Systems of Computer Mathematics for Educational Purposes as the Means of Reception of Procedural Knowledge

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Abstract. This article describes systems of computer mathematics for educational purposes with intelligent properties oriented to support practical activities of users - students and teachers.
The article is presented general theoretical and methodological bases, functional requirements to systems of computer mathematics for educational purposes, models of systems of computer mathematics for educational purposes as systems of support of learning processes

Keywords. Systems of computer mathematics for educational purposes, computer software, procedural knowledge
Key Terms. Educational process, computer software.

1 Introduction

One of the key technological problems in the development of informative society in Ukraine is the informatization of social activities, including the computerization of educational and research activities [1, 2].

Over the last years, Ukraine has intensified processes of the informatization of education. In this regard, special relevance acquire general scientific, methodological and technological problems related to the organization of the processes of creation, maintenance and effective use of software for educational purposes (SEP).

Supporting professional mathematical activity is carried out with the help of modern professional systems of computer mathematics (PSCM). Their use provides specialists in theoretical and applied mathematics the ability to solve a wide spectrum mathematics tasks [3-5].

Practice of applications of PSCM in the learning process has shown that they do not solve all the problems related to the efficiency of the learning process.

Activities aimed at the assimilation of the courses in mathematics have a certain specificity. Mathematical practical activities of the student consist in solving mathematical tasks.

Market analysis of SEP in mathematics shows that there are several types of specialized software functional learning tools in mathematics [6-9]:

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- educational programs such as e-textbooks, which offer the users of didactic materials and systems testing acquired knowledge;
- software support practical user experience on computing, graphics and geometric constructions;
- step by step solvers of educational tasks.

There are specific tasks that require the development of new specialized SEP in mathematics, based on the technologies of symbolic transformations and algorithms of computer algebra [10-15].

For this reason, it is necessary to create systems of computer mathematics for educational purposes (SCMEP) in which intelligent capabilities PSCM focused on specific tasks support the learning process.

We understand SCMEP as software for educational purposes for exact sciences and other disciplines that use mathematical models and methods of subject domains based on the technologies of character conversion and methods of computer algebra.

General model of SCMEP is the model of intelligent software that corresponds to the forms of the educational process organization. It is focused on all participants of the educational process and on all kinds of learning activities and based on the knowledge of the subject domain.

SCMEP provides the formation of a declarative (factual) and procedural (algorithmic) knowledge.

Thus, the purpose of the given work is the research of general theoretical and methodological bases, the formulation of the functional requirements to SCMEP and the development of a model of SCMEP as systems of support of learning processes based on the analysis of the topical forms and the peculiarities of the processes of learning of the exact disciplines.

2 The outline of the problem

The concept of programmed learning was introduced in the automated training systems and complexes. On the basis of this concept created a lot of computer courses in mathematics, physics and other disciplines.

In most SEP in mathematics [6-9] the lecture part of the course is the maximally advanced and sophisticated one from both methodical and technical points of view. As a rule, the educational material of the lecture is accompanied by a system of control questions and tests. Such computer system provides incomplete, indirect knowledge control. A mathematician should be able to solve the problem; a programmer should be able to write computer programs. These professional skills are formed during active forms of learning: practical exercises and laboratory work. So, this part of the curriculum in many disciplines (mathematics, physics, computer science ...) is central.

The functionality of software for educational purposes in mathematics is limited only by graphical and computing tasks. They do not perform functions which require character conversion and computer algebra methods. From our point of view, these

properties are crucial for creating effective intelligent SEP in mathematics, physics and other exact and natural sciences.

Market introduction of PSCM led to their intensive implementation into the learning process and numerous educational research on the application of PSCM in learning mathematics. Generally, supervisors recommend using of universal computer algebra system - Mathematics, Maple, Mathcad, Derive and etc. to support workshops [3-5, 16-17]. However, from our point of view, the use of PSCM in the learning process is somewhat limited.

