Diversification of Stock Portfolio Structure under Market Restrictions

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Abstract

This article describes the mathematical formulation of the problems of managing a portfolio of securities on the stock market. The problem of managing a stock portfolio is considered as a mathematical problem of optimal management. Mathematical statements are formulated for problems with two fixed ends of the trajectory and for one fixed and one free end of the trajectory. For the correct formulation of optimal management problems, mathematical models are describing the dynamics of the market value formation of one share and the portfolio are applied. The corresponding models are written in the class of ordinary differential equations with parameters. The procedure for building a dynamic model of the formation of the market value of one share is based on the application of the market model of W. Sharpe and the fundamental theory of H. Markowitz. The principles of H. Markowitz theory make it possible to determine the optimal values of the portfolio's expected profitability and riskiness when applying the procedure for building an optimal portfolio of risky securities. The application of optimal management theory methods in the optimization of the stock portfolio involves an iterative procedure for determining the optimal structure. The work also considers an important applied problem of applying the theory of H. Markowitz to solve the problem of optimal diversification of a portfolio of risky investments in the presence of restrictions that are formed by the stock market at each moment of time. The presence of market restrictions significantly affects the decision-making procedure regarding optimal portfolio diversification. This scientific study presents an algorithm for optimal diversification of a portfolio of risky securities in the presence of market restrictions.

Keywords [1]

Portfolio optimization, mathematical control theory, portfolio diversification

1. Introduction

The problem of mathematical description of the dynamics of market value formation of one share and a portfolio of shares has long been relevant and attracts the attention of scientists and practitioners. According to H. Markowitz, mathematically, such a problem is formulated as a two-criterion problem of nonlinear optimization. The criteria are mutually contradictory and it does not have a single solution. Despite this, its practical importance is extremely great. Taking into account the complexity of mathematical statements and their practical implementation, researchers have identified important principled approaches to decision-making on optimizing the structure of the investment portfolio. Among such approaches, the most famous can be distinguished:

Dynamical System Modeling and Stability Investigation (DSMSI-2023), December 19-21, 2023, Kyiv, Ukraine
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- H. Markowitz approach, using a valid and effective set, investor indifference curves; The theory of H. Markowitz is classic and its principles are the basis of many decision-making strategies in the stock market.
- Splitting two-criterion investment portfolio optimization problem into two one-criterion; The mathematical two-criterion problem of optimizing a portfolio of risky securities involves maximizing the expected return and minimizing the risk of the investment portfolio [16, 25].
- Methods of technical analysis; The most common technology for making practical decisions involves the use of known statistical information about the dynamics of the market value of one share.
- Methods of fundamental analysis. The theory of fundamental analysis is actively developing and includes new mathematical models and methods for describing the dynamics and procedures for making decisions about the optimal structure of an investment portfolio [17]. In the researches of K. Ito, F. Black, M. Scholes, and R. Merton, new fundamental results are formulated, The results are actively developed and implemented in the practice of decision-making on the stock market.

In this study an algorithm for identifying the parameters of the mathematical model of the dynamics of the market value of one share and a portfolio of shares is proposed. The principle of operation of the algorithm is based on the application of an iterative procedure, at each step of which model parameters are calculated that improve the value of the selected quality criterion [22]. It is worth noting that the choice of model parameter optimization procedure may depend on the selected quality criterion.

1.1. Development of Portfolio Investment Model

The portfolio model was first proposed by Markowitz [1, 2]. Based on the relationship between income and risk, Markowitz [1, 2] constructs the mean-variance model. He put forward the risk measurement and the modern principles of portfolio investment. This has laid a theoretical foundation for the researches of securities investment portfolio. Sharpe's (1964) [3,4] capital asset pricing model have been known and are used by many investors when solving the problems of investing in shares.
Garashchenko, Kulian and Rutitskaya [5] perform qualitative and quantitative analysis of investment management models. Porter [6], Huang [7], Li [8] have studied the mean-variance model.

