Temporalizing Epistemic Logic L-DINF*

Stefania Costantini[1,3], Andrea Formisano[2,3,*] and Valentina Pitoni[1]

[1] DISIM - Università dell'Aquila, via Vetoio–loc. Coppito, 67100 L'Aquila, Italy
[2] DMIF - Università di Udine, via delle Scienze 206, 33100 Udine, Italy
[3] GNCS-INdAM, piazzale Aldo Moro 5, 00185 Roma, Italy

Abstract

Agents and Multi-Agent Systems (MAS) are a technology that has many fields of application, which extend also to human sciences and where Computational Logic has been widely applied. In this paper, we join together two of our long-lasting lines of work in this field. In particular, we introduce time and time intervals into the epistemic logic *L-DINF*, that copes with group dynamics in MAS.

Keywords

Epistemic Logic Programs, Agents and Multi-Agent Systems, Temporal Reasoning

1. Introduction

Agents and Multi-Agent Systems are a technology that has many fields of application, which extend also to human sciences (cf., e.g., the recent book [1]). The applications of Computational Logic in the field of agents and MAS are several, as can be seen in the surveys [2, 3] (the latter being very recent). Logic is, in fact, often used to model such kind of systems, as it (at least potentially) provides verifiability and explainability. In this paper, we join together two of our long-lasting lines of work in this field.

The first one [4, 5, 6] was aimed at introducing a treatment of time in agents, so that, upon reception of new perceptions that led to acquire new beliefs, the agent would not have to override old beliefs, but rather to update the time interval where they resulted to hold. In order not to restrict the application of our approach only to certain agent-oriented frameworks, we defined in [6] a "time module" suitable to add time in an easy way into many logic representations of agents. This module is in practice a particular kind of function, that we called T, that assigns a "timing" to atoms, in terms of either single instants or time intervals. We drew inspiration for this work from methods to design agent memorization mechanisms inspired, in turn, by models of human memory [7, 8], that have been developed in cognitive science.

The second line of work [9, 10, 11] has been aimed to formally model via epistemic logic (aspects of) the group dynamics of cooperative agents. Our overall objective has been to devise

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*Corresponding author.

stefania.costantini@univaq.it (S. Costantini); andrea.formisano@uniud.it (A. Formisano); valentina.pitoni@univaq.it (V. Pitoni)

0000-0002-5686-6124 (S. Costantini); 0000-0002-6755-9314 (A. Formisano); 0000-0002-4245-4073 (V. Pitoni)

agent-oriented logical frameworks that allows a designer to formalize and formally verify Multi-Agent Systems, modelling the capability to construct and execute joint plans within a group of agents. We devote a special attention on explainability, in the perspective of Trustworty AI: to enable human-level explanations to be generated, the syntax of our logic is especially devised to make it possible to transpose a proof into a natural language. All along, we have taken into particular account the connection between theory and practice, so as to make our logic actually usable by a system's designers. So, we care about aspects related to enabling and performing physical actions, and to agent's memory of the action performed, where those aspects are often neglected in related work. We have proposed in particular a framework called the Logic of "Inferable" *L-DINF*, based on epistemic logic, where a group of cooperative agents can jointly perform actions. I.e., at least one agent of the group can perform the action, either with the approval of the group or on behalf of the group. We have taken into consideration actions' *cost* [10], and the preferences that each agent can have for what concerns performing each action [11]. We have recently introduced agents' *roles* within a group, in terms of the actions that each agent is enabled by its group to perform. *L-DINF* has a fully-defined semantics, and a proof of strong completeness w.r.t. canonical models. In this paper we incorporated the T function into *L-DINF*, so that actions, goals, plans, of groups of agents are now temporalized, and thus refer to time instants or time intervals. We have joined the two semantic approaches, preserving the strong completeness of the axiomatic framework.

The paper is organized as follows. In Section 2 we present syntax, an example of application to a simple planning problem, namely, a group of researches co-authoring a paper to be submitted, and semantics of the enhanced epistemic logic. Section 3 presents a revised definition of canonical model to take into account the temporal aspect. Finally, in Section 4 we shortly conclude.

2. Logical Framework

L-DINF is a logic which consists of a static component and a dynamic one. The static component, called *L-INF*, is a logic of explicit beliefs and background knowledge. The dynamic component, called *L-DINF*, extends the static one with dynamic operators capturing the consequences of the agents' inferential actions on their explicit beliefs as well as a dynamic operator capturing what an agent can conclude by performing some inferential action in its repertoire.

2.1. Syntax

Let be $Atm = \{p(t_1, t_2), q(t_3, t_4), \ldots, h(t_i, t_j), \ldots\}$ where p, q, h are predicate symbols and each $t_\ell \in \mathbb{N}$. Here an atomic proposition of the form $p(t_1, t_2)$ stands for *"p is true from the time instant t_1 to t_2"* with $t_1 \leqslant t_2$ (*Temporal Representation* of the external world); as a special case we can have $p(t_1, t_1)$ which stands for *"p is true in the time instant t_1"*. We also admit predicate symbols of higher arity, but in that case we assume that the first two arguments are those that identify the time duration of the belief (e.g., the atomic proposition $open(1, 3, \text{door})$ means "the agent knows that the door is open from time 1 to time 3"). By $Prop$ we denote the set of all propositional formulas, i.e. the set of all Boolean formulas built out of the set of atomic propositions Atm. The set Atm_A represents the physical actions that an agent can

perform, including "active sensing" actions (e.g., "let's check whether it rains", "let's measure the temperature"). Let Agt be a set of agents. In what follows, I is a MTL "time-interval" [12] which is a closed finite interval $[t, l]$ or an infinite interval $[t, \infty)$ (considered open on the upper bound), for any expressions/values t, l such that $0 \leq t \leq l$.
The language of *L-DINF*, denoted by $\mathcal{L}_{\textit{L-DINF}}$, is defined by the following grammar:

$$\begin{array}{lll} \varphi, \psi & ::= & p(t_1, t_2) \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{B}_i\, \varphi \mid \mathbf{K}_i\, \varphi \mid \Box_I\, \varphi \mid do_i(\phi_A, I) \mid can\_do_i(\phi_A, I) \mid \\ & & do_G(\phi_A, I) \mid can\_do_G(\phi_A, I) \mid pref\_do_i(\phi_A, d, I) \mid pref\_do_G(i, \phi_A, I) \mid \\ & & exec_i(\alpha) \mid exec_G(\alpha) \mid [G : \alpha]\, \varphi \mid intend_i(\phi_A, I) \mid intend_G(\phi_A, I) \\ \alpha & ::= & \vdash(\varphi,\psi) \mid \cap(\varphi,\psi) \mid \downarrow(\varphi, \psi) \mid \dashv(\varphi, \psi) \end{array}$$

where $p(t_1, t_2)$ ranges over Atm, $d \in \mathbb{N}$, $i \in Agt$ and $G \subseteq Agt$. (Other Boolean operators are defined from $\neg$ and $\wedge$ in the standard manner.) The language of *inferential actions* of type α is denoted by $\mathcal{L}_{\mathsf{ACT}}$. The static part *L-INF* of *L-DINF*, includes only those formulas not having sub-formulas of type α.