First, PSCM are designed for solving mathematical tasks, while SEP in mathematics should support the process of solving mathematical tasks. This specificity is known as the principles of black and white boxes (Fig. 1).

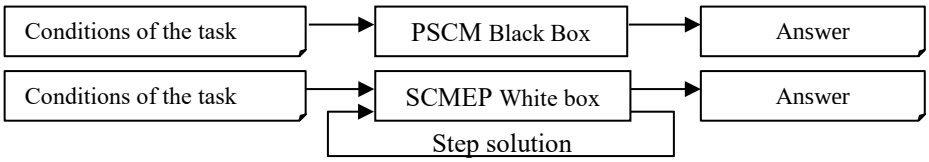

Fig. 1. The principle of black and white boxes.

Second, PSCM does not contain didactic materials. User Manual of PSCM can replace neither a textbook nor a book of mathematical tasks.

Third, the interface of PSCM is not focused on high school students. PSCM usually do not even use the specialized mathematical editor, only a line of editor with limited programming syntax.

Software products for educational purposes are based on the principle of white box do not generate the course of solving a task according to its conditions.

1. Answer Ace, Formulae 1, Universal Math Solver - step by step algebraic tasks' solvers.
2. Bagatrix Geometry Software, Bagarix Algebra Software, Bagatrix Trigonometry Software, Bagatrix Graphing Software - step by step solvers of mathematical problems with capabilities of graphics support.
3. KwikTrig – step by step trigonometric problems' solver.
4. WebAlmir - an integrated software environment for learning the basics of linear algebra [18-20].

In the summary overview of educational software for mathematical disciplines, we are highlighting the following.

There are several functional types of specialized software for learning mathematics on the global market of software. First of all, these are teaching programs such as electronic books, which offer users didactic materials and systems of testing of acquired knowledge. Second, there are software tools of support a user's practical work with computing, graphics and geometric constructs. Third, there are step by step solvers of educational tasks. The most specialized software tools can combine only few of these functional types. Yet, each of these software tools is focused on different stages and on different categories of participants of the educational process.

The universal computer algebra systems, which are positioned as being suitable for use in the educational process, are operating on the principle of black box. They contain no electronic teaching materials (textbook, taskbook, etc.). Besides, they focus more on the application during the studying of applied sciences.

The practice of using PSCM in the learning process has shown that they do not solve all the problems related to the efficiency of the learning process. There are such specific tasks that require the development of new algorithms and specialized software systems for educational purposes, based on the technologies of character conversion and algorithms of computer algebra [10-15].

3 The model of learning process

The rapid growth of the professional requirements for professional widespread complex professionally-oriented information systems penetration into all areas of the global information and communication networks and other factors determine the following fundamental changes in educational technology [21]:

- a wide use of effective information technologies focused not only on classroom work - the work of the teacher and the students, but also on the teacher's individual work with a student, the student's individual work, both in the classroom and beyond;
- a wide use of distance learning not only in higher education but also in secondary and vocational schools;
- an individualization, increased internationalization of the learning process;
- widely used forms of remote access to knowledge, creating entirely new conditions for the organization of independent work.

The concept of information systems for educational purposes (ISEP) is in a comprehensive automation of support of the learning process, the main participants of which are the teacher as a professional, a carrier of knowledge in the field of didactics and teaching methods, in the subject area, and the student as an object of study.

The main goal of ISEP is a comprehensive providing of learning process.

The main tasks of ISEP are:

MT1. Ensure the relevance and accessibility of learning tools.
MT2. Support the process of transferring of new knowledge.
MT3. Support the knowledge control (feedback).

In accordance with the general schemes of the application of ICT in the learning process, we offer conceptual methodological requirements for ISEP:

MR1. ISEP must match the form of the learning process.
MR2. ISEP should focus on all participants of the learning process.
MR3. ISEP should focus on all kinds of training activities.
MR4. ISEP must be based on knowledge of the subject area.

