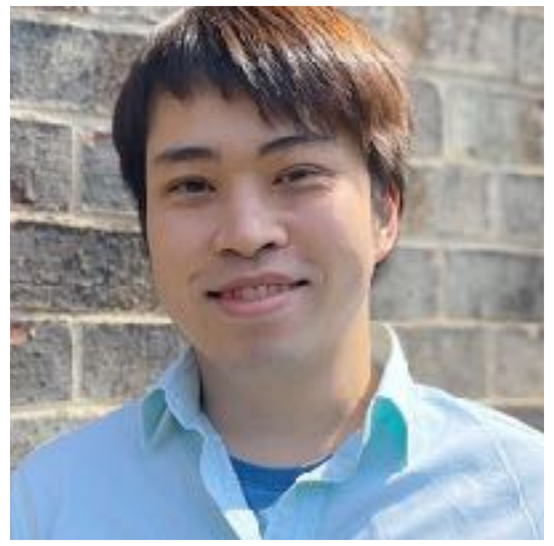
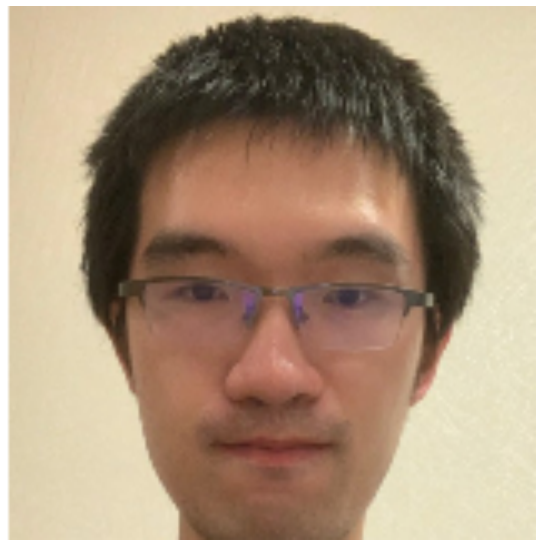
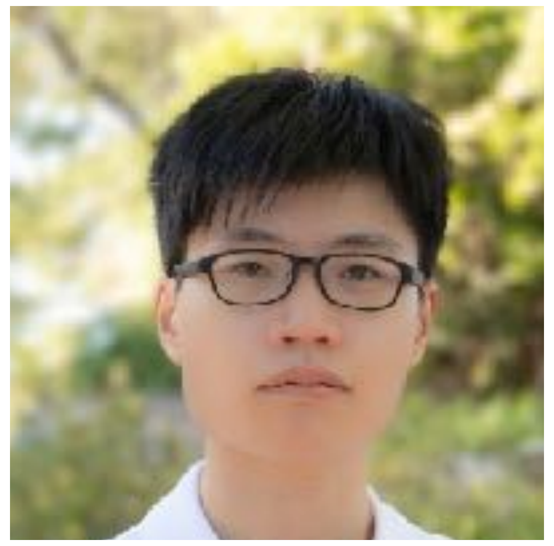
Quantum Meets the Minimum Circuit Size Problem



Nai-Hui Chia — IUB

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Ruizhe Zhang — UT Austin

ITCS 2022

arXiv:2108.03171

Motivation

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Study quantum computation and complexity

through the lens of meta complexity!

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The complexity of complexity!

Meta Complexity

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In the classical setting

Meta Complexity

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Study the complexity of computational problems about complexity.

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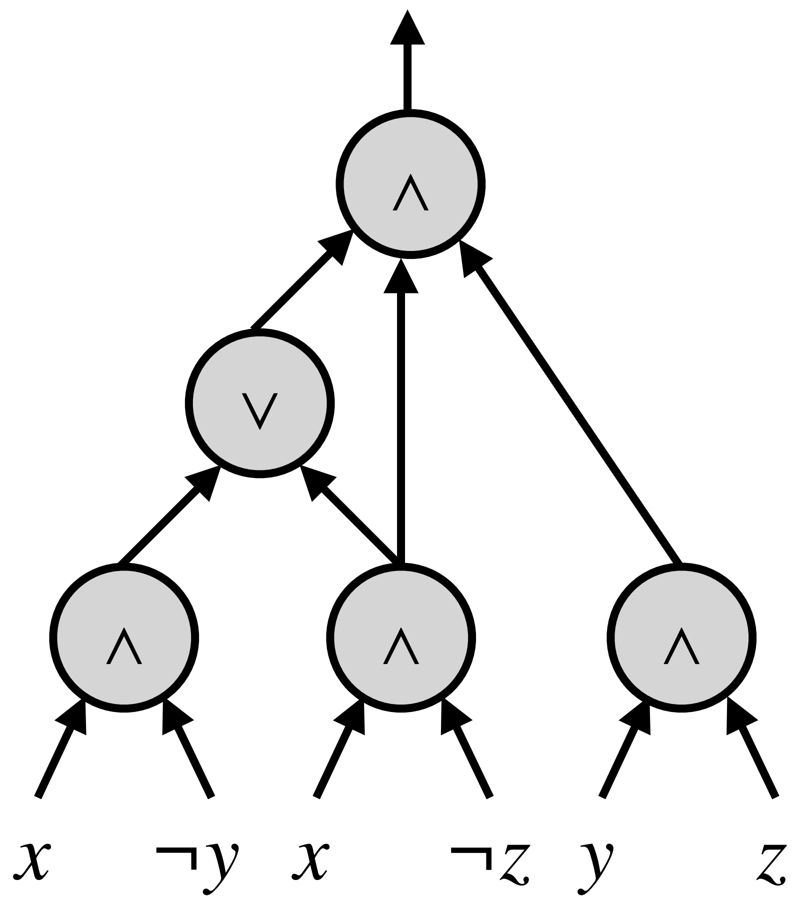
Complexity	Meta Complexity Problem
Circuit Complexity	Minimum Circuit Size Problem (MCSP)



