Hesitant Fuzzy Information Processing Based on the Generalized Aggregation of Resulting Trapezoidal Linguistic Terms

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Abstract. This paper discuss the results of the analysis of multi-criteria decision-making algorithms based on expert evaluations, which are presented in the form of hesitant linguistic terms (LTs). Authors propose to form the resulting trapezoidal linguistic terms for any pairs of hesitant triangular fuzzy numbers. To increase the efficiency of the multi-criteria decision making process with hesitant input data, the authors suggest a new approach for fuzzy aggregation of generalized trapezoidal LTs based on combination of pessimistic and optimistic views of decision-makers. Simulation results proves high efficiency of the proposed hesitant fuzzy information processing approach, in particular in solving multi-criteria problem for selection of the most efficient transport company from the set of the existing alternatives.

Keywords: hesitant fuzzy set, linguistic term, aggregation, pessimistic position, optimistic position, transport company.

1 Introduction

The process of multi-criteria decision-making consists in choosing the best solution among alternatives according to a certain list of criteria. Modeling on the basis of expert knowledge of the system is an approach based on the knowledge and experience of the person who acts as an expert in matters related to the real system [1]. Finding a solution to a multi-criterial problem does not pose any particular difficulties, if the advantage of one of the criteria leads to the same advantage by another criterion, that is, if the criteria are co-operated [2]. Difficulty arises when evaluating the decision according to the relevant criteria is not comparable. In this case, situations arise in which it is difficult for an expert to evaluate an alternative solution using only a quantitative assessment scale. This is due to the fact that the expert's judgment in most cases takes the linguistic form in the form of fuzzy sets, rules and grammar [3]. For example, it is easier for an expert to evaluate a transport company by the criterion of "insurance level of cargo" in a fuzzy form (with the help of linguistic estimates - terms), for example, "the level of cargo insurance - low or lower than average" than giving an exact quantification. Solving the problems of multi-criteria decision-making under uncertainty conditions, in which expert evaluations fluctuate within the limits of several linguistic values of the estimating parameter, is an urgent problem for today. This is due to the complexity of developing models that take into account the relevant conditions of uncertainty, since expert assessments in most cases are presented as fuzzy interval intervals [2, 3]. In this case, the expert usually indicates an assessment within the limits of several TLs for each alternative solution according to the relevant criterion.

2 Related Works and Problem Statement

More and more urgent is the need for the processing of fuzzy, that is, qualitative information, the process of formalization which is quite complicated [1, 2]. In addition, the automation of decision support processes in conditions of uncertainty becomes especially important in conditions of rapid and dynamic growth of the functional capabilities [3].

Different estimation methods are used to measure customer expectations, including questionnaires, expert assessments, statistical methods, etc. The difficulty lies in the fact that most of the parameters of the system cannot be quantified, that is, it is difficult to obtain formalized estimates. An instrument for formalizing the fuzzy expectations of consumers is a mathematical apparatus based on the theory of fuzzy sets [4, 5]. The motivation to use fuzzy logic to solve multi-criteria decision-making tasks is the possibility of a convenient and understandable linguistic interpretation of the processes of constructing models of human judgment, which facilitates their implementation in interactive computer decision support systems [2].

In the decision-making process, expert assessments may vary within several LTs. In paper [6] authors presented several approaches to assess the degree of comparison of vibrational fuzzy sets. The authors of the study [7] proposed the use of various types of distances between vague fuzzy sets. In research [8], the authors proposed a method of multi-criteria decision-making based on a comparison of the values of the probability of fuzzy vibrational sets. Y. Tang and J. Zheng [9] introduced the concept of the sequence of LT in multi-criteria decision-making problems and presented a fuzzy model in which expert estimates are expressed in terms of several LTs. The process of transformation of LT triangular form into a trapezoidal way of their previous aggregation is considered [10], while the transfer of trapezoidal LTs to fuzzy intervals is carried out on the basis of α-cross models, and the choice of the best solution is based on a comparative analysis of alternatives in terms of the probability that the interval is longer than (or equal to) the pessimistic and optimistic position of the decision maker (DM) [3, 11, 12].

The purpose of this study is to develop a new approach for fuzzy aggregation of generalized trapezoidal LTs based on combination of pessimistic and optimistic views of DM and prove high efficiency of the proposed approach, in particular in solving multi-criteria problem for selection of the most efficient transport company from the set of the existing alternatives.