Thus, the problem of the study can be defined as the study of general theoretical and methodological bases, the functional requirements of mathematical models and methods, technologies and tools of the SCMEP creation, that satisfy conceptual and methodological requirements 1-4 and solving the tasks MT1-MT3.

4 The information models for providing a process of learning a discipline

The class-form of organization of learning process for a certain discipline is regulated by curriculum course. This document is the basis for determining the content of documents and didactic materials such as:

- Thematic lesson's plan of discipline,
- Textbooks and educational manuals,
- Taskbooks,
- Libraries of supporting compendiums (visual aids),
- Summaries of lessons,
- Exercisebooks,
- Examination tasks (tickets),
- Methodical manuals,
- Glossaries and reference books.

An important role in informational support of a learning process, regardless of the form of organization, plays technical means of training. There are a blackboard, a video projector, a ruler, a calculator, laboratory equipment, computer, etc.

Models of didactic content ISEP. Information basis for modeling software ISEP is structural and logical schemes (SLS) of subject domain - speciality, discipline, a training module.

SLS of subject domain is a directed graph without cycles, which is the basic unit of domain knowledge and logical connections between them. The last one determine, in particular, the sequence of the study of knowledge units. In our opinion, the SLS should represent a three-level hierarchy:

"speciality" - "discipline" - "training module"

Models of a representation of the level of a learning module are described in knowledge engineering. They are semantic networks, frame models, object-oriented models. A well-known model of mathematical knowledge is semantic (algebraic calculating) Tiugu network [22]. Next, we review the mathematical models of representation of the procedural knowledge in SCMEP.

Model of information support of mathematical discipline is shown on Fig. 2.

The basic structural unit of informational support of a learning process is a training module. SLS of module in mathematical discipline defines:

- The contents of didactic materials;
- Signature of a learning module;
- A list of mathematical models of a learning module;
- A list of elementary transformations of models of a learning module;
- A list of types of training tasks of a learning module.

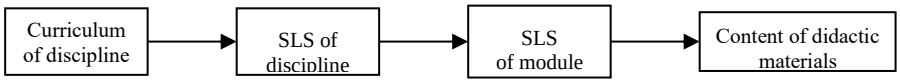

Fig 2. The structure of a knowledge representation of an academic discipline

The signature of learning module. Mathematical theories, which are taught in the module, use mathematical symbols (operations, predicates, functions). For example, Trigonometry module introduces the characters of trigonometric functions and inverse trigonometric functions, a symbol of the constant π. The list of these symbols consists of the signature of a learning module. The complete signature of this module consists of own signature of the module and signatures of modules which this module depends on in SLS of discipline.

Mathematical models of learning module. The subject of the study is the formal definitions of mathematical objects. In the training module Trigonometry it is, for example, the formal definition of trigonometric expressions, trigonometric equations, etc. Graphical tools play an important role in the presentation of mathematical models. Generally mathematical models of applied learning modules can and should be presented in the form of graphs, figures, charts, etc. Therefore, the formal definition of mathematical objects includes the information about the parameters of their graphical representations.

Educational tasks of learning module. The main subject of the study in the learning module of mathematics and exact disciplines is educational tasks; the list of their types is defined in the curriculum of a discipline. These are standard tasks. To solve them is the requirement of the state standard (the educational professional program. The formal definition of an educational task P includes the model of task $M(x_1,\ldots,x_n)$, the condition of task $\varphi(x_1,\ldots,x_n)$ and a question $Q(x_{j_1},\ldots,x_{j_m})$ to it ($P = \langle M, \varphi, Q\rangle$). The definition of educational task can be interpreted as:

Given $M(x_1,\ldots,x_n)$ and $\varphi(x_1, \ldots, x_n)$. Find $Q(x_{j_1},\ldots,x_{j_m})$.