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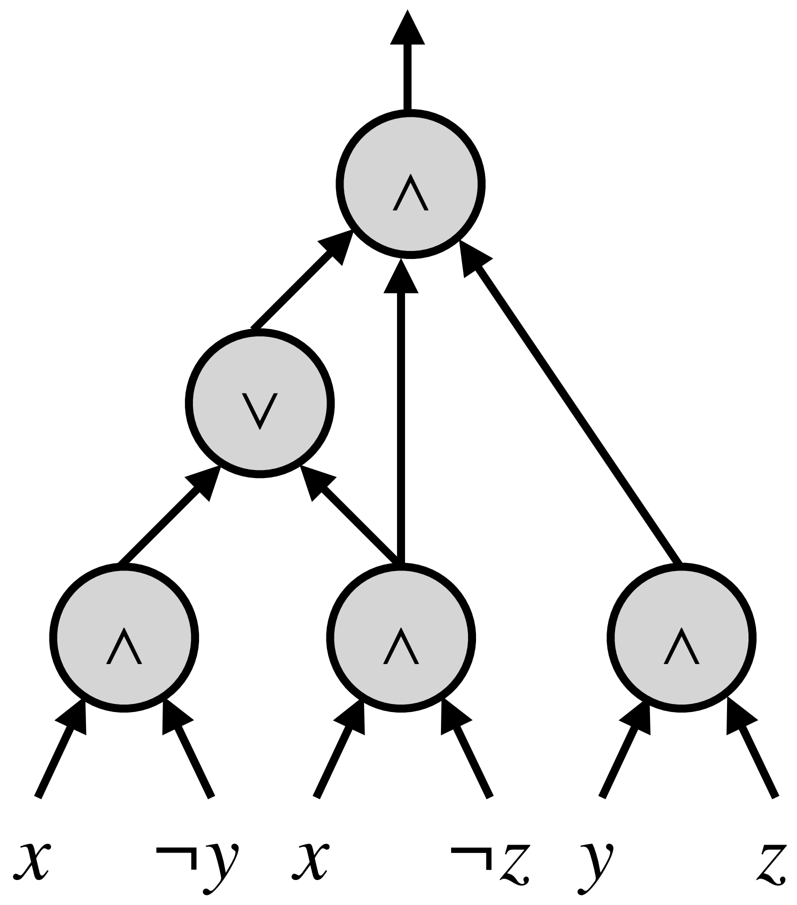