3 Modeling Results

Let's consider the solution of the multi-criteria decision-making problem on the example of the task of choosing the best transportation company for delivery of cargo [13-15]. The method proposed by the authors will be illustrated by the example of choosing the best solution from 4 alternatives according to 8 criteria. In particular, the expert is asked to assess the quality of transport services of the relevant 4 transport companies (x_1 is a transport company A, x_2 is a transport company B, x_3 is a transport company C and x_4 is a transport company D) according to the following criteria: C_1 is a company image, C_2 is a cargo support, C_3 is an insurance level, C_4 is a vehicle traffic monitoring, C_5 is a conservancy of cargo by quantity, C_6 is a consistency of cargo in quality, C_7 is a timeliness of delivery, C_8 is a flexibility of payment service system. The assessment scale is presented in the form of fuzzy triangular LTs {L – low, LM – lower than medium, M – medium, HM – higher than medium, H – high}. Grammar of the formation of expert assessments allows the use of operators {within, lower, and higher} to represent judgments of experts within the limits of several triangular forms.

Let’s formulate in more detail the main steps of the method of the generalized aggregation of resulting trapezoidal LTs proposed by the authors.

Step 1. Formation of the matrix of expert assessments. At this step, the expert evaluates each alternative solution in relation to a certain list of criteria according to the linguistic scale of assessment, presented in the form of a corresponding LTs, for example, a triangular form. In this case, the expert score may vary within several LTs (Table 1).

Table 1. Expert evaluation of alternatives by a certain criteria according to the linguistic scale

	Criteria				
	C_1	C_2	C_3		C_8
x_1	M	within HM and H	HM		within M and HM
x_2	LM	within LM and M	LM	…	within LM and M
x_3	M	within M and HM	M		within M and HM
x_4	within LM and M	LM	lower LM		M

Step 2. Transformation of the matrix of expert estimates (Table 1) into the matrix of interval estimates. At this stage, expert assessments that were within defined limits of LT were transformed into interval-type estimates. If, for example, the expert evaluated the decision x_1 on the criterion C_8 as "within M and HM," then the corresponding estimate is transformed into an interval {M, HM}.

Step 3. Aggregation of LTs into generalized trapezoidal terms. In this case, the combination of interval estimates (LTs triangular form) is combined into generalized trapezoidal terms. The model of LT trapezoidal form can be represented in the form

$S_i^j = (a_1, a_2, a_3, a_4)$, where i is the number of the alternative; j is the number of the criterion (Table 2), for example $S_1^8 = (0.25, 0.5, 0.75, 1)$ (Fig. 1).

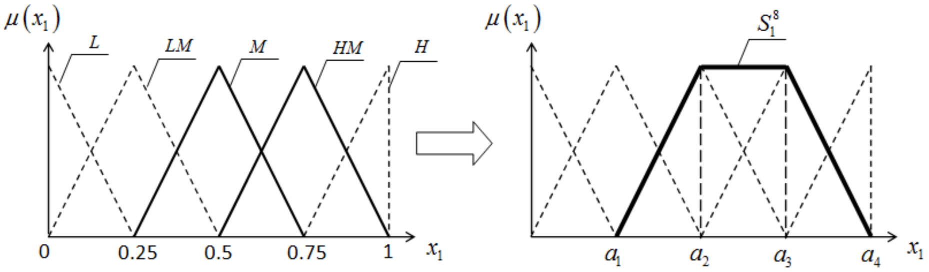

Fig. 1. Aggregation of LTs for the alternative x_1 according to the criterion C_8 into a generalized trapezoidal LT S_1^8

Table 2. The matrix of generalized trapezoidal LTs

	Criteria				
	C_1	C_2	C_3		C_8
x_1	(0.25, 0.5, 0.5, 0.75)	(0.5, 0.75, 1, 1)	(0.5, 0.75, 0.75, 1)	…	(0.25, 0.5, 0.75, 1)
x_2	(0, 0.25, 0.25, 0.5)	(0, 0.25, 0.5, 0.75)	(0, 0.25, 0.25, 0.5)		(0, 0.25, 0.5, 0.75)
x_3	(0.25, 0.5, 0.5, 0.75)	(0.25, 0.5, 0.75, 1)	(0.25, 0.5, 0.5, 0.75)		(0.25, 0.5, 0.75, 1)
x_4	(0, 0.25, 0.5, 0.75)	(0, 0.25, 0.25, 0.5)	(0, 0, 0.25, 0.5)		(0.25, 0.5, 0.5, 0.75)

Step 4. Aggregation of generalized trapezoidal LTs S_i^j into averaged (combined) trapezoidal LTs GS_i. This allows to take into account both the minimum (pessimistic position) and the maximum (optimistic position) expert assessments simultaneously (Fig. 2). It eliminates the need to define intervals for all individual generalized trapezoidal LTs, in particular for the pessimistic and optimistic positions of DM. Averaged trapezoidal LTs GS_i for all alternatives: $GS_1 = (0.25,\ 0.5,\ 1,\ 1)$; $GS_2 = (0,\ 0.25, 0.75, 1)$; $GS_3 = (0.25,\ 0.5, 0.75, 1)$; $GS_4 = (0,\ 0, 0.5, 0.75)$.