For example, the task: Build a tangent to the graph F of the function $y = \frac{x+1}{x}$ at point A with the abscissa $x_1 = 1$ is submitted by the model:

$$M = F(y = f(x)) \,\&\, A(x_0, y_0) \,\&\, L(y - y_0 = f'(x_0)(x - x_0)) \,\&\, (y_0 = f(x_0)),$$

condition $\varphi = (f(x) = \frac{x+1}{x} \;\&\; (x_0 = 1)$ and question $Q = ?L$.

Let's note, that the model and questions of the task contain the model of the function $y = f(x)$, the points $A(x_0, y_0)$ and the straight (tangent) $y - y_0 = f'(x_0)(x - x_0)$. Letter designations F, A, L mean that these models have graphic images designated by the appropriate letters on the chart.

Elementary transformations of models. The process of solving of an educational task is defined as a sequence of steps, above each of which one of the elementary transformations $M\&\varphi\&Q$ is carried out. For each learning module specific transformations are defined. A complete list of elementary transformations of this learning module consists of specific transformations of this module and transformations of the modules which this module depends on in SLS of discipline.

The model of didactic content is shown on Fig. 3.

Thus, the formal definition of the learning module (Subject Domain) is defined by the quartet $SD = \langle \Sigma, MM, ET, Task\rangle$.

If the learning module in the SLS discipline depends on the modules $SD_1,\ldots, SD_k$, then $\overline{\Sigma}_{SD} = \bigcup_{j=1}^{k} \overline{\Sigma}_j$. Similarly $\overline{ET}_{SD} = \bigcup_{j=1}^{k} \overline{ET}_j$. $\overline{\Sigma}_{SD}$, $\overline{\Sigma}_j$ are the complete signatures of modules (set of all symbols of functions and operations defined in this module). These parities are a formal definition of the modular structure of the learning discipline and speciality.

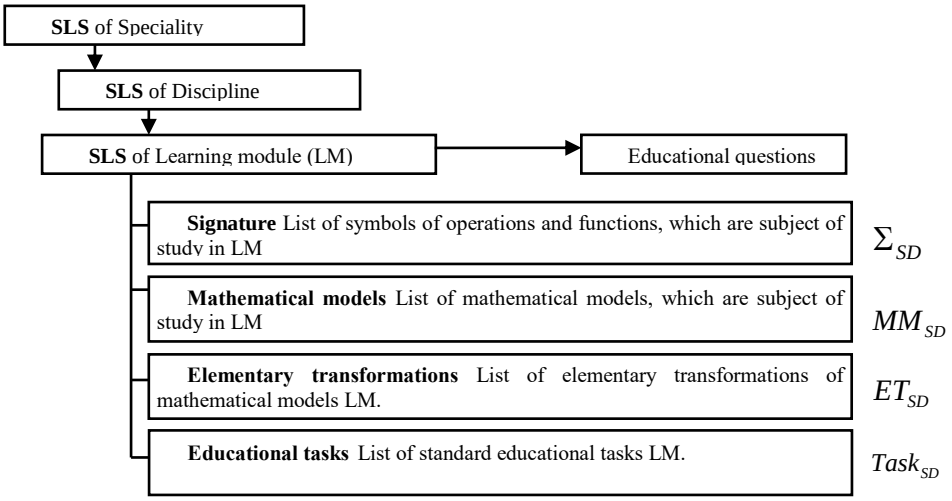

Fig. 3. Model of didactic content of ISEP for speciality

5 Models of the management of a learning process

The analysis of the learning process allows building basic models of learning management: lessons (Fig. 4), lectures, workshops, control works, editing didactic material and consultation.