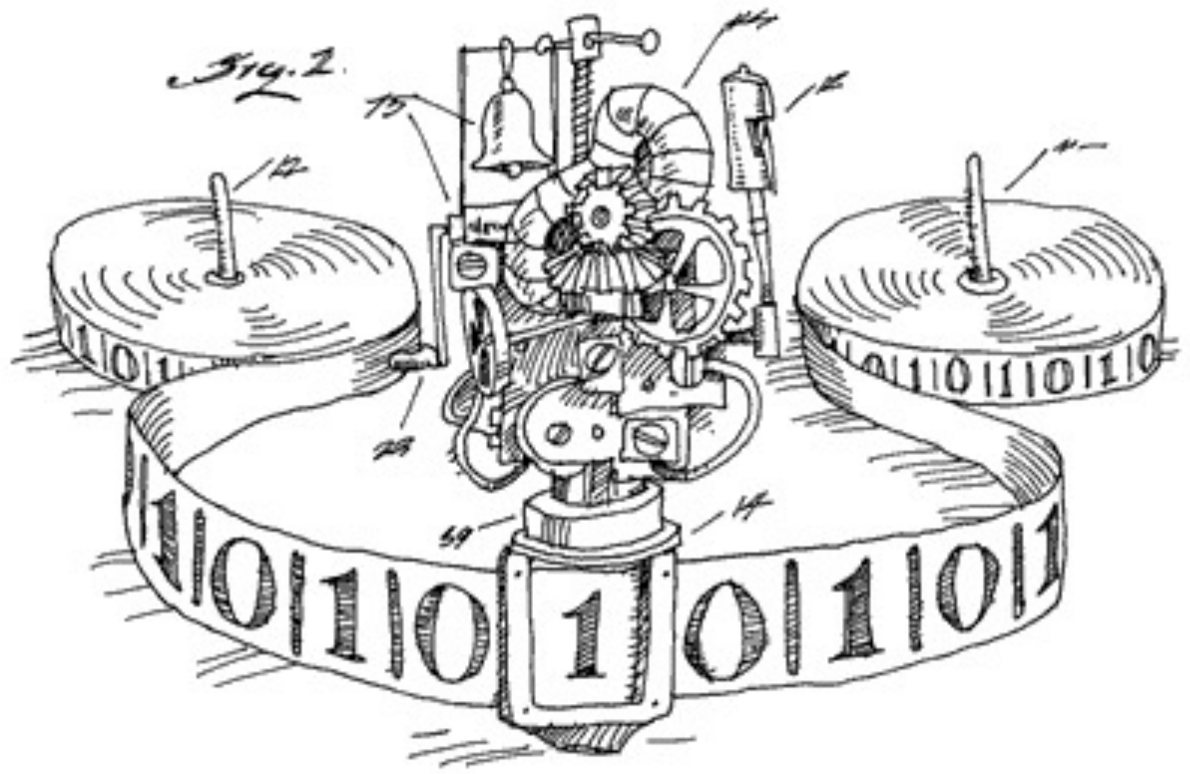

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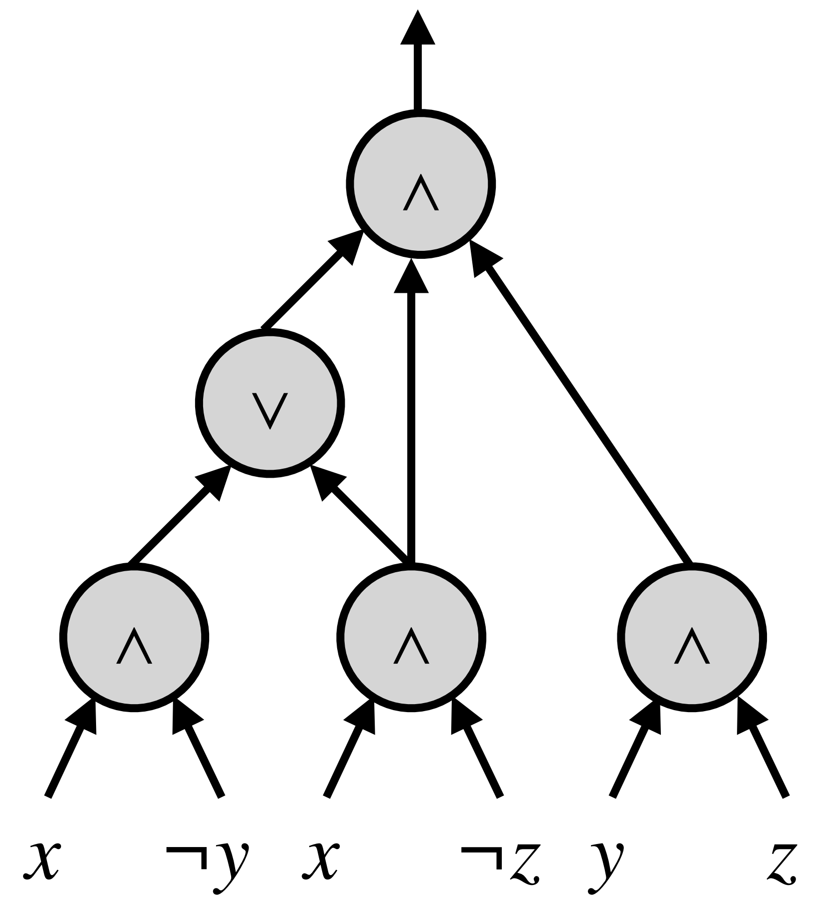