Notice the expression $intend_i(\phi_A, I)$, where it is required that $\phi_A \in Atm_A$ and I is a time interval. This expression indicates the intention of agent i to perform action ϕ_A in the interval I in the sense of the BDI agent model [13]. This intention can be part of an agent's knowledge base from the beginning, or it can be derived later. In this paper we do not cope with the formalization of BDI, for which the reader may refer, e.g., to [14]. So, we will treat intentions rather informally, assuming also that $intend_G(\phi_A, I)$ holds whenever all agents in group G intend to perform action ϕ_A in the interval I.

The formula $do_i(\phi_A, I)$, indicates *actual execution* of action ϕ_A by agent i. By precise choice, do (and similarly do_G, that indicates the actual execution of ϕ_A by the group of agents G) are not axiomatized. In fact, they are realized by what has been called in [15] a *semantic attachment*, i.e., a procedure which connects an agent with its external environment in a way that is unknown at the logical level. The axiomatization concerns only the relationship between doing and being enabled to do.

The expressions $can\_do_i(\phi_A, I)$ and $pref\_do_i(\phi_A, d, I)$ (where, as before, $\phi_A \in Atm_A$ and I is a time interval) are closely related to $do_i(\phi_A, I)$. In fact, $can\_do_i(\phi_A, I)$ is to be seen as an enabling condition, indicating that agent i is enabled to execute action ϕ_A in the interval I, while instead $pref\_do_i(\phi_A, d, I)$ indicates the level d of preference/willingness of agent i to perform that action in the time interval I. $pref\_do_G(i, \phi_A, I)$ indicates that agent i exhibits the *maximum level* of preference on performing action ϕ_A within all group members in the time interval I. Notice that, if a group of agents intends to perform an action ϕ_A, this will entail that the entire group intends to do ϕ_A, that will be enabled to be actually executed only if at least one agent $i \in G$ can do it, i.e., it can derive $can\_do_i(\phi_A, I)$.

Unlike explicit beliefs, i.e., facts and rules acquired via perceptions during an agent's operation and kept in the *working memory*, an agent's background knowledge is assumed to satisfy *omniscience* principles, such as closure under conjunction and known implication, and closure under logical consequence, and introspection. In fact, $\mathbf{K}_i$ is actually the well-known S5 modal operator often used to model/represent knowledge. The fact that background knowledge is closed under logical consequence is justified because we conceive it as a kind of stable reliable *knowledge base*, or *long-term memory*. We assume the background knowledge to include: facts (formulas) known by the agent from the beginning, and facts the agent has later decided to store in its long-term memory (by means of some decision mechanism not treated here) after having processed them in its working memory. We therefore assume background knowledge to be irrevocable, in the sense of being stable over time.

In the formula $\Box_I \phi$ the MTL Interval "always" operator is applied to a formula, which means that ϕ is always true in the interval I. $\Box_{[0,\infty)}$ will sometimes be written simply as $\Box$.

A formula of the form $[G{:}\alpha]\, \varphi$, with $G \subseteq Agt$, and where α must be an inferential action, states that "φ holds after action α has been performed by at least one of the agents in G, and all agents in G have common knowledge about this fact".

Borrowing from [11, 16], we distinguish four types of inferential actions α which allow us to capture some of the dynamic properties of explicit beliefs and background knowledge: $\downarrow(\varphi, \psi)$, $\cap(\varphi,\psi)$, $\dashv(\varphi, \psi)$, and $\vdash(\varphi,\psi)$, These actions characterize the basic operations of forming explicit beliefs via inference:

- $\downarrow(\varphi, \psi)$: this action infers ψ from φ, where ψ is an atom, say $p(t_1, t_2)$: an agent, believing that φ is true and having in its long-term memory that φ implies ψ (in some suitable time interval including $[t_1, t_2]$), starts believing that $p(t_1, t_2)$ is true.
- $\cap(\varphi,\psi)$: this action closes the explicit beliefs φ and ψ under conjunction. I.e., $\cap(\varphi,\psi)$ characterizes the inferential action of deducing $\varphi \wedge \psi$ from the explicit belief φ and the explicit belief ψ.
- $\dashv(\varphi, \psi)$: this action performs a simple form of "belief revision", where φ and ψ are atoms, say $p(t_1, t_2)$ and $q(t_3, t_4)$ respectively: an agent, believing $p(t_1, t_2)$ and having in the long-term memory that $p(t_1, t_2)$ implies $\neg q(t_3, t_4)$, removes the timed belief $q(t_3, t_4)$ if the intervals match. Notice that, should q be believed in a wider interval I such that $[t_1, t_2] \subseteq I$, the belief $q(.,.)$ is removed concerning intervals $[t_1, t_2]$ and $[t_3, t_4]$, but it is left for the remaining sub-intervals (so, its is "restructured").
- $\vdash(\varphi,\psi)$: let ψ be an atom, say $p(t_1, t_2)$. An agent, believing φ and that φ implies $p(t_1, t_2)$ in the working memory (in some suitable time interval including $[t_1, t_2]$), starts believing $p(t_1, t_2)$. This last action operates directly on the working memory without retrieving anything from the background knowledge.

Formulas of the forms $exec_i(\alpha)$ and $exec_G(\alpha)$ express executability of inferential actions either by agent i, or by a group G of agents (which is a consequence of any of the group members being able to execute the action). It has to be read as: "α is an inferential action that agent i (resp. an agent in G) can perform".

2.2. Problem Specification and Inference: An Example

In this section, we propose an example to explain the usefulness of this kind of logic and to help the reader's understanding. Consider a group G of three agents, who are the authors of a paper that has to be submitted to a conference: the first author a deals with the drafting of the introduction and finding the references, the second b deals with the experiments and the third c deals with the formalization part. The second is the only one who can perform the experiments because he has the required certifications; the others are enabled to perform different tasks, such as, e.g., write the abstract, search references, check the correctness of the formal part, and so on.

The group receives notification of a deadline for a paper, so they decide to organize themselves for submitting it. The group will reason, and devise the intention/goal $\mathbf{K}_i(\Box_I\, intend_G(submit\_fullpaper(t_0, t_2), I))$: the group intends to submit their paper within the indicated time I. Here t_0 is the time instant when the group begins to organize to write the paper, $I = [t_0, t_1]$ where t_1 is the deadline and t_2 is the time instant when they really submit the paper and $t_2 \leq t_1$.