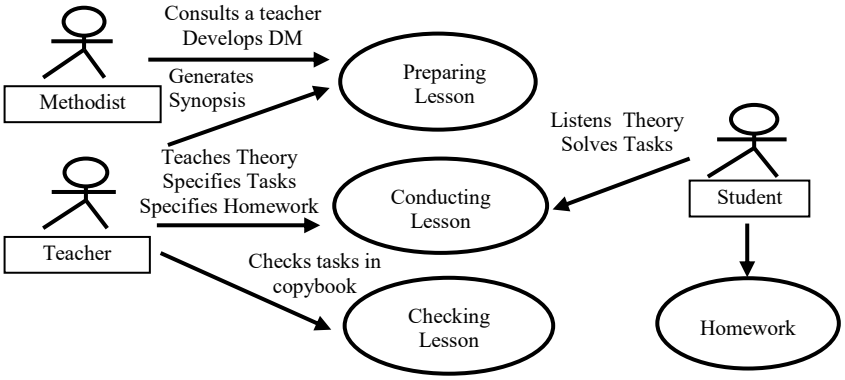

Fig. 4. Model business precedents “Lesson”

First of all, these models take into account all stages of learning - from the preparation to the supply of a relevant precedent's results, secondly, they show participants' learning activities.

The teacher's responsibility is the organizing of a lesson as a business-precedent. It means preparation and conducting of the lesson, reviewing the results of it. A student is responsible for homework, participation and completion of the lesson.

6 Activity environments as a means of support of practical mathematical activity

The content of an educational discipline for a class-lesson form of organization of a learning process has the following structure:

Academic discipline (*name*)
Subject (*number*, *name*)
Theoretical questions (*number*, *name*):
Content // Definition, Properties, Theorems, Proof, Examples …
Practical learning objectives:
Content // Types of educational tasks. Methods for solving of educational tasks.
Thematic tests (*Number*, *Name*)
Content // Options for thematic control work

It is possible to extract the group of materials that contain educational information on the corresponding area of knowledge from all the (electronic) didactic materials in the learning discipline. These are a Textbook, a Taskbook, a Library of Supporting Compendiums, a Solving Environment, an Educational Manual, a Guide, Summaries of Lessons, an Exercisebook, a Library of Practical Lessons, a Testing Environment, etc. All of these didactic materials is intended to provide a certain form of a lesson, a type of educational activity and oriented on a specific category of users as well. The functionality of these modules is described in [23-25].

Practical mathematical student's activity is the main form of educational activity in the study of subjects, based on mathematical models and methods. Its educational goal is to build the course of solving educational tasks without getting answers. That is why object-oriented ISEP in mathematics, such as SCMEP, must support the process of mathematical problem solving. From the technological point of view, the information support of the solving of educational tasks is possible, only if its solution is in a specialized software module - activity environment (AE). AE is the main software modules of SCMEP. There are some works devoted to the mathematical methods of its construction [23, 26, 27-29, 30, 31, 32, 33].

The main function of the Solving Environment (SE) is to support the solving process of mathematical task in different modes.

Here are some basic functional requirements to the SE: the ability to export of a learning task from Taskbook or Exercisebook; the ability to save solving process of the task (fully or partially) in a Exercisebook; the ability of keyboard input of condition of an educational task of one of the standard types; the ability to select one

of the modes of solving the educational task; the ability to review a solving process of the educational task.

One of the most important aspects of practical mathematical support of the student is the validation of the performance of his actions at various stages of solving task - from the stage of the construction of mathematical models to the finishing stage of checking of the process solutions or answers. The second important aspect of the support is the automation of routine activities associated with the calculations. The third aspect is providing a convenient system of tips to a student in various stages of solving the problem like a generation of a mathematical model of a problem, process or step of its solution, answers.

Practical mathematical activity of the teacher should also be supported. The first aspect of this support is checking of the process of the problem solving. The system has to check the correctness of the process of the problem solving, done earlier by the student (mode of control work).

The second aspect of teacher's support is the automation of testing of students' knowledge. The specific activity environment is to verify the knowledge of basic mathematical rules and formulas (special test that uses mathematical tests).

The system should also ensure an effective management of the learning process in whole, supporting the interaction of a teacher and a student.

SCMEP must also provide the user with the appropriate mathematical tools (calculator, graphing tools, etc.).

Testing Environment. Testing is the main most common technology of knowledge control in systems for educational purposes. However, in practice the system of testing verification are oriented on declarative knowledge. The problem of checking the procedural knowledge needs to be resolved.