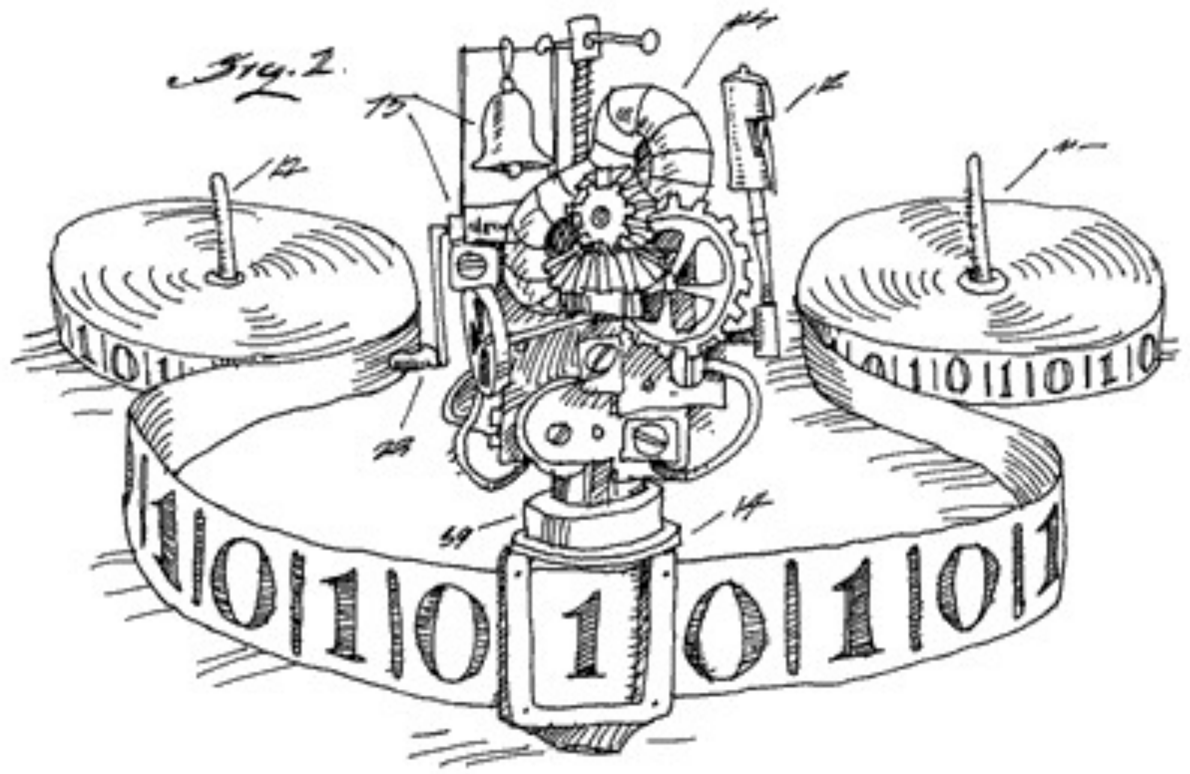

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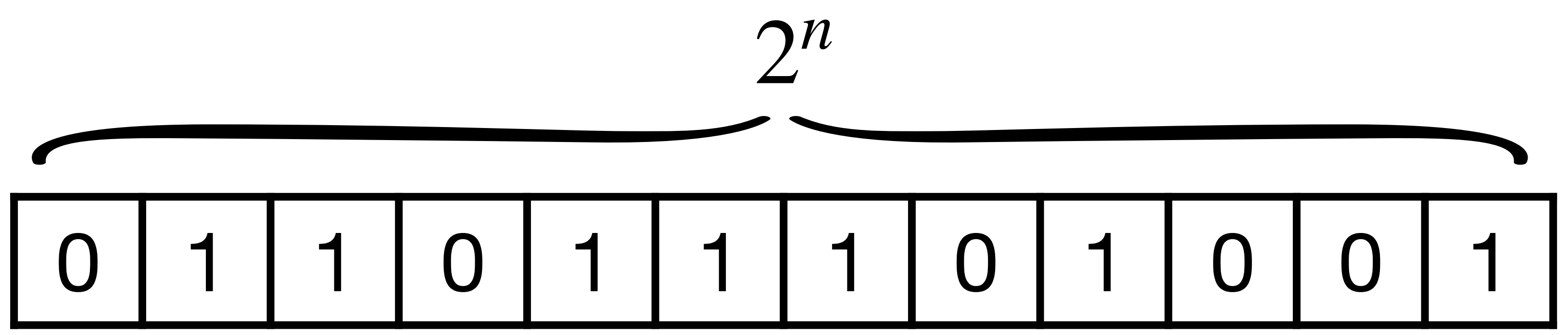