Among the physical actions that agents in the group can perform are for instance the following: $submit\_abstract$, $do\_experiment$, $write\_introduction$,$write\_formal\_part$ and $write\_experiment\_results$.

The group will now be required to perform a planning activity. Assume that, as a result of the planning phase, the knowledge base of each agent i contains the following rule, that specifies how to reach the intended goal in terms of actions to perform and sub-goals to achieve (listed after the " $\rightarrow$ "):

$$\begin{aligned}\mathbf{K}_i\big(\Box_I\, intend_G(submit\_fullpaper(t_0, t_2), I) \rightarrow \Box_{I_1}\, intend_G(submit\_abstract(t_0, t_3), I_1) \\ \wedge \Box_{I_2}\, intend_G(do\_experiment(t_0, t_4), I_2) \\ \wedge \Box_I\, intend_G(write\_formal\_part(t_0, t_5), I)\big)\end{aligned}$$

where $I_1, I_2 \subseteq I$, t_3 is the time instant when the author submit the abstract and $t_3 \leq t_1$, t_4 is the time instant when the author b has finished his experiment and he has written the results at $t_4 \leq t_1$, finally t_5 is the time instant when the other agent has finished to write the formal part. Thanks to the axiomatization, which we are going to explain in Section 2.5, we have that $intend_G(\phi_A, I) \leftrightarrow \forall i \in G\; intend_i(\phi_A, I)$, each agent has the specialized rule (for $i \leq 3$):

$$\begin{aligned}\mathbf{K}_i\big(\Box_I\, intend_i(submit\_fullpaper(t_0, t_2), I) \rightarrow \Box_{I_1}\, intend_i(submit\_abstract(t_0, t_3), I_1) \wedge \\ \Box_{I_2}\, intend_i(do\_experiment(t_0, t_4), I_2) \wedge \\ \Box_I\, intend_i(write\_formal\_part(t_0, t_5), I)\big)\end{aligned}$$

Therefore, the following is entailed for each of the agents:

$$\begin{aligned}&\mathbf{K}_i\big(\Box_I\, intend_i(submit\_fullpaper(t_0, t_2), I) \rightarrow \Box_{I_1}\, intend_i(submit\_abstract(t_0, t_3), I_1)\big) \\ &\mathbf{K}_i\big(\Box_I\, intend_i(submit\_fullpaper(t_0, t_2), I) \rightarrow \Box_{I_2}\, intend_i(do\_experiment(t_0, t_4), I_2)\big) \\ &\mathbf{K}_i\big(\Box_I\, intend_i(submit\_fullpaper(t_0, t_2), I) \rightarrow \Box_I\, intend_i(write\_formal\_part(t_0, t_5), I)\big).\end{aligned}$$

Assume now that the knowledge base of each agent i contains also the following general rules, stating that the group is available to perform each of the necessary actions. Which agent will in particular perform each action ϕ_A? According to items (t4) and (t7) in the definition of truth values, listed in the next section, for *L-DINF* formulas, this agent will be chosen as the one which best prefers to perform this action, among those that can do it. Formally, in the present situation, $pref\_do_G(i, \phi_A, I)$ identifies the agent i in the group with the highest degree of preference on performing ϕ_A, and $can\_do_G(\phi_A, I)$ is true if there is some agent i in the group which is able and allowed to perform ϕ_A, i.e., $\phi_A \in A(i, w) \wedge \phi_A \in H(i, w)$.

$$\mathbf{K}_i\big(\Box_{I_1}(intend_G(submit\_abstract(t_0,t_3),I_1)\wedge can\_do_G(submit\_abstract(t_0,t_3),I_1)\wedge$$
$$pref\_do_G(i,submit\_abstract(t_0,t_3),I_1))\rightarrow \Box_{I_1} do_G(submit\_abstract(t_0,t_3),I_1)\big)$$
$$\mathbf{K}_i\big(\Box_{I_2}(intend_G(do\_experiment(t_0,t_4),I_2)\wedge can\_do_G(do\_experiment(t_0,t_4),I_2)\wedge$$
$$pref\_do_G(i,do\_experiment(t_0,t_4),I_2))\rightarrow \Box_{I_2} do_G(do\_experiment(t_0,t_4),I_2)\big)$$
$$\mathbf{K}_i\big(\Box_{I}(intend_G(write\_formal\_part(t_0,t_5),I)\wedge can\_do_G(write\_formal\_part(t_0,t_5),I)\wedge$$
$$pref\_do_G(i,write\_formal\_part(t_0,t_5),I))\rightarrow \Box_{I} do_G(write\_formal\_part(t_0,t_5),I)\big)$$

As before, such rules can be specialized to each single agent.

$$\mathbf{K}_i\big(\Box_{I_1}(intend_i(submit\_abstract(t_0,t_3),I_1)\wedge can\_do_i(submit\_abstract(t_0,t_3),I_1)\wedge$$
$$pref\_do_i(i,submit\_abstract(t_0,t_3),I_1))\rightarrow \Box_{I_1} do_i(submit\_abstract(t_0,t_3),I_1)\big)$$
$$\mathbf{K}_i\big(\Box_{I_2}(intend_i(do\_experiment(t_0,t_4),I_2)\wedge can\_do_i(do\_experiment(t_0,t_4),I_2)\wedge$$
$$pref\_do_i(i,do\_experiment(t_0,t_4),I_2))\rightarrow \Box_{I_2} do_i(do\_experiment(t_0,t_4),I_2)\big)$$
$$\mathbf{K}_i\big(\Box_{I}(intend_i(write\_formal\_part(t_0,t_5),I)\wedge can\_do_i(write\_formal\_part(t_0,t_5),I)\wedge$$
$$pref\_do_i(i,write\_formal\_part(t_0,t_5),I))\rightarrow \Box_{I} do_i(write\_formal\_part(t_0,t_5),I)\big)$$

So, for each action ϕ_A required by the plan, there will be some agent (let us assume for simplicity only one), for which $do_i(\phi_A, I)$ will be concluded. In our case, the agent a will conclude $do_a(submit\_abstract(t_0,t_3),I_1)$; the agent b will conclude $do_b(do\_experiment(t_0,t_4),I_2)$ and the agent c will conclude $do_c(write\_formal\_part(t_0,t_5),I)$.

