equilibrium of the system.

These results are consistent with empirically established ideas about significant positive role of fanfictions of medium size (**MID**) in a functioning of social networks of this category and their close relation to short-sized fanfictions (**MIN**) representing a reaction of the most dynamic part of users. Differences in role of **OTHER** correspond to significance of "offtopic" as an index of deterioration in work of dedicated web-sites.

Our next example concerns the system relations of anthropometric parameters of adolescents suffering diseases of cardiovascular system [13].

Anthropometry is important in school medical, in particular, for determining the factors of predisposition of adolescents to cardiovascular disorders. At the same time, among other drawbacks of currently used anthropometric methods they often refer to insufficient use of systematic approach, among other things, in description of regularities in formation of body's proportions in the individual development of adolescents.

Here we present a demo of application of DMDS for this purpose, calculated on the material of adolescents anthropometry with arterial hypertension and other forms of cardiovascular disorders. Body compositions related to overweight plays an important role in development of arterial hypertension. Taking that into account, the models for four following components were built: hip circumference, waist circumference, chest circumference, and shoulder breadth divided by height of a subject. The Spearman correlation and Liebig's approach with $K = 3$ levels of components were used in modeling.

Comparison of these graphs has revealed a different role of such anthropometric parameters as the hip circumference for two group of adolescents under investigation. In the group with disorders different from arterial hypertension high values of hip circumference increase other three components. Simultaneously, shoulder breadth negatively affects hip circumference, that should form a proportion of male's future body perceiving by subconscious as harmonious on the base on evolutionary history and recognized as such by modern physiology and medicine — the proportions of male "triangle" directed beneath by edge. The structure of relationships in the group with hypertension prevents the formation of such a standard and associates with the accumulation of a depot fat in

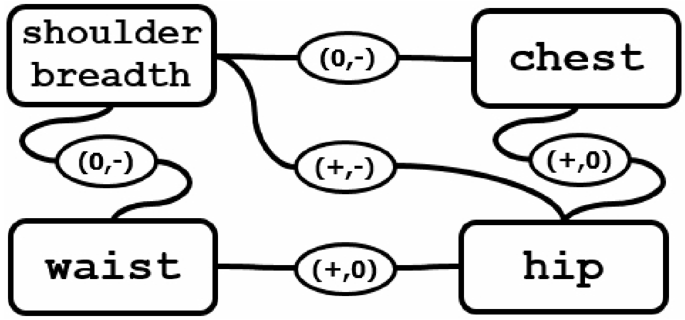

Fig. 3. The structure of relationships for adolescents without arterial hypertension.

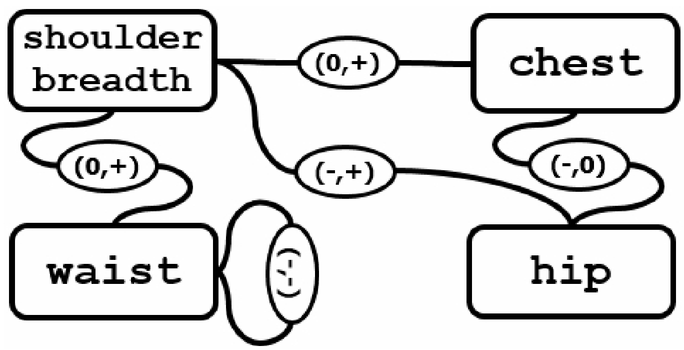

Fig. 4. The structure of relationships for adolescents with arterial hypertension.

certain parts of a human body: relatively high values of the hip circumference negatively affects shoulders breadth and chest circumference, not directly affecting waist circumference, on which shoulders breadth positively influences.

These results, regarded by authors as preliminary, do not contradict known facts about the impact of anthropometric parameters on the risk of development of hypertension in adolescents groups.

Our last example was taken from industrial fishery of Atlantic cod (*Gadus morhua*) at North Sea. The fishery of the cod plays important role in the economy of several countries and provokes considerable interest to use of mathematical models in industrial ichthyology describing large fluctuations of catching [14] (well-known example of this kind is collapse of the Atlantic northwest cod fishery in 1992).