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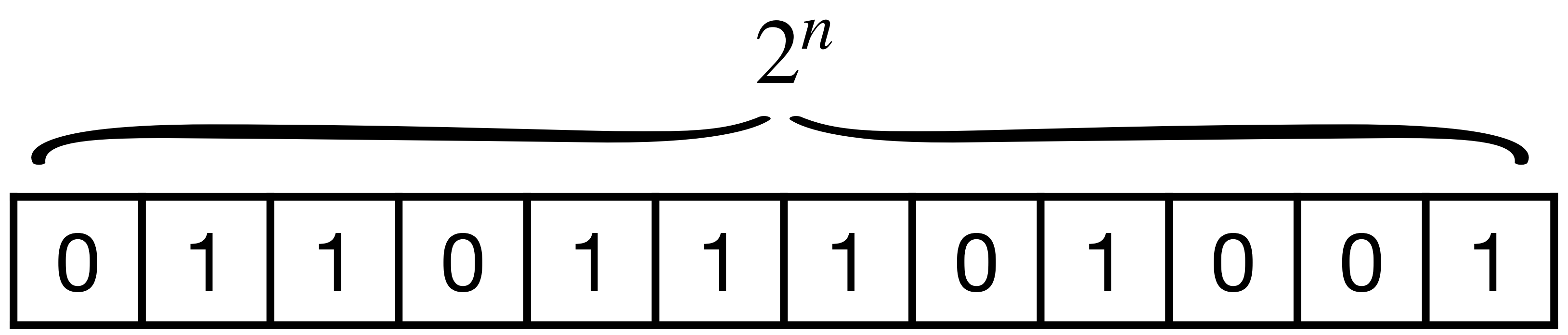

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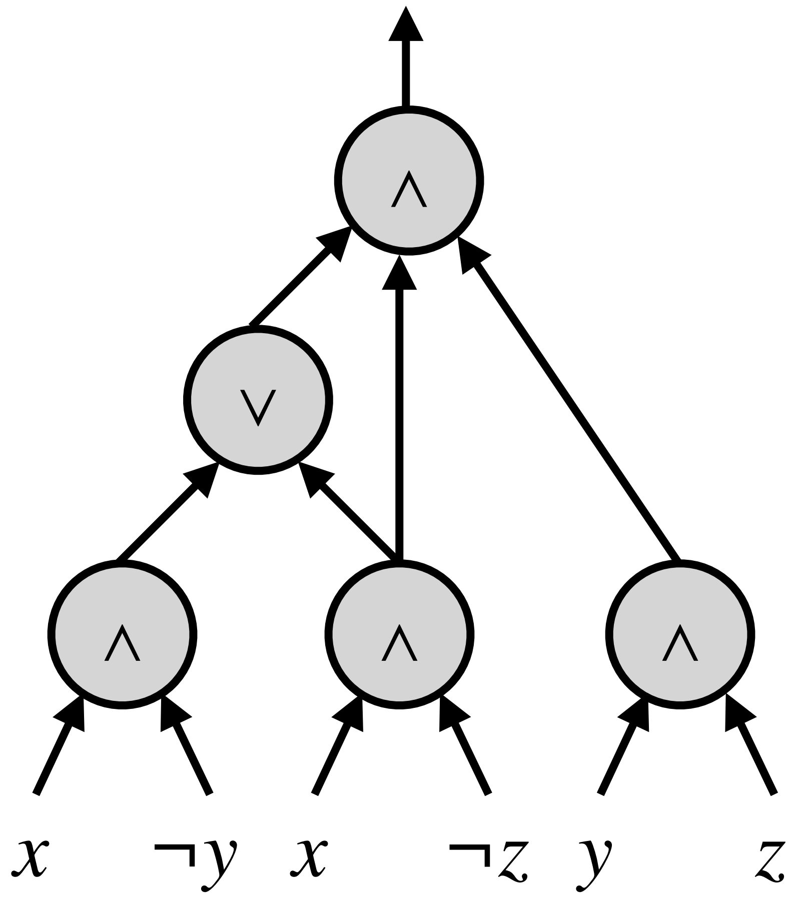