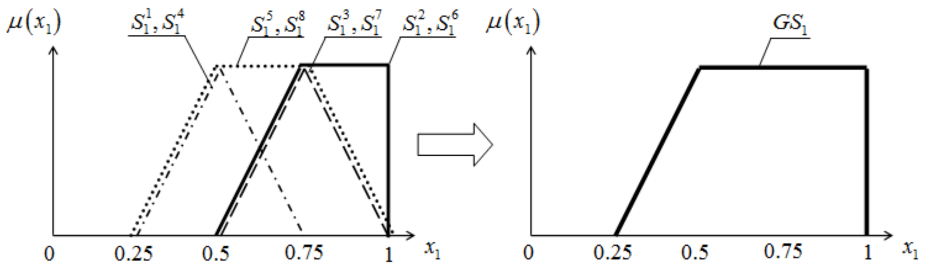

Fig. 2. Aggregation of generalized trapezoidal LTs S_1^j for the alternative x_1 according to all criteria into the averaged trapezoidal LT GS_1

Step 5. Formulating the average trapezoidal LTs of each alternative to fuzzy intervals. At this stage, the formula (1) transforms the averaged trapezoidal LTs into fuzzy intervals [2, 15, 17] with the appropriate choice of parameter $\alpha \in [0,1]$.

$$I(x_i) = [I_L, I_R] = [\alpha(a_2 - a_1) + a_1, a_4 - \alpha(a_4 - a_3)],\ i \in (1,...,m). \quad (1)$$

Step 6. Determine the probability indicator [9, 10, 16] for each alternative (2). Ranking decisions by the appropriate indicator.

$$p(I(x_i) \geq [0,1]) = \max\left(1 - \max\left(\frac{1 - I_L}{I_R - I_L + 1}, 0\right), 0\right),\ i \in (1,...,m). \quad (2)$$

According to Step 5 and Step 6 fuzzy intervals $I(x_i)$ and probability indicators $p(I(x_i) \geq [0,1])$ for all alternatives: $I(x_1) = [0.375,\ 1]$ and $p(I(x_1) \geq [0,1]) = 0.62$; $I(x_2) = [0.125,\ 0.875]$ and $p(I(x_2) \geq [0,1]) = 0.5$; $I(x_3) = [0.375,\ 0.875]$ and $p(I(x_3) \geq [0,1]) = 0.58$; $I(x_4) = [0,\ 0.625]$ and $p(I(x_4) \geq [0,1]) = 0.38$.

As a result of the implementation of the proposed method of the generalized aggregation of resulting trapezoidal LTs, the best alternative is x_1 (company A), $p(I(x_1) \geq [0,1]) > p(I(x_3) \geq [0,1]) > p(I(x_2) \geq [0,1]) > p(I(x_4) \geq [0,1])$, since ranking of the relevant probability indicators allows to determine the next order of priority of the alternativess $x_1 > x_3 > x_2 > x_4$.

The results of the application of existing methods for multi-criteria decision-making based on the pessimistic ($x_1 = x_3 > x_2 > x_4$, execution time is 2300 ms) and optimistic ($x_1 > x_2 = x_3 > x_4$, execution time is 2300 ms) positions of DM [6, 10] and the proposed method of aggregation of generalized trapezoidal LTs ($x_1 > x_3 > x_2 > x_4$, execution time is 1350 ms) prove the performance and effectiveness of the proposed method.

4 Conclusions

The proposed method of aggregation of generalized trapezoidal LTs makes it possible to simplify the process of choosing the best alternative (in comparison with existing methods) and to increase the efficiency, in particular, the speed of the processes of multi-criteria decision-making. This statement is based on a comparative analysis of the time duration of computing operations in the implementation of the appropriate methods. In particular, the implementation time of the program code for the implementation of the method of aggregation of generalized trapezoidal LTs is 1350 microseconds, and for the method of multi-criteria decision-making based on hesitant fuzzy terms using pessimistic and optimistic positions of DM is 2300 microseconds.

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