2.3. Semantics

Now we can go into the details of semantics, definition 2.1 introduces the notion of *L-INF model*, which is then used to introduce semantics of the static fragment of the logic. Before that we define the "time" function T that associates to each formula the time interval in which this formula is true and operates as follows:

- $T(p(t_1,t_2)) = [t_1,t_2]$, which stands for *"p is true in the time interval* $[t_1,t_2]$*"* where $t_1,t_2 \in \mathbb{N}$; as a special case we have $T(p(t_1,t_1)) = t_1$, which stands for *"p is true in the time instant* t_1*"* where $t_1 \in \mathbb{N}$ (time instant);
- $T(\neg p(t_1,t_2)) = T(p(t_1,t_2))$, which stands for *"p is not true in the time interval* $[t_1,t_2]$*"* where $t_1,t_2 \in \mathbb{N}$;
- $T(\varphi \text{ op } \psi) = T(\varphi) \uplus T(\psi)$ with $op \in \{\vee,\wedge,\rightarrow\}$, which is the unique smallest interval including both $T(\varphi)$ and $T(\psi)$;
- $T(\mathbf{B}_i\varphi) = T(\varphi)$;
- $T(\mathbf{K}_i\varphi) = T(\varphi)$;
- $T(\Box_I\varphi) = I$ where I is a time interval;
- $T([(G:\alpha)]\varphi)$ there are different cases depending on the inferential action α:

  1. $T([G:\downarrow(\varphi,\psi)]\,\psi) = T(\psi)$;
  2. $T([G:\cap(\varphi,\psi)]\,(\varphi\wedge\psi)) = T(\varphi) \uplus T(\psi)$, the smallest interval including $T(\varphi)$ and $T(\psi)$;
  3. $T([G:\dashv(\varphi,\psi)]\,\psi)$ returns the "restructured" interval where ψ is true;
  4. $T([G:\vdash(\varphi,\psi)]\,\psi) = T(\psi)$;

- $T(do_i(\phi_A, I)) = T(do_G(\phi_A, I)) = I$;
- $T(can\_do_i(\phi_A, I)) = T(can\_do_G(\phi_A, I)) = I$;
- $T(intend_i(\phi_A, I)) = T(intend_G(\phi_A, I)) = I$;
- $T(pref\_do_i(\phi_A, d, I)) = T(pref\_do_G(i, \phi_A, I)) = I$;
- $T(exec_i(\alpha)) = T(exec_G(\alpha)) = T([(G : \alpha)]\varphi)$.

Definition 2.1, below, depends on a given *set of world* W and a *valuation function*, namely a mapping $V : W \longrightarrow 2^{Atm}$. For each world $w \in W$, let t_1 the minimum time instant of $T(p(t_1, t))$ where $p(t_1, t) \in V(w)$ and let t_2 be the supremum time instant (we can have $t_2 = \infty$) w.r.t. the atoms $p(t, t_2)$ in $V(w)$. Whenever useful, we denote w as w_I where $I = [t_1, t_2]$, which identifies the world in a given interval.

Notice that many relevant aspects of an agent's behaviour are specified in the definition of *L-INF model*, including which mental and physical actions an agent can perform, which is the cost of an action and which is the budget that the agent has available, which is the preference degree of the agent to perform each action. This choice has the advantages of keeping the complexity of the logic under control, and of making these aspects modularly modifiable. As before let Agt be the set of agents.

Definition 2.1. *A model is a tuple* $M = (W, N, \mathcal{R}, E, B, C, A, H, P, V, T)$ *where:*

- W *is a set of worlds (or situations);*
- $\mathcal{R} = \{R_i\}_{i \in Agt}$ *is a collection of equivalence relations on* W*:* $R_i \subseteq W \times W$ *for each* $i \in Agt$*;*
- $N : Agt \times W \longrightarrow 2^{2^W}$ *is a neighborhood function such that, for each* $i \in Agt$*, each* $w_I, v_I \in W$*, and each* $X \subseteq W$ *these conditions hold:*

  (C1) *if* $X \in N(i, w_I)$ *then* $X \subseteq \{v_I \in W \mid w_I R_i v_I\}$*,*

  (C2) *if* $w_I R_i v_I$ *then* $N(i, w_I) = N(i, v_I)$*;*
- $E : Agt \times W \longrightarrow 2^{\mathcal{L}_{\mathsf{ACT}}}$ *is an executability function of mental actions such that, for each* $i \in Agt$ *and* $w_I, v_I \in W$*, it holds that:*

  (D1) *if* $w_I R_i v_I$ *then* $E(i, w_I) = E(i, v_I)$*;*
- $B : Agt \times W \longrightarrow \mathbb{N}$ *is a budget function such that, for each* $i \in Agt$ *and* $w_I, v_I \in W$*, the following holds*

  (E1) *if* $w_I R_i v_I$ *then* $B(i, w_I) = B(i, v_I)$*;*
- $C : Agt \times \mathcal{L}_{\mathsf{ACT}} \times W \longrightarrow \mathbb{N}$ *is a cost function such that, for each* $i \in Agt$*,* $\alpha \in \mathcal{L}_{\mathsf{ACT}}$*, and* $w_I, v_I \in W$*, it holds that:*

  (F1) *if* $w_I R_i v_I$ *then* $C(i, \alpha, w_I) = C(i, \alpha, v_I)$*;*
- $A : Agt \times W \longrightarrow 2^{Atm_A}$ *is an executability function for physical actions such that, for each* $i \in Agt$ *and* $w_I, v_I \in W$*, it holds that:*

  (G1) *if* $w_I R_i v_I$ *then* $A(i, w_I) = A(i, v_I)$*;*
- $H : Agt \times W \longrightarrow 2^{Atm_A}$ *is an enabling function for physical actions such that, for each* $i \in Agt$ *and* $w_I, v_I \in W$*, it holds that:*

(G2) *if* $w_I R_i v_I$ *then* $H(i, w_I) = H(i, v_I)$*;*

- $P : Agt \times W \times Atm_A \longrightarrow \mathbb{N}$ *is a preference function for physical actions* ϕ_A *such that, for each* $i \in Agt$ *and* $w_I, v_I \in W$*, it holds that:*

  (H1) *if* $w_I R_i v_I$ *then* $P(i, w_I, \phi_A) = P(i, v_I, \phi_A)$*;*
- $V : W \longrightarrow 2^{Atm}$ *is a valuation function;*
- *T is the "Time Function", defined before.*

To simplify the notation, let $R_i(w_I) = \{v_I \in W \mid w_I R_i v_I\}$, for $w_I \in W$. The set $R_i(w_I)$ identifies the situations that agent i considers possible at world w_I. It is the *epistemic state* of agent i at w_I. In cognitive terms, $R_i(w_I)$ can be conceived as the set of all situations that agent i *can retrieve* from its long-term memory and reason about.

While $R_i(w_I)$ concerns background knowledge, $N(i, w_I)$ is the set of all facts that agent i explicitly believes at world w_I, a fact being identified with a set of worlds. Hence, if $X \in N(i, w_I)$ then, the agent i has the fact X under the focus of its attention and believes it. We say that $N(i, w_I)$ is the explicit *belief set* of agent i at world w_I.