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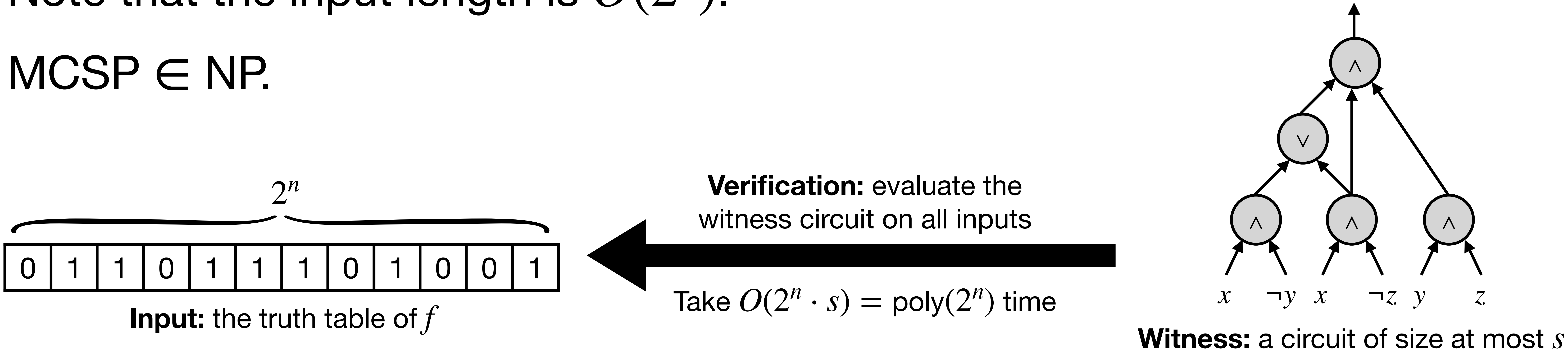

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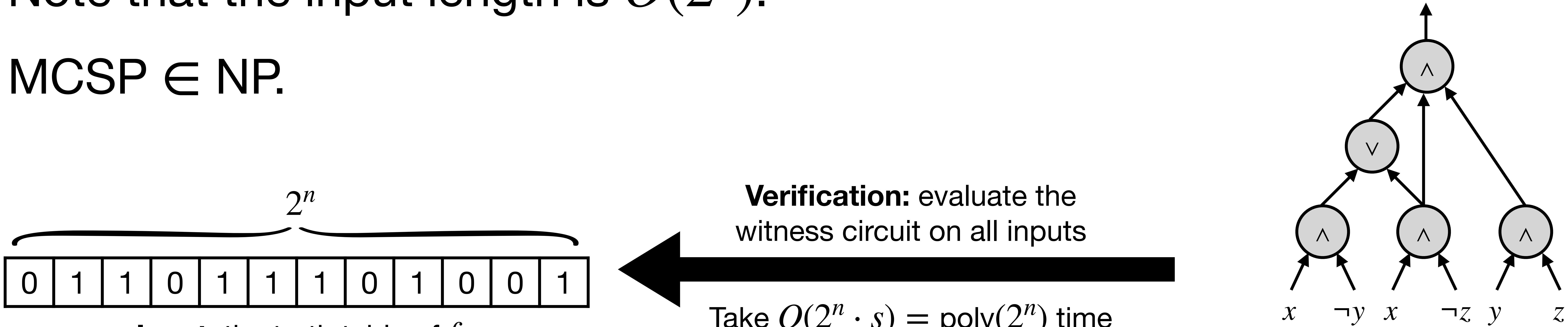

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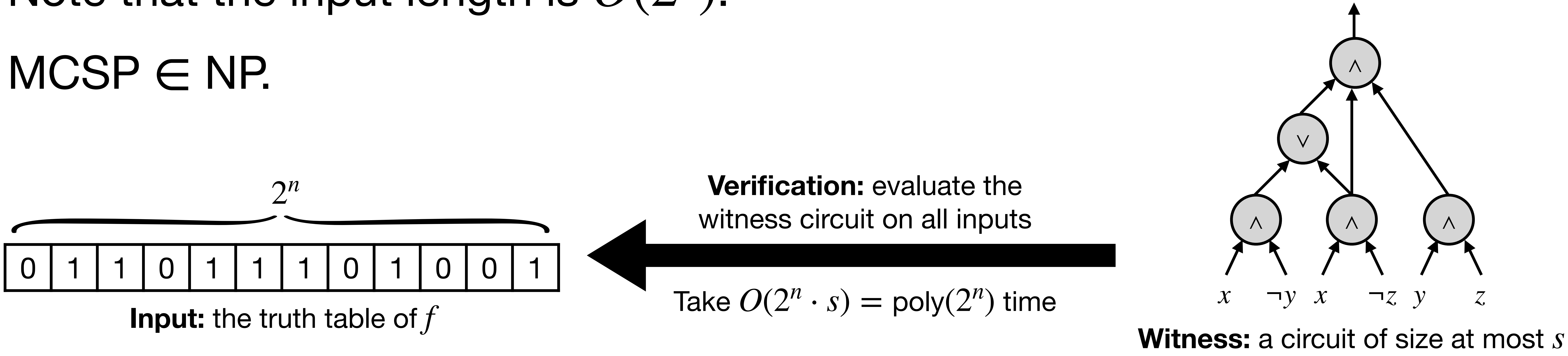

- But proving MCSP $\in$ P or MCSP being NP-hard require new techniques!
- Perebor conjecture: brute-force search is the best algorithm!?

Why MCSP?