A characteristic feature of mathematical test is that the response must be in the form of mathematical (logical) expression, and the checking of answers is the checking of the semantic correctness of the expression.

Graphs. An important methodological role in the study of mathematics has graphic construction. The topic "Charts of a function" is a major cross-cutting theme in the course of school algebra. That is why graphical constructions' environments should be included as well in the SCMEP.

Calculator. The main function of the Calculator is to support the solving of learning mathematical task. It is the generation of the response or process of the solution and the answer. This environment implements the classic function Solve(), Simplify() professional systems of computer mathematics. The environment should also support algebraic and arithmetic calculations, including approximate calculations for processing of the results of laboratory experiments.

7 Implementation of the concept of systems of computer mathematics for educational purposes

The scientists of the department of computer science, software engineering and economic cybernetics of Kherson State University are engaged in the implementation of the concept SCMEP.

The researches were carried out during the process of the scientific and technical work on government contracts and state program "Information and communication technologies in education and science". They were based on the practical experience received during the development of ISEP and performed by the order of the Ministry of Education and Sciences (copyright certificates [34-38]):

- Program-methodical complex «TerM VII» of the support of a practical mathematical learning activity.
- Software tool "Library of Electronic Visual Aids Algebra 7-9 grade for secondary schools in Ukraine."
- Pedagogical software tool "Algebra, Grade 7".
- Software tool for educational purposes "Algebra, Grade 8".

The system approach was offered for the construction of models and program realizations of SCMEP as for the development of SCMEP that are listed above The approach can be used at the construction of a wide class of program systems for educational and scientific purposes.

Currently several software products are developed using the same technology. «TerM VII» is one that has English interface.

Fig. 5. General view of the window «TerM VII».