The executability of inferential actions is determined by the function E. For an agent i, $E(i, w_I)$ is the set of inferential actions that agent i can execute at world w_I in time interval I. The value $B(i, w_I)$ is the budget the agent has available to perform inferential actions in time interval I. Similarly, the value $C(i, \alpha, w_I)$ is the cost to be paid by agent i to execute the inferential action α in the world w_I in time interval I. The executability of physical actions is determined by the function A. For an agent i, $A(i, w_I)$ is the set of physical actions that agent i can execute at world w_I in time interval I. $H(i, w_I)$ instead is the set of physical actions that agent i is enabled by its group to perform always in I. Which means, H defines the *role* of an agent in its group, via the actions that it is allowed to execute.

Agent's preference on executability of physical actions is determined by the function P. For an agent i, and a physical action ϕ_A, $P(i, w_I, \phi_A)$ is an integer value d indicating the degree of willingness of i to execute ϕ_A at world w_I.

Constraint (**C1**) imposes that agent i can have explicit in its mind only facts which are compatible with its current epistemic state. Moreover, according to constraint (**C2**), if a world v_I is compatible with the epistemic state of agent i at world w_I, then agent i should have the same explicit beliefs at w_I and v_I. In other words, if two situations are equivalent as concerns background knowledge, then they cannot be distinguished through the explicit belief set. This aspect of the semantics can be extended in future work to allow agents make plausible assumptions. Analogous properties are imposed by constraints (**D1**), (**E1**), and (**F1**). Namely, (**D1**) imposes that agent i always knows which inferential actions it can perform and those it cannot. (**E1**) states that agent i always knows the available budget in a world (potentially needed to perform actions). (**F1**) determines that agent i always knows how much it costs to perform an inferential action. (**G1**) and (**H1**) determine that an agent i always knows which physical actions it can perform and those it cannot, and with which degree of willingness, where (**G2**) specifies that an agent also knows whether its group gives it the permission to execute a certain action or not, i.e., if that action pertains to its *role* in the group.

Given a model $M = (W, N, \mathcal{R}, E, B, C, A, H, P, V, T)$, $i \in Agt$, $G \subseteq Agt$, $w_I \in W$, and a formula $\varphi \in \mathcal{L}_{L\text{-}INF}$, we introduce the following shorthand notation:

$$\|\varphi\|^M_{i,w_I} = \{v_I \in W : w_I R_i v_I \text{ and } M, v_I \models \varphi\}$$

whenever $M, v_I \models \varphi$ is well-defined (see below). Then, truth values of *L-DINF* formulas are inductively defined as follows:

(t1) $M, w_I \models p(t_1, t_2)$ iff $p(t_1, t_2) \in V(w_I)$ and $T(p(t_1, t_2)) \subseteq I$
(t2) $M, w_I \models exec_i(\alpha)$ iff $\alpha \in E(i, w_I)$ and $T(exec_i(\alpha)) \subseteq I$
(t3) $M, w_I \models exec_G(\alpha)$ iff $\exists i \in G$ with $\alpha \in E(i, w_I)$ and $T(exec_G(\alpha)) \subseteq I$
(t4) $M, w_I \models can\_do_i(\phi_A, J)$ iff $\phi_A \in A(i, w_I) \cap H(i, w_I)$ and $J \subseteq I$
(t5) $M, w_I \models can\_do_G(\phi_A, J)$ iff $\exists i \in G$ with $\phi_A \in A(i, w_I) \cap H(i, w_I)$ and $J \subseteq I$
(t6) $M, w_I \models pref\_do_i(\phi_A, d, J)$ iff $\phi_A \in A(i, w_I)$, $P(i, w_I, \phi_A) = d$ and $J \subseteq I$
(t7) $M, w_I \models pref\_do_G(i, \phi_A, J)$ iff $M, w \models pref\_do_i(\phi_A, d, J)$ for $d = \max\{P(j, w, \phi_A) \mid j \in G \wedge \phi_A \in A(j, w) \cap H(j, w)\}$ and $J \subseteq I$
(t8) $M, w_I \models \neg\varphi$ iff $M, w \not\models \varphi$ and $T(\neg\varphi) \subseteq I$
(t9) $M, w_I \models \varphi \wedge \psi$ iff $M, w \models \varphi$ and $M, w \models \psi$ with $T(\varphi), T(\psi) \subseteq I$
(t10) $M, w_I \models \mathbf{B}_i\, \varphi$ iff $||\varphi||^M_{i,w} \in N(i, w)$ with $T(\varphi) \subseteq I$
(t11) $M, w_I \models \mathbf{K}_i\, \varphi$ iff $M, v \models \varphi$ for all $v \in R_i(w)$ with $T(\varphi) \subseteq I$
(t12) $M, w_I \models \Box_J\, \varphi$ iff $T(\varphi) \subseteq J \subseteq I$ and for all $v_I \in R_i(w_I)$ it holds $M, w_I \models \varphi$

As seen above, a physical action can be performed by a group of agents if at least one agent of the group can do it, and the level of preference for performing this action is set to the maximum among those of the agents enabled to do this action. For any inferential action α performed by any agent i, we set:
$M, w \models [G : \alpha]\varphi$ iff $M^{[G:\alpha]}, w \models \varphi$
where $M^{[G:\alpha]} = \langle W, N^{[G:\alpha]}, \mathcal{R}, E, B^{[G:\alpha]}, C, A, H, P, V, T\rangle$, is the model representing the fact that the execution of an inferential action α affects the sets of beliefs of agent i and modifies the available budget in a certain time interval I. Such operation can add new beliefs by direct perception, by means of one inference step, or as a conjunction of previous beliefs. Hence, when introducing new beliefs (i.e., performing mental actions), the neighborhood must be extended accordingly.

The following condition characterizes the circumstances in which an action may be performed, and by which agent(s):

$$enabled_{w_I}(G, \alpha) : \;\; \exists j \in G\, (\alpha \in E(j, w) \wedge \tfrac{C(j,\alpha,w_I)}{|G|} \leq \min_{h \in G} B(h, w_I))$$

with $T([G{:}\alpha]\varphi) \subseteq I$. This condition states when an inferential action is enabled. In the above particular formulation (that is not fixed, but can be customized to the specific application domain) if at least an agent can perform it and if the "payment" due by each agent (obtained by dividing the action's cost equally among all agents of the group) is within each agent's available budget. In case more than one agent in G can execute an action, we implicitly assume the agent j performing the action to be the one corresponding to the lowest possible cost. Namely, j is such that $C(j, \alpha, w_I) = \min_{h \in G} C(h, \alpha, w_I)$. Other choices might be viable, so variations of this logic can be easily defined simply by devising some other enabling condition and, possibly, introducing differences in neighborhood update. Notice that the definition of the enabling function basically specifies the "concrete responsibility" that agents take while concurring with their own resources to actions' execution. Also, in case of specification of various resources, different corresponding enabling functions might be defined.