Pyle and Turnovsky [9], H.Levy and M. Levy [10], Chiu and Li [11] have studied the safety-first model. Malkiel [12] have studied the negative effective investment strategy in the efficient market. He find that in the market value of stocks and small cap stocks, the market in international markets, in the bond or stock market, passive investment strategy is an effective investment strategy. Dierkes [13] uses the Cumulative Prospect Theory to describe the investor preference. Based on the investor preference perspective, he analyzes the investment strategy in different investment period, and fined that bond investment strategy is more popular in the short term. In [14] studies investment horizon effect on asset allocation between value and growth strategies. Zhang [15] proposes the multi-period fuzzy portfolio model, and then uses genetic algorithm, hybrid intelligent algorithm and differential approximation algorithm to solve the model.

2. Mathematical modeling in portfolio analysis

2.1. Construction of a mathematical model of a stock portfolio

Let's consider a mathematical model of the market value of the investment portfolio and build appropriate analytical trajectory. At the time interval $t \in [t_0, T]$ equation describing the profitability of the portfolio shares r_p, has the form

$$r_p(t) = \sum_i x_i(t) r_i(t) , \qquad (1)$$

where x_i – proportion of shares and type-portfolio; r_i – expected return on equity. Having differentiated both parts of (1), we get

$$\frac{dr_p(t)}{dt} = \sum_i (r_i(t)\frac{dx_i(t)}{dt} + x_i(t)\frac{dr_i(t)}{dt}) . \qquad (2)$$

Consider normalizing expressions that the content is inverse response time of the system to change the dynamics of expected profitability and structure of portfolio

$$\sum_i \frac{\dot{r}_i(t)}{r_i(t)} = 1, \ \sum_i \frac{\dot{x}_i(t)}{x_i(t)} = 1, \ t \in [t_0, t_1] .$$

Given that there is a property

$$\text{for } i \neq j \ \text{ and } \sum_j b_j = 1$$

$$\sum_i a_i b_i = \sum_i a_i + \sum_i \sum_j a_i b_j \ .$$

In this way occurring ratio

$$\sum_i x_i(t) r_i(t) \frac{f_i}{r_i(t)} = \sum_i x_i(t) r_i(t) - \sum_i \sum_j x_i(t) \times r_i(t) \frac{f_j}{r_j(t)},$$

$$\sum_i \frac{x_i(t) r_i(t)}{x_i(t)} \frac{dx_i(t)}{dt} = \sum_i x_i(t) r_i(t) - \sum_i \sum_j x_i(t) \times r_i(t) \frac{dx_j(t)}{dt} \frac{1}{x_j(t)}.$$

The dynamic model of the market value of the stock portfolio will have the following form

$$\frac{dr_p(t)}{dt} = 2r_p(t) - \sum_i \sum_j x_i(t) r_i(t) (\frac{f_j}{r_j(t)} \times \frac{dx_j(t)}{dt} \frac{1}{x_j(t)}) \ . \tag{3}$$

This is a first order linear differential equation. Applying the method of variation of arbitrary constants and getting its general solution

$$r_p(t) = -\ell^{2t} \int \ell^{-2t} \sum_i \sum_j x_i(t) r_i(t) (\frac{f_j}{r_j(t)} \times \frac{dx_j(t)}{dt} \frac{1}{x_j(t)}) dt, \tag{4}$$

where

$$f_j(t) = (\alpha_1 SM_{ind}(t) + \alpha_2 I(t)) r_j(t) + \sum_{i=1}^{N} \beta_{ij} r_j(t) \ .$$

In latter ratio the assumptions made phenomenon that describes the dynamics of the formation of the market value of the portfolio of risky securities. A more detailed analysis indicates two important properties that characterize the market value of the portfolio: dynamics depends on the dynamics of both the expected profitability of shares and changes in the structure of the portfolio. In relation (3) f_j is the right-hand side of the differential equation that describes the dynamics of the formation of the market value of the shares [2]. Its structure and content corresponds to a market model of W. Sharpe [3].

We were able to construct a sequence paths at selected time intervals, allowing you to create the initial structure of the investment portfolio.

However, this approach can not effectively influence the structure of investment and not fully using the mathematical description of the properties listed in mathematical models (4).

Among other, it contains a factor

$$\frac{dx_j(t)}{dt} * \frac{1}{x_j(t)} ,$$

that describes the possible changes in the structure of investment.