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Circuit Complexity

- [Razborov-Rudich'00]: MCSP $\in$ P $\Rightarrow$ natural property against P/poly$\Rightarrow$ no PRG.
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MCSP has connections to many sub-fields in TCS!

Quantum Meets MCSP

Roadmap

Roadmap

finition & Basic
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Roadmap

A Bird-Eye View on Our Results

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Special
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Definitions & Basic Complexity Results

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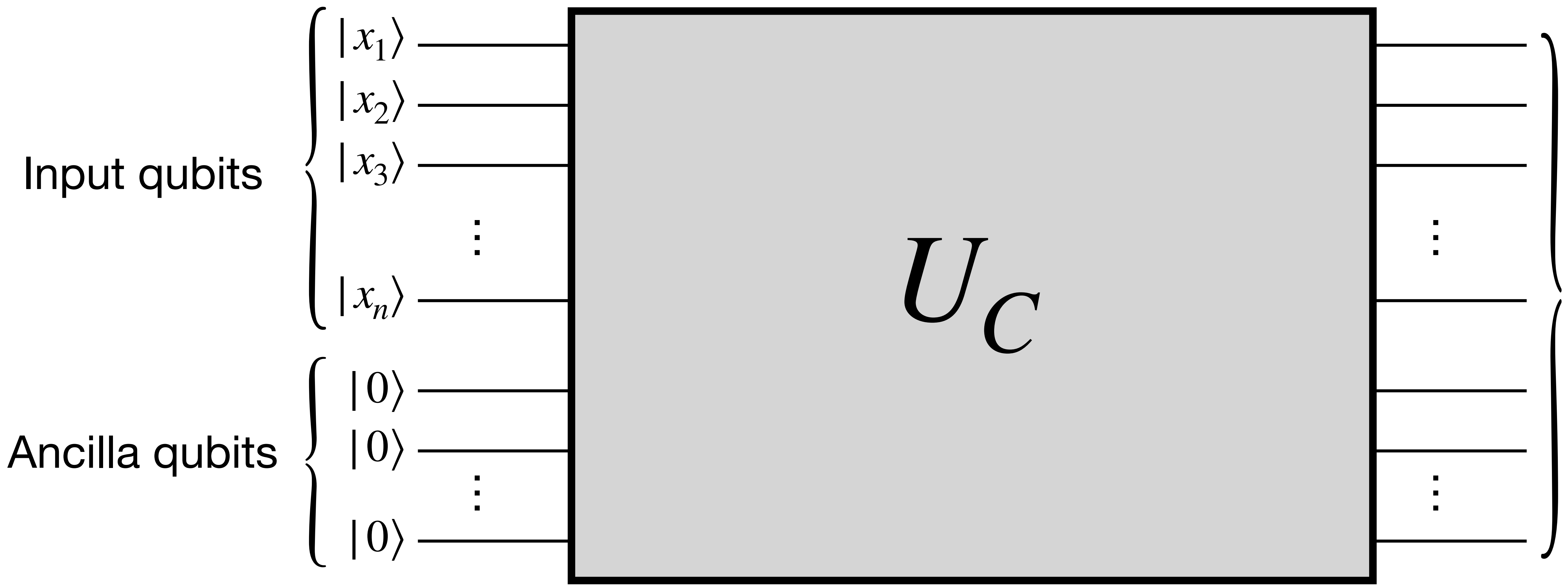