2.4. Belief Update

In this kind of logic, updating an agent's beliefs accounts to modify the neighborhood of the present world. The updated neighborhood $N^{[G:\alpha]}$ resulting from execution of a mental action α by a group G of agents is as follows.

$$N^{[G:\downarrow(\psi,\chi)]}(i,w_I) = \begin{cases} N(i,w_I) \cup \{||\chi||^M_{i,w_I}\} & \text{if } i \in G \text{ and } T([G:\downarrow(\psi,\chi)]\chi) \subseteq I \text{ and} \\ & \mathit{enabled}_{w_I}(G,\downarrow(\psi,\chi)) \text{ and} \\ & M,w_I \models \mathbf{B}_i\psi \wedge \mathbf{K}_i(\psi \rightarrow \chi) \\ N(i,w_I) & \text{otherwise} \end{cases}$$

$$N^{[G:\cap(\psi,\chi)]}(i,w_I) = \begin{cases} N(i,w_I) \cup \{||\psi \wedge \chi||^M_{i,w_I}\} & \text{if } i \in G \text{ and} \\ & T([G:\cap(\psi,\chi)](\psi \wedge \chi)) \subseteq I \text{ and} \\ & \mathit{enabled}_{w_I}(G,\cap(\psi,\chi)) \text{ and} \\ & M,w_I \models \mathbf{B}_i\psi \wedge \mathbf{B}_i\chi \\ N(i,w_I) & \text{otherwise} \end{cases}$$

$$N^{[G:\vdash(\psi,\chi)]}(i,w_I) = \begin{cases} N(i,w_I) \cup \{||\chi||^M_{i,w_I}\} & \text{if } i \in G \text{ and } T([G:\dashv(\psi,\chi)]\chi) \subseteq I \text{ and} \\ & \mathit{enabled}_{w_I}(G,\vdash(\psi,\chi)) \text{ and} \\ & M,w_I \models \mathbf{B}_i\psi \wedge \mathbf{B}_i(\psi \rightarrow \chi) \\ N(i,w_I) & \text{otherwise} \end{cases}$$

Notice that, after an inferential action α has been performed by an agent $j \in G$, all agents $i \in G$ see the same update in the neighborhood. Conversely, for any agent $h \notin G$ the neighborhood remains unchanged (i.e., $N^{[G:\alpha]}(h,w) = N(h,w_I)$). However, even for agents in G, the neighborhood remains unchanged if the required preconditions, on explicit beliefs, knowledge, and budget, do not hold (and hence the action is not executed). Notice also that we might devise variations of the logic by making different decisions about neighborhood update to implement, for instance, partial visibility within a group.

For formulas of the form $[G:\dashv(\psi,\chi)]\chi$, we take in account the following ground case: given $Q = q(j,k)$ such that $T(q(j,k)) = T(q(t_1,t_2)) \cap T(q(t_3,t_4))$ with $j,k \in \mathbb{N}$ and $P \equiv \Big((M,w_I \models \mathbf{B}_i(p(t_1,t_2)) \wedge \mathbf{B}_i(q(t_3,t_4)) \wedge \mathbf{K}_i(p(t_1,t_2) \rightarrow \neg q(t_3,t_4))\Big)$ and $\big(T([G:\dashv (p(t_1,t_2),q(t_3,t_4))]q(t_5,t_6)) \subseteq I\big)$ and there is no interval $J \supsetneq T(p(t_1,t_2))$ s.t. $\mathbf{B}_i(q(t_5,t_6))$

where $T(q(t_5, t_6))=J\Big)$:

$$N^{[G:\dashv(p(t_1,t_2),q(t_3,t_4))]}(i, w_I) = \begin{cases} N(i, w_I) \setminus \{||Q||^M_{i,w_I}\} & \text{if } P \text{ holds} \\ N(i, w_I) & \text{otherwise} \end{cases}$$

The following update of the budget function determines how each agent in G contributes to cover the costs of execution of an action, by consuming part of its available budget. We assume, however, that only inferential actions that add new beliefs have a cost. I.e., forming conjunction and performing belief revision are actions with no cost. As before, for an action α, we require $enabled_{w_I}(G, \alpha)$ to hold and assume that $j \in G$ executes α. Then, depending on α, we have:

$$B^{[G:\downarrow(\psi,\chi)]}(i, w_I) = \begin{cases} B(i, w_I) - \frac{C(j,\downarrow(\psi,\chi),w_I)}{|G|} & \text{if } i \in G \text{ and } T([G : \downarrow(\psi, \chi)]\chi) \subseteq I \text{ and} \\ & enabled_{w_I}(G, \downarrow(\psi, \chi)) \text{ and} \\ & M, w_I \models \mathbf{B_I}_i\psi \wedge \mathbf{K}_i(\psi \rightarrow \chi) \\ B(i, w_I) & \text{otherwise} \end{cases}$$

$$B^{[G:\vdash(\psi,\chi)]}(i, w_I) = \begin{cases} B(i, w_I) - \frac{C(j,\vdash(\psi,\chi),w_I)}{|G|} & \text{if } i \in G \text{ and } T([G : \vdash(\psi,\chi)]\chi) \subseteq I \text{ and} \\ & enabled_{w_I}(G, \vdash(\psi,\chi)) \text{ and} \\ & M, w_I \models \mathbf{B}_i\psi \wedge \mathbf{B}_i(\psi \rightarrow \chi) \\ B(i, w_I) & \text{otherwise} \end{cases}$$

We write $\models_{\textit{L-DINF}} \varphi$ to denote that $M, w_I \models \varphi$ holds for all worlds w_I of every model M.