To simplify further calculations mathematical model (4) is presented in more general terms

$$\dot{r}_p = f^p(r_p, x_i, \dot{x}_i, r_i, \dot{r}_i), \ i = \overline{1, l} \tag{5}$$

2.2. Choosing of optimal portfolio structure

When solving optimal control problem for (5), we use the method of successive approximations. As a phase state we consider the market value of equity portfolio r_p and as the control parameters in accordance with the model - vector

$$u(t) = (x_i(t), r_i(t), \frac{dx_i(t)}{dt}, \frac{dr_i(t)}{dt}). \qquad (6)$$

We consider a mathematical model of the dynamic formation of the market value of investment portfolio (2); the desired level of the market value of the portfolio at the time T, $r_p(T) = r_P^T$; time interval $t \in [t_0, T]$; control constraints at each time point $x(t) \in U(t)$; quality criterion

$$J(x(t)) = \int_{t_0}^{T} (r_p(t) - r_p(T))^2 dt + \Phi(r_p(t_0)) \to \min_{x(t)\in U(t)}.$$

Here $\Phi(r_p(t_0))-$ given function.

We need to identify $x(t_0)$ and, respectively, $r_p(t_0)$.

At the same time, we recall that the vector x describes the proportion of different types of shares in the portfolio, $x = (x_1, x_2, ..., x_n)$. Here n - number of shares in the investment portfolio types.

To solve the optimal control problem stated above with one fixed end of the trajectory and fixed time using the maximum principle. The result is the ability to identify $x(t_0)$, and on its basis $r_p(t_0)$. The Hamilton function has the form

$$H(r_p(t), \psi(t), t, x(t)) = -(r_p(t) - r_p(T) + \psi(t) * f(r_p, x_i(t), \dot{x}_i(t), r_i, \dot{r}_i).$$

A necessary condition for its optimality is

$$\frac{\partial(-(r_p(t) - r_p(T))^2 + \psi(t) * f(r_p, x_i, \dot{x}_i, r_i, \dot{r}_i))}{\partial x_i} = 0, \quad i = \overline{1, n}.$$

A solution of the equation is portfolio management function $x^*(\psi(t), r_p(t), t)$.

We construct a conjugated system in the form of

$$\dot{\psi}(t) = -\frac{\partial H(r_p(t), \psi(t), t, x(t))}{\partial r_p},$$

$$\dot{\psi}(t) = 2(r_p(t) - r_p(T)) - \psi(t) * f'_{r_p}(r_p, x_i, \dot{x}_i, r_i, \dot{r}_i).$$

Let's formulate the transversality condition at the left end of the trajectory

$$\psi(t_0) = -\frac{\partial \Phi(r_p(t_0))}{\partial r_p}.$$

A boundary value problem the maximum principle has the form

$$\begin{cases} \dot{\psi}(t) = 2(r_p(t) - r_p(T)) - \psi(t) * f'_{r_p}((r_p, x_i, \dot{x}_i, r_i, \dot{r}_i) \\ \dot{r}_p(t) = f(r_p, x_i, \dot{x}_i, r_i, \dot{r}) \end{cases}$$

under conditions $\psi(t_0) = \psi_0$, $r_p(T) = r_p^T$.

The right side of the equations of the model depends on the parameters $\alpha_1, \alpha_2 \in A$, where $A-$a closed bounded set of model parameters. For the correct mathematical formulation of the problem we formulate additional equations

$$\frac{\partial H(r_p(t),\psi(t),t,x(t,\alpha))}{\partial \alpha_j}=0, \quad j=1,2.$$

Let's use the statement of the theorem for the optimal control problem with one fixed end of the trajectory and fixed time.

We decide to build a system of ordinary differential equations that determine the function $r_p(t)$, $\psi(t)$, which, when $x^*(\psi(t),r_p(t),t)$ is substituted in the decision, will determine the optimal structure of the investment portfolio in selected time interval.

The analysis algorithm described above makes it possible to note some shortcomings, the most important of which is the parametrically defined objective function and significant limitations on the functions of the control. The algorithm for constructing a vector of unknown parameters $\alpha=(\alpha_1,\alpha_2)$ will be given below.