As the demo the additive stochastic model of relationships structure between dimensional parameters of cod populations was considered. The average fish body length (**L**), the difference between Upper Length Bound and Lower Length Bound (**vL**), the average stomachs weight (**M**), and the average weight of preys of cod (**dM**) were taken as components of the model. Additive stochastic models were built according to data of International Council for the Exploration of the Sea for two years (1984 and 1989) preceding to rapid changes of CPUE (the catch per unit effort). We used the model with $K = 4$ levels of components values. In the matrix

Table 1. Relationships matrices for 1984 and 1989

1984					1989				
	L	**vL**	**M**	**dM**		**L**	**vL**	**M**	**dM**
L	-0.936	-0.379	0.893	0.581	**L**	-0.843	0.137	0.953	0.059
vL	0.325	-0.737	-0.405	0.519	**vL**	0.963	-0.721	-0.997	0.494
M	-0.184	-0.868	0.969	-0.081	**M**	-0.882	-0.054	0.788	0.224
dM	0.016	-0.941	0.999	-0.028	**dM**	0.091	0.066	0.941	-0.995

corresponding to 1984, which precedes significant (till 1990) decrease of catching, there are large (above 0.85) negative effects of high values of **vL** on **M** and **dM**. That is, increasing the diversity of dimensional characteristics of the cod population, that improves the consumption possibilities of forage reserve by the cod, leads to exhaustion of food resources (reducing the number of available preys) and deterioration of preys quality (reducing the average size of forage organisms), and results in deterioration of food supply of the cod, that lowers the values of **M** and **dM**.

In the matrix corresponding to 1989, which precedes sharp increase of CPUE, recorded a year later, in 1991, there exist extremely small (below 0.07) negative effects of high values of **vL** on **M** and **dM**. In this case, the increasing diversity of sizes, that enhances abilities of consumption of forage reserve, does not lead to exhaustion and deterioration of the latter. This result of modeling explains differences described above in the dynamics of catching in accordance with modern concepts of industrial ichthyology. We also note the difference in the value of positive influence of **L** on **vL**: 0.325 and 0.963 for 1984 and 1989 correspondingly.

Presented results bring hope for the possibility of developing methods for a forecast of cod catching with use of the stochastic models of this class, built on the base of actual material on size structure of the population.

4 Conclusion

In the paper we followed the established framework in model development, appropriated for natural sciences. Typical approach in development, among others, comprises the data selection, specification of assumptions and simplifications, selection of a mathematical modeling framework, estimation of parameter values, model diagnostics, model validation, model refinements and model application. It's clear, that all these stages of building mathematical models for biological systems are too complicated, but the most difficult task among them is the model parameters estimation for identifying structure in the underlying biological networks.

The models presented in the paper are created for description of biological and ecological systems, based on pairwise relationships characterized by the direction (positive, negative, or neutral) for both models and by the strength varied from 0 to 1 in the stochastic model only.

The task of parameter estimation is a true challenging problem for both models and requires development of special algorithms of numerical optimization. For example, if the system has N components and the number of levels is to be assumed K, for the first deterministic model the number of initial states is equal to K^N and the number of possible relationships' structures is equal to 3^{N^2}. For solving the stated optimization problem (6), one should built the minor (2) with use of an initial state and a relationships' structure, calculate correlation matrix P and calculate the distance (5). So, the exhaustive search of both initial states and relationships' structures jointly gives us $K^N 3^{N^2}$ variants, that is a huge number for even moderate N and K.

The case studies presented in the paper, considered by the authors as preliminary and illustrating, offer the prospects of applications of proposed models.

The results of modeling of system aspects in anthropometry of adolescents present the approaches to use of this simple and cheap method for identifying the risk groups of the progress of arterial hypertension. These approaches may be applied in school medicine an, if necessary, in extreme situations for mass screening as well.

The investigation of system factors of functioning of web-site dedicated to fiction about characters from original works about Harry Potter, due to use of components of the system, that are invariant to the content of the web-site, may have a broader meaning in analysis of the social networks performance.

The model of the cod population as a whole does not contradict known facts on the role of fish size and state of a forage reserve in the population dynamics. At the same time, these results reveal some promises and can be used in the development of approximate methods for prediction of populations of commercial fish with use of relatively simple and inexpensive methods of data acquisition, including even the commercial reports concerning the assortment of fish products.

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