We introduce below relevant consequences of our formalization. For lack of space we omit the proof, that can be developed analogously to what done in previous work [10]. For any set of agents G and each $i \in G$, we have the following:

- $\models_{\textit{L-INF}} (\mathbf{K}_i(\varphi \rightarrow \psi)) \wedge \mathbf{B}_i\, \varphi) \rightarrow [G : \downarrow(\varphi, \psi)]\, \mathbf{B}_i\, \psi$.
  Namely, if an agent has φ among beliefs and $\mathbf{K}_i(\varphi \rightarrow \psi)$ in its background knowledge, then as a consequence of the action $\downarrow(\varphi, \psi)$ the agent and any group G to which it belongs start believing ψ.
- $\models_{\textit{L-INF}} (\mathbf{K}(p(t_1, t_2) \rightarrow \neg q(t_3, t_4)) \wedge \mathbf{B}_i p(t_1, t_2) \wedge \mathbf{B}_i q(t_3, t_4)) \rightarrow$
  $[(G : \dashv(p(t_1, t_2), q(t_3, t_4)))]\mathbf{B}_i q(t_5, t_6)$,
  where $T(q(t_5, t_6)) = T(q(t_3, t_4)) \setminus T(q(t_1, t_2))$.
  Namely, if agent i has $q(t_3, t_4)$ as one of its beliefs, q is not believed outside $T(q(t_3, t_4))$, the agent perceives $p(t_1, t_2)$ where $T(p(t_1, t_2)) \subseteq T(q(t_3, t_4))$, and has $\mathbf{K}_i(p(t_1, t_2) \rightarrow \neg q(t_3, t_4))$ in its background knowledge. Then after the mental operation $\dashv(p(t_1, t_2), q(t_3, t_4))$ the agent starts believing $q(t_5, t_6))$ where $T(q(t_5, t_6)) = T(q(t_3, t_4)) \setminus T(q(t_1, t_2))$.
- $\models_{\textit{L-INF}} (\mathbf{B}_i\varphi \wedge \mathbf{B}_i\psi) \rightarrow [G : \cap(\varphi, \psi)]\mathbf{B}_i(\varphi \wedge \psi)$.
  Namely, if an agent has φ and ψ as beliefs, then as a consequence of the action $\cap(\varphi, \psi)$ the agent and any group G to which it belongs start believing $\varphi \wedge \psi$.

- $\models_{\textit{L-INF}} (\mathbf{B}_i(\varphi \rightarrow \psi)) \wedge \mathbf{B}_i\,\varphi) \rightarrow [G : \vdash(\varphi, \psi)]\,\mathbf{B}_i, \psi$.
  Namely, if an agent has φ among its beliefs and $\mathbf{B}_i(\varphi \rightarrow \psi)$ in its working memory, then as a consequence of the action $\vdash(\varphi, \psi)$ the agent and any group G to which it belongs start believing ψ.

2.5. Axiomatization

Below we introduce the axiomatization of our logic. The *L-INF* and *L-DINF* axioms and inference rules are the following:

1. $(\mathbf{K}_i\,\varphi \wedge \mathbf{K}_i(\varphi \rightarrow \psi)) \rightarrow \mathbf{K}_i\,\psi$;
2. $\mathbf{K}_i\,\varphi \rightarrow \varphi$;
3. $\neg\mathbf{K}_i(\varphi \wedge \neg\varphi)$;
4. $\mathbf{K}_i\,\varphi \rightarrow \mathbf{K}_i\,\mathbf{K}_i\,\varphi$;
5. $\neg\mathbf{K}_i\,\varphi \rightarrow \mathbf{K}_i\,\neg\mathbf{K}_i\,\varphi$;
6. $\mathbf{B}_i\,\varphi \wedge \mathbf{K}_i\,(\varphi \leftrightarrow \psi) \rightarrow \mathbf{B}_i\,\psi$;
7. $\mathbf{B}_i\,\varphi \rightarrow \mathbf{K}_i\,\mathbf{B}_i\,\varphi$;
8. $\Box_I\varphi \wedge \Box_I(\varphi \rightarrow \psi) \rightarrow \Box_I(\psi)$;
9. $\Box_I\varphi \rightarrow \Box_J\varphi$ with $J \subseteq I$;
10. $\frac{\varphi}{\mathbf{K}_i\,\varphi}$;
11. $[G : \alpha]p \leftrightarrow p$;
12. $[G : \alpha]\neg\varphi \leftrightarrow \neg[G : \alpha]\varphi$;
13. $exec_G(\alpha) \rightarrow \mathbf{K}_i\,(exec_G(\alpha))$;
14. $[G : \alpha](\varphi \wedge \psi) \leftrightarrow [G : \alpha]\varphi \wedge [G : \alpha]\psi$;
15. $[G : \alpha]\mathbf{K}_i\,\varphi \leftrightarrow \mathbf{K}_i\,([G : \alpha]\varphi)$;
16. $[G : \downarrow(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \mathbf{B}_i\,([G : \downarrow(\varphi, \psi)]\chi) \vee [G : \downarrow(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \big((\mathbf{B}_i\,\varphi \wedge \mathbf{K}_i\,(\varphi \rightarrow \psi)) \wedge [G : \downarrow(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \mathbf{K}_i\,([G : \downarrow(\varphi, \psi)]\chi \leftrightarrow \psi)\big)$;
17. $[G : \cap(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \mathbf{B}_i\,([G : \cap(\varphi, \psi)]\chi) \vee [G : \cap(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \big((\mathbf{B}_i\,\varphi \wedge \mathbf{B}_i\,\psi) \wedge [G : \cap(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \mathbf{K}_i\,[G : \cap(\varphi, \psi)]\chi \leftrightarrow (\varphi \wedge \psi)\big)$;
18. $[G : \vdash(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \mathbf{B}_i\,([G : \vdash(\varphi, \psi)]\chi) \vee [G : \vdash(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \big((\mathbf{B}_i\,\varphi \wedge \mathbf{B}_i\,(\varphi \rightarrow \psi)) \wedge [G : \vdash(\varphi, \psi)]\mathbf{B}_i\,\chi \leftrightarrow \mathbf{K}_i\,([G : \vdash(\varphi, \psi)]\chi \leftrightarrow \psi)\big)$;
19. $[G : \dashv(\varphi, \psi)]\neg\mathbf{B}_i\,\chi \leftrightarrow \mathbf{B}_i\,([G : \dashv(\varphi, \psi)]\chi) \vee [G : \dashv(\varphi, \psi)]\neg\mathbf{B}_i\,\chi \leftrightarrow \big((\mathbf{B}_i\,\varphi \wedge \mathbf{K}_i\,(\varphi \rightarrow \neg\psi)) \wedge [G : \dashv(\varphi, \psi)]\neg\mathbf{B}_i\,\chi \leftrightarrow \mathbf{K}_i\,([G : \dashv(\varphi, \psi)]\chi \leftrightarrow \psi)\big)$;
20. $intend_G(\phi_A, I) \leftrightarrow \forall i \in G\; intend_i(\phi_A; I)$;
21. $do_G(\phi_A, I) \rightarrow can\_do_G(\phi_A, I)$;
22. $do_i(\phi_A, I) \rightarrow can\_do_i(\phi_A, I) \wedge pref\_do_G(i, \phi_A, I)$;
23. $\frac{\psi \leftrightarrow \chi}{\varphi \leftrightarrow \varphi[\psi/\chi]}$.