2.3. Identification of parameters in the task of the structure optimizing of portfolio investments

When solving the problems of modeling the dynamics of trajectories of complex systems, one often has to face problems related to the fact that the equations of motion are known with precision to the parameters. It is obvious that before proceeding to the solution of the problems of constructing the trajectories of the system's movement, it is necessary to determine the values of the parameters of such models, or the sets to which they belong. Modern applied mathematical modeling uses different approaches [18] to the calculation of parameters with a defined structure of mathematical models. The most well-known and common methods of identifying parameters of discrete and continuous models include [20]:

- correlation-dispersion analysis;
- approaches based on the principles of the theory of stability;
- approaches based on methods of analyzing the sensitivity of solutions.

With different statements of the problem, vectors can be considered as a vector of parameters $\alpha=(\alpha_1,\alpha_2)$. At the same time, we will use known statistical information about the dynamics of the market value of the relevant shares $\overline{r_i}\ (t)$.

Based on this dynamics, let's divide the integration interval into subintervals $t_0<t_1<t_2<...<t_T$.

We will look for optimal parameter values on subintervals α. For this on the initial subinterval for $i-$share and for the selected value of the parameter we will solve the Cauchy problem $r_i(t)=f(r_i,t,\alpha),\quad r_i(t_0)=r_{i0,},\ \ t\in(t_0,t_1)$.

Next, in order to obtain the optimal value of the parameter, we will formulate an optimization problem

$$\alpha_1^*=arg\min_{\alpha}(r_i^0(r(t_0),t,\alpha)-\overline{r_i}(t))^2.$$

Here as $r^0(r(t_0),t,\alpha)$ denotes the solution of the Cauchy problem on the first interval. Thus, we can determine the optimal, in the sense of the quality criterion, the value of the parameters of the model (1) on the first interval. We will formulate and solve similar problems on other partition intervals. At $k-$ step of the algorithm, the procedure for calculating the optimal values of the parameters of the dynamic model is as follows:

We solve the Cauchy problem

$$\dot{r}_i(t)=f(r_i,t,\alpha_{k-1}),\ r_i(t_{k-1})=r_{i_{k-1}}\quad t\in(t_{k-1},t_k),\ \ i=\overline{1,n}.$$

1. We build the trajectory of the system from a point t_{k-1} to point t_k at the value of the parameter α_{k-1}^*. In this case, the value of the function at the moment of time t_k is be r_k.

1. We optimize the system parameter on this interval. To do this, we will formulate and solve an optimization problem

$$\alpha^*_{k-1} = arg \min_{\alpha}(r^0(r(t_{k-1}), t, \alpha^*_{k-2}) - \overline{r}_{k-1}(t))^2$$

The solution of the problem for the chosen k one of them will be the value of the parameter α^*_{k-1} that translates the system from point r_{k-1} to point r_k.

The above procedure makes it possible on the basis of known statistical information, build a sequence of values of the parameters of a mathematical model, which allows to simulate the behavior and forecast the expected profitability of the share r_i at selected moments of the time interval specified by the investor. It is worth noting that when dividing the integration interval, it is also necessary to take into account the values of the components of the system state vector for the correct application of the found parameters when solving the corresponding trajectory problems. Let's move on to the problem of constructing a guaranteed multiple estimation of parameters for a mathematical model of the general form (5). Let us formulate the procedure for constructing the optimal ellipsoidal estimation of the parameters of the mathematical model (5) in the parameter space.

2.3.1. Algorithm for constructing an admissible set of parameters of the mathematical model

We will consider the vector of model parameters in the form

$$\alpha = (\alpha_1, \alpha_2) \text{ at } x(t_0) = x_0,\ t \in [t_0, t_1].$$

Consider a point α_0 in the parameter space

$$\alpha_0 = (\alpha_{0_1}, \alpha_{0_2}).$$

And let this parameter value be the solution to the problem of parametric identification of the model (1). Let's describe a circle of unit radius around the point α_0

$$(\alpha_1 - \alpha_{0_1})^2 + (\alpha_2 - \alpha_{0_2})^2 = 1$$

With the center in $(\alpha_{0_1}, \alpha_{0_2})$. The length of the circle will be $l = 2\pi$. Let's divide this line into n equal parts, and the value n is chosen depending on the accuracy of the obtained solution to the problem of constructing a guaranteed multiple estimation of parameters. Consider an arbitrary point $\alpha_k = (\alpha_{k_1}, \alpha_{k_2})$.