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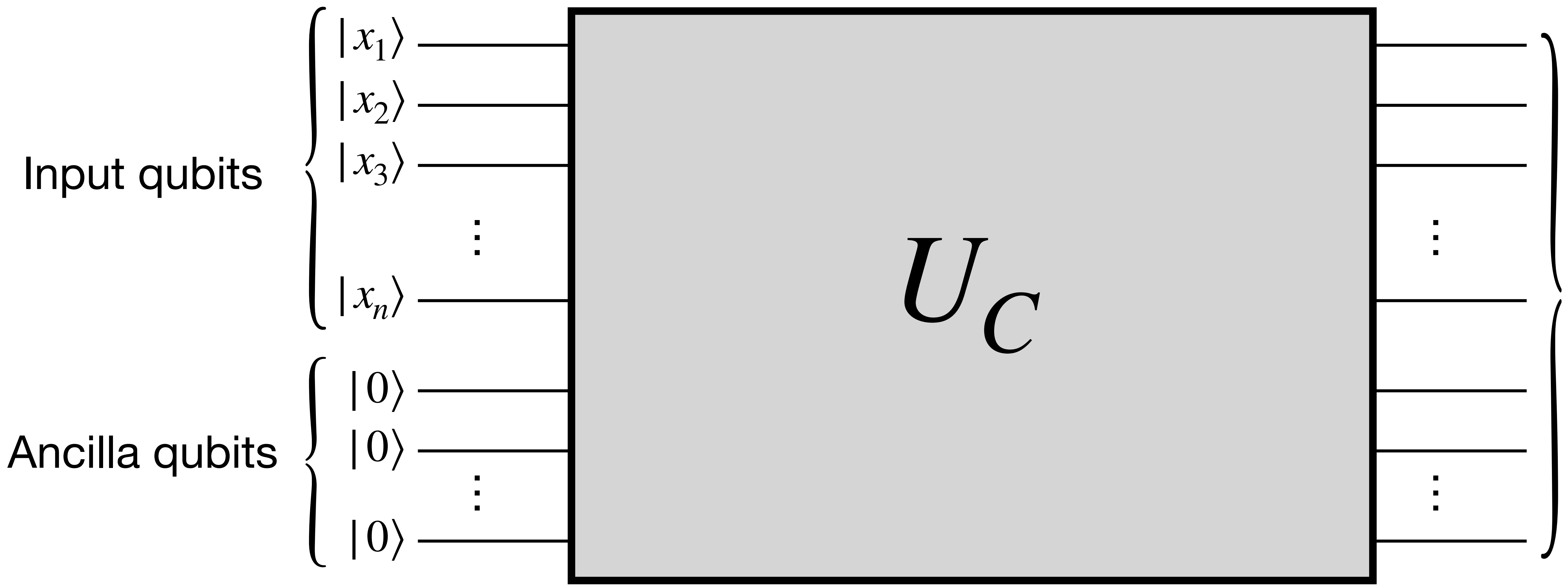

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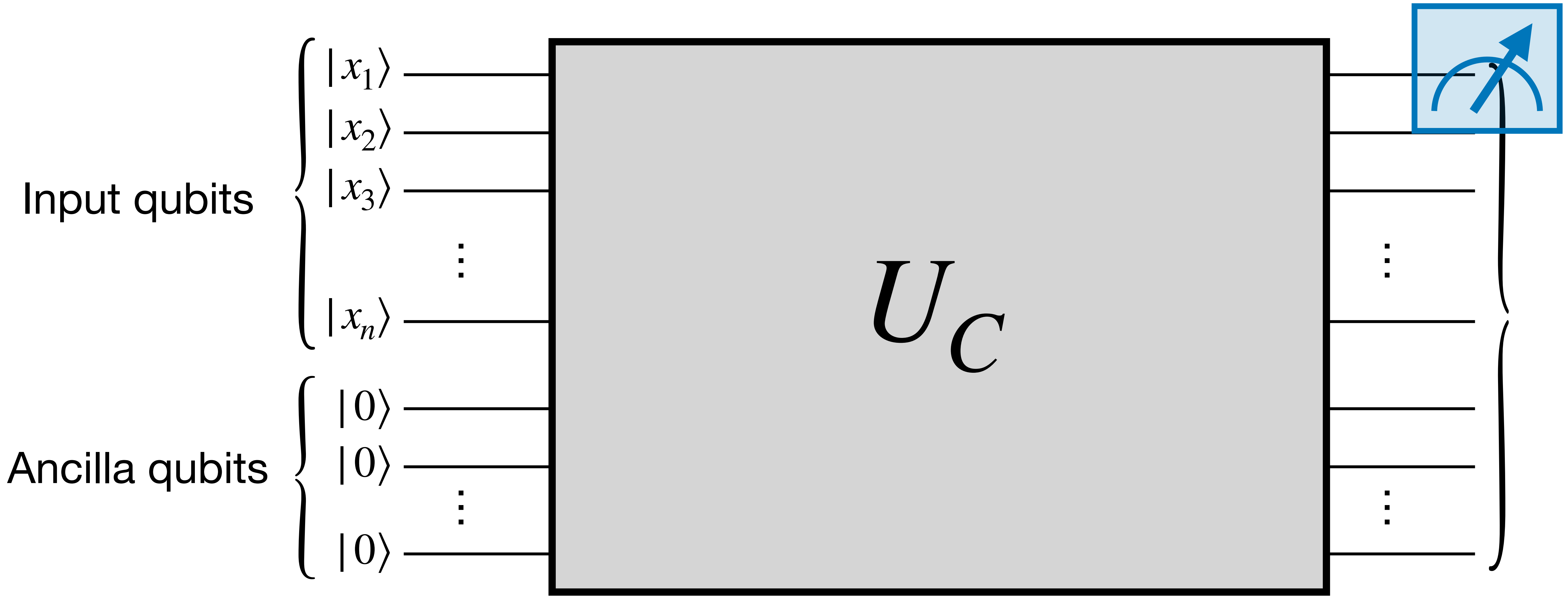

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