We write *L-DINF* $\vdash \varphi$ to denote that φ is a theorem of *L-DINF*. It can be verified that the above axiomatization is sound for the class of *L-INF* models, namely, all axioms are valid and inference rules preserve validity. In particular, soundness of axioms 16–19 follows from the semantics of $[G{:}\alpha]\varphi$, for each inferential action α, as previously defined. Notice that, by abuse of notation, we have axiomatized the special predicates concerning intention and action enabling. Axioms 20–22 concern in fact physical actions, stating that: what is intended by a group of agents is intended by them all; and, neither an agent nor a group of agents can do what it is not enabled to do. Such axioms are not enforced by the semantics, but are supposed to be enforced by a designer's/programmer's encoding of parts of an agent's behaviour. In fact, axiom 20 enforces agents in a group to be cooperative. Axioms 21 and 22 ensure that agents will attempt to perform actions only if their preconditions are satisfied, i.e., if they can do them. We do not handle such properties in the semantics as done, e.g., in dynamic logic, because we want agents' definition to be independent of the practical aspect, so we explicitly intend to introduce flexibility in the definition of such parts.

3. Canonical Model and Strong Completeness

In this section we adapt the notion of canonical model for *L-INF* introduced in [10] to deal with the time component. The proof of strong completeness of the framework directly exploits the notion of canonical model by applying a standard argument. Time is handled in the semantics by means of the time function T and the definition of canonical *L-INF* model is immediately obtained from the one in [10], as follows:

Definition 3.1. *Let* Agt *be a set of agents. The* canonical *L-INF model is a tuple* $M_c = \langle W_c, N_c, \mathcal{R}_c, E_c, B_c, C_c, A_c, H_c, P_c, V_c, T_c \rangle$ *where:*

- W_c *is the set of all maximal consistent subsets of* $\mathcal{L}_{\textit{L-INF}}$*;*
- $\mathcal{R}_c = \{R_{c,i}\}_{i \in Agt}$ *is a collection of equivalence relations on* W_c *such that, for every* $i \in Agt$ *and* $w_I, v_I \in W_c$*,* $w_I R_{c,i} v_I$ *if and only if (for all* φ*,* $\mathbf{K}_i\, \varphi \in w_I$ *implies* $\varphi \in v_I$*);*
- *For* $w \in W_c$*,* $\varphi \in \mathcal{L}_{\textit{L-INF}}$ *let* $A_\varphi(i, w_I) = \{v \in R_{c,i}(w_I) \mid \varphi \in v\}$*. Then, we put* $N_c(i, w_I) = \{A_\varphi(i, w_I) \mid \mathbf{B}_i\, \varphi \in w_I\}$*;*
- $E_c : Agt \times W_c \longrightarrow 2^{\mathcal{L}_{\mathsf{ACT}}}$ *is such that, for each* $i \in Agt$ *and* $w_I, v_I \in W_c$*, if* $w_I R_{c,i} v_I$ *then* $E_c(i, w_I) = E_c(i, v_I)$*;*
- $B_c : Agt \times W_c \longrightarrow \mathbb{N}$ *is such that, for each* $i \in Agt$ *and* $w_I, v_I \in W_c$*, if* $w_I R_{c,i} v_I$ *then* $B_c(i, w_I) = B_c(i, v_I)$*;*
- $C_c : Agt \times \mathcal{L}_{\mathsf{ACT}} \times W_c \longrightarrow \mathbb{N}$ *is such that, for each* $i \in Agt$*,* $\alpha \in \mathcal{L}_{\mathsf{ACT}}$*, and* $w_I, v_I \in W_c$*, if* $w_I R_{c,i} v_I$ *then* $C_c(i, \alpha, w_I) = C_c(i, \alpha, v_I)$*;*
- $A_c : Agt \times W_c \longrightarrow 2^{Atm_A}$ *is such that, for each* $i \in Agt$ *and* $w_I, v_I \in W_c$*, if* $w_I R_{c,i} v_I$ *then* $A_c(i, w_I) = A_c(i, v_I)$*;*
- $H_c : Agt \times W_c \longrightarrow 2^{Atm_A}$ *is such that, for each* $i \in Agt$ *and* $w_I, v_I \in W_c$*, if* $w_I R_{c,i} v_I$ *then* $H_c(i, w_I) = H_c(i, v_I)$*;*
- $P_c : Agt \times W_c \times Atm_A \longrightarrow \mathbb{N}$ *is such that, for each* $i \in Agt$ *and* $w_I, v_I \in W$*, if* $w_I R_{c,i} v_I$ *then* $P_c(i, w_I, \phi_A) = P_c(i, v_I, \phi_A)$*;*
- $V_c : W_c \longrightarrow 2^{Atm}$ *is such that* $V_c(w_I) = Atm \cap w_I$*;*
- T_c *: the time function defined as before.*

Analogously to what done before, let $R_{c,i}(w_I)$ denote the set $\{v_I \in W_c \mid w_I R_{c,i} v_I\}$, for each $i \in Agt$. M_c is an *L-INF* model as defined in Definition 2.1, since, it satisfies conditions

(**C1**),(**C2**),(**D1**),(**E1**),(**F1**),(**G1**),(**G2**),(**H1**). Hence, it models the axioms and the inference rules 1–19 and 23 introduced before (while, as mentioned in Section 2.5, axioms 20–22 are assumed to be enforced by the specification of agents behaviour). Consequently, the following properties hold too. Let $w_I \in W_c$, then:

- given $\varphi \in \mathcal{L}_{\textit{L-INF}}$, it holds that $\mathbf{K}_i\,\varphi \in w_I$ if and only if $\forall v_I \in W_c$ such that $w_I R_{c,i} v_I$ we have $\varphi \in v$;
- for $\varphi \in \mathcal{L}_{\textit{L-INF}}$, if $\mathbf{B}_i\,\varphi \in w_I$ and $w_I R_{c,i} v$ then $\mathbf{B}_i\,\varphi \in v_I$;

Thus, $R_{c,i}$-related worlds have the same knowledge and N_c-related worlds have the same beliefs, i.e. there can be $R_{c,i}$-related worlds with different beliefs.

By proceeding similarly to what done in [16], we obtain the proof of strong completeness. For lack of space, we list the main theorems but omit lemmas and proofs, that we have however developed analogously to what done in previous work [10].

Theorem 3.1. *L-INF is strongly complete for the class of L-INF models.*

Theorem 3.2. *L-DINF is strongly complete for the class of L-INF models.*

4. Conclusions

In this paper we proposed a possible way to enrich the epistemic logic introduced in [9, 10, 11], originally designed to express group dynamics of cooperative agents, with the possibility of specifying time intervals to express the time periods in which agents' acting takes place. Hence, by adapting the treatment introduced in [6], an enriched semantics for formulas, as well as a new belief update mechanism, has been suitably designed for the new temporalized logic. The approach appears promising and its usefulness has been shown by outlining a not trivial example.

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