Let's draw a tangent to the circle at this point. Its equation will be

$$F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 - \alpha_{k_1}) = F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_2 - \alpha_{k_2}).$$

The equation of the normal to the tangent at this point will have the form

$$F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})(\alpha_2 - \alpha_{k_2}) = F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 - \alpha_{k_1}).$$

We will look for new values of parameters that satisfy the conditions of software functioning of the system on the normal

$$\alpha_2 = \frac{F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 - \alpha_{k_1})}{F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})} + \alpha_{k_2}.$$

We determine the new position of the coordinate α_2 by changing the position of the coordinate α_1

$$\alpha_{N_1} = \alpha_1 + \delta\alpha_1, \qquad \delta > 0.$$

$$\alpha_{N_2} = \frac{F'_{\alpha_2}(\alpha_{k_1}, \alpha_{k_2})(\alpha_1 + \delta\alpha_1 - \alpha_{k_1})}{F'_{\alpha_1}(\alpha_{k_1}, \alpha_{k_2})} + \alpha_{k_2}.$$

Thus, a new position of the parameter value is obtained $\alpha_N \in A$, де A – a limited closed set of parameters of a mathematical model

$$\alpha_N = (\alpha_{N_1}, \alpha_{N_2}).$$

We present some quality criteria for checking whether a parameter belongs to an admissible set

$$\sum_i (x^0(x_0, t_i, \alpha_\delta) - x_{\exp}(t_i))^2 < \varepsilon \quad ,$$

$$\max_t (x^0(x_0, t, \alpha_\delta) - x_{\exp}(t)) < \varepsilon, \quad t \in [t_0, t_1],$$

The above procedure implements the movement of the α parameter along the normal in one direction or the other as long as one of the selected criteria above is met. After performing the following steps for each of the previously defined points, we will get a new position of the points in the parameter space of the mathematical model.

2.4. Optimal portfolio diversification

In this study, attention is focused on the possibility of applying the theory of effective and admissible sets by H. Markowitz for the diversification of the portfolio of risky investments. The real market involves a dynamic change in the market values of shares, which prompts the investor to actively diversify the portfolio [21]. According to the algorithm for constructing the admissible set, the portfolio that is more effective in terms of expected profitability and riskiness for the investor will be located higher and to the left on the "expected profitability - riskiness" plane. According to H. Markowitz, portfolios that are elements of the effective set [24] will be optimal. Such portfolios are Pareto-optimal. The classic problem of portfolio optimization is formulated without taking into account restrictions on changing the portfolio structure. Such a feature significantly affects the possibilities of real investment. The algorithms proposed in this study make it possible to take into account such limitations and form a portfolio that is closest to the optimal one that is potentially possible at the moment on the market [19-23]. The developed method can be effectively applied to computer programs ("trading robots") that perform automated selection of the optimal portfolio structure [25-27].