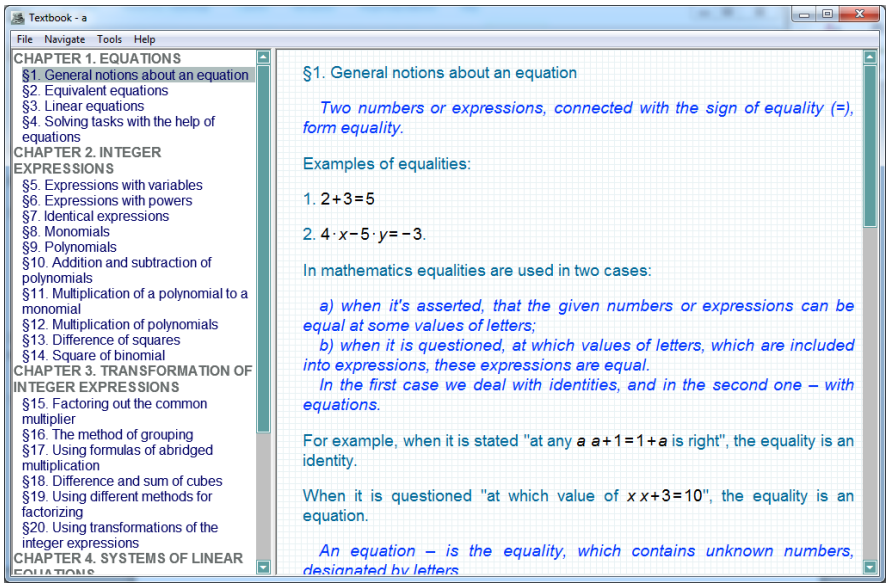

Fig. 6. General view of the page of Textbook

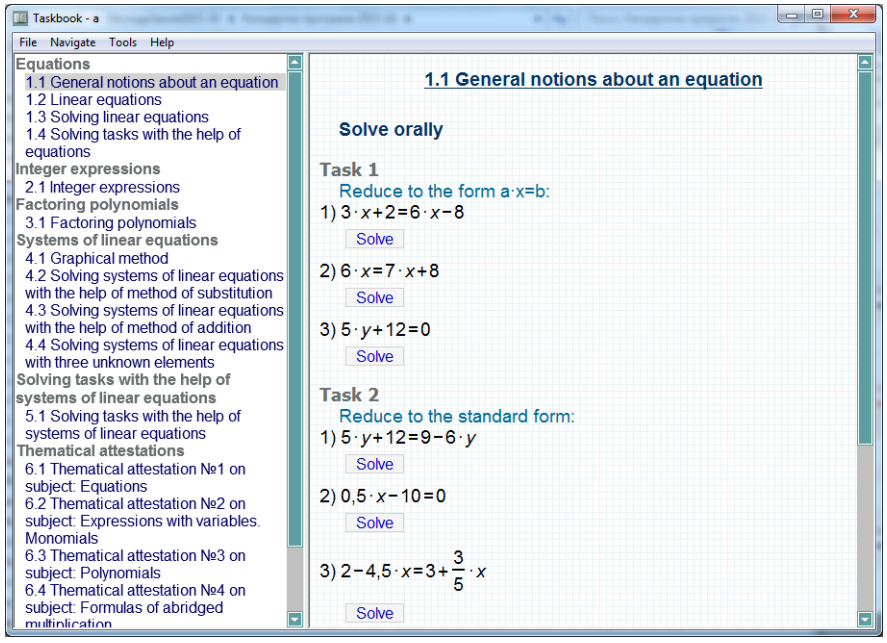

Fig. 7. General view of the window of TaskBook

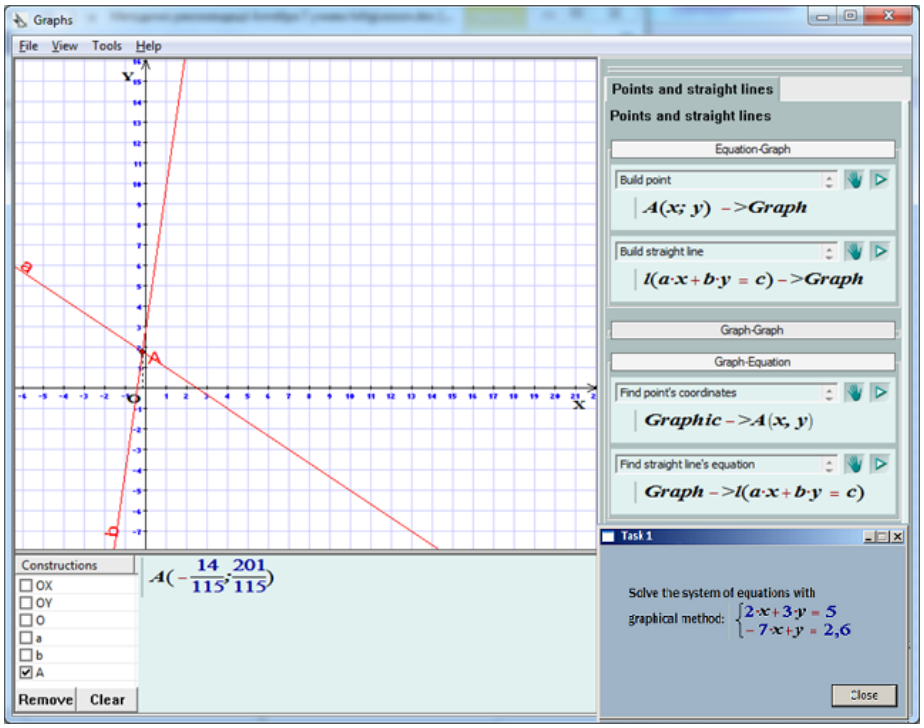

Fig. 8. Solving task in the window of AE "Graphs"