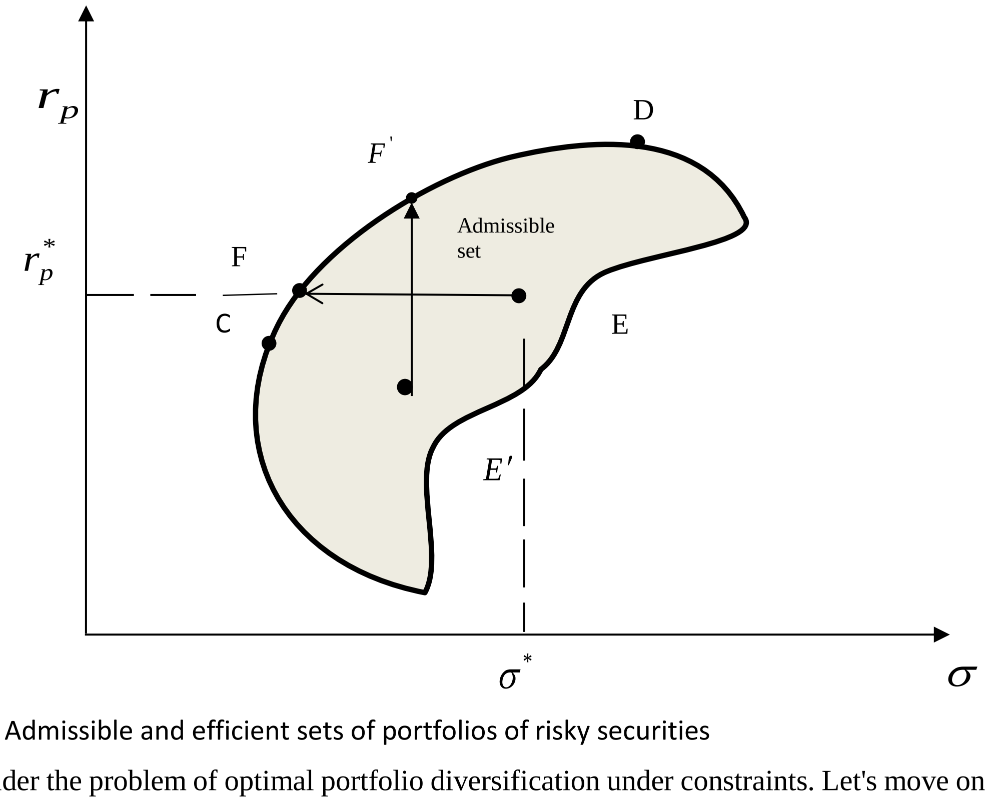

Figure 1: Admissible and efficient sets of portfolios of risky securities

We consider the problem of optimal portfolio diversification under constraints. Let's move on to the second problem in the general formulation of H. Markowitz about optimizing the risk of the portfolio of shares that is optimal in terms of expected profitability. For this, we will use sets of admissible and efficient portfolios corresponding to the selected set of shares. The risk optimization procedure for the optimal expected return portfolio consists in choosing at each step admissible portfolios that lie on the EF line. This line connects point E, which corresponds to the optimal market value of the portfolio, and point F, which belongs to the efficient set. This line is parallel to the portfolio's riskiness axis. The peculiarity of this selection of the optimal portfolio is that on this straight line, according to the definition, each of the portfolios corresponds to the same expected return, but the riskiness decreases in the direction of the axis. This property of the admissible set of investment portfolios allows, on the one hand take into account the restrictions

$$x_i(t) \in X(t), \quad i = \overline{1,n}$$

and on the other hand - to determine the portfolio of "optimal" expected return with less risk.

If the specified portfolio is located at point E', that is, it is one for which it is not possible to reduce the riskiness according to the rule proposed above, then the "optimal portfolio" is determined by moving it from E', to F', which is an element of the effective set of portfolios. In effect, this means identifying a portfolio of stocks with higher expected returns. At the same time, this procedure allows you to constructively take into account existing limitations when diversifying the portfolio. Another mathematical formulation of the problem of optimization of the expected profitability $r_p(T)$ of the investment portfolio at a certain time T level of its risk τ is as follows [28]:

$$\left\{\begin{array}{l} r^{\mathrm{T}}(T)x(T) \to \max\limits_{x} \\ x^{\mathrm{T}}(T)Vx(T) = \tau \\ I^{\mathrm{T}}x(T) = 1 \\ x_i(t) \geq 0, i = \overline{1,n}, t \in [t_0, T] \\ x_i(t) \in X(t), i = \overline{1,n}, t \in [t_0, T] \end{array}\right\}.$$

The procedure for optimizing the portfolio's expected return r_p for a certain level of its risk consists in choosing at each step admissible portfolios that lie on the line EG connecting point E, which corresponds to the optimal portfolio calculated by expected return, and point G, which belongs to the efficient set. This line is parallel to the axis of market value r_p. The peculiarity of this selection of the optimal portfolio is that on this straight line, according to the definition, each of the portfolios corresponds to the same riskiness, but the market value r_p increases.

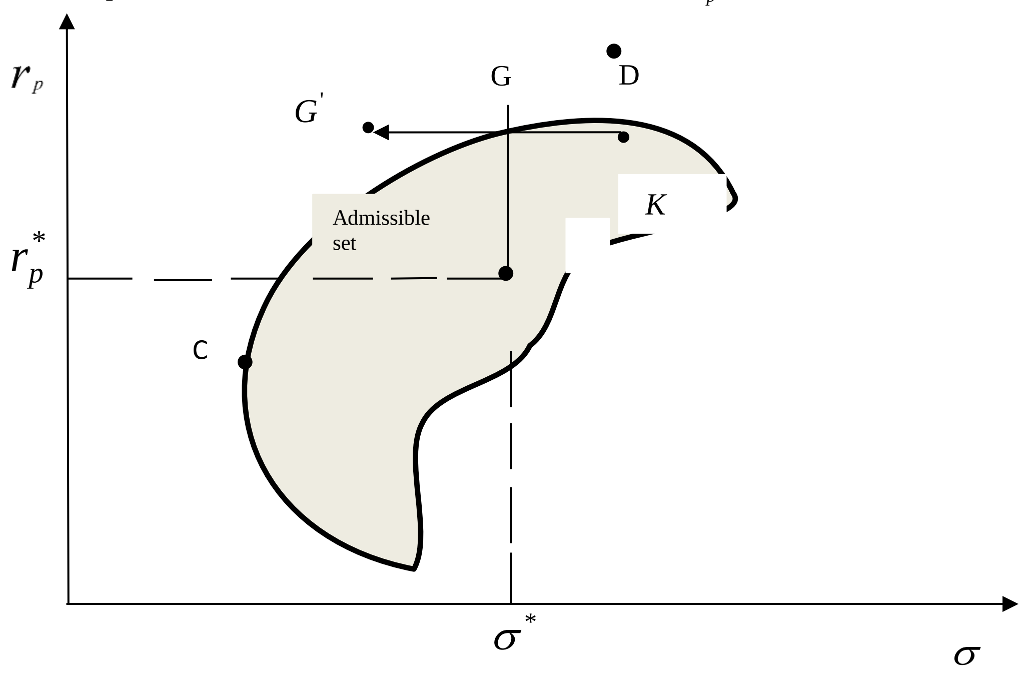

Figure 2: Optimizing the market value of the stock portfolio

This property of the admissible set of investment portfolios, as in the previous case, allows, on the one hand, to take into account restrictions $x_i(t) \in X(t)$, $i = \overline{1,n}$ and on the other hand, to determine the portfolio with the "optimal" risk and higher expected profitability.

If the defined portfolio is located at point K, that is, one for which there is no possibility to increase the expected return, according to the rule proposed above, then the "optimal portfolio" is determined by moving it from K to G', which is an element of the effective set of portfolios. In fact, this means reducing the riskiness of the stock portfolio. The effective set or the set of effective portfolios in figures 1,2 is on the arc. It is a Pareto set [1] for a set of shares existing on the market.

3. Conclusion

In this study new mathematical formulations of the optimization problems of the stock portfolio structure are given and methods of their solution are developed. Mathematical problems formulated on the basis of models of the dynamics of the market value of one share (4) and a portfolio of shares (5) make it possible to solve the problem of optimal diversification of the investment portfolio, taking into account quantitative and qualitative market restrictions on the portfolio structure.

The formulation of the problem and built algorithm greatly expand the possibilities of investing, as in every moment of diversification it is possible to construct a set of alternatives that are equivalent in terms of selected quality criteria. The investor, as in the classical approach, Mr. Markowitz has the ability to take into account when deciding additional factors that arise in the course of practical investment and are associated with the peculiarities of the system dynamics modeling. Mathematical portfolio diversification procedure makes it possible for the above-mentioned models of the dynamics of the market value of a share and the stock portfolio to solve the problem of constructing an initial portfolio with a known value of its expected market value at the selected future date.

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