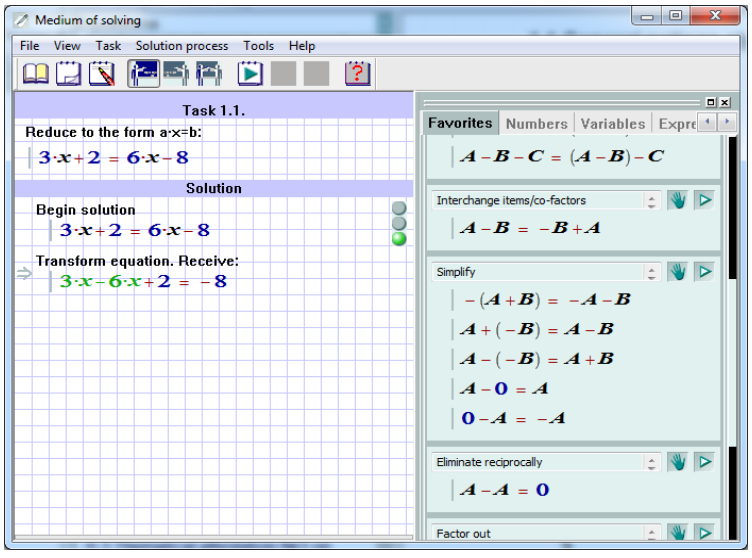

Fig. 9. View of the AE “Solving Environment” with a loaded task

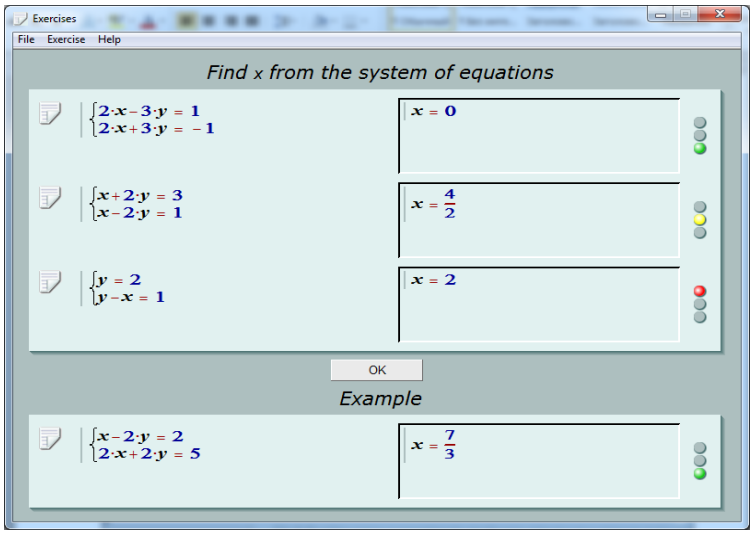

Fig. 10. View of the AE "Testing Environment" with a loaded test.

All SCMEP developed in the framework of this study are used in the educational process in Ukrainian schools. They successfully passed the procedure of the certification, the relevant Commission of Ministry of Education and Science of Ukraine, the established procedures of approbation in secondary schools of Ukraine and recommended for the use in the educational process.

8 Conclusion

ISEP is a system of the learning process support. Its main objectives are to ensure the relevance and accessibility of learning tools, support the processes of new knowledge transferring and the process of knowledge control (feedback).

ISEP should correspond to the contemporary forms of learning process in schools of various levels, be focused on all participants in the learning process, all forms of workshops and activities of participants of the learning process. ISEP must be subject-oriented, based on the knowledge of a corresponding subject domain.

The system of training SCMEP contains appropriate didactic materials and the activity environment for disciplines based on mathematical models and methods.

Practical mathematical activity of a student is the main form of learning activities in the study of subjects, based on mathematical models and methods. Its aim is solving of mathematical learning problems.

Activity environments are computer learning tools designed to support practical mathematical activities of participants of the learning process.

ISEP is the system distributed to the working areas of the participants of the learning process. Each of the area in its composition has appropriate tools of control of the training sessions and types of activities, personalized system of learning tools and tools of the interaction of participants of the learning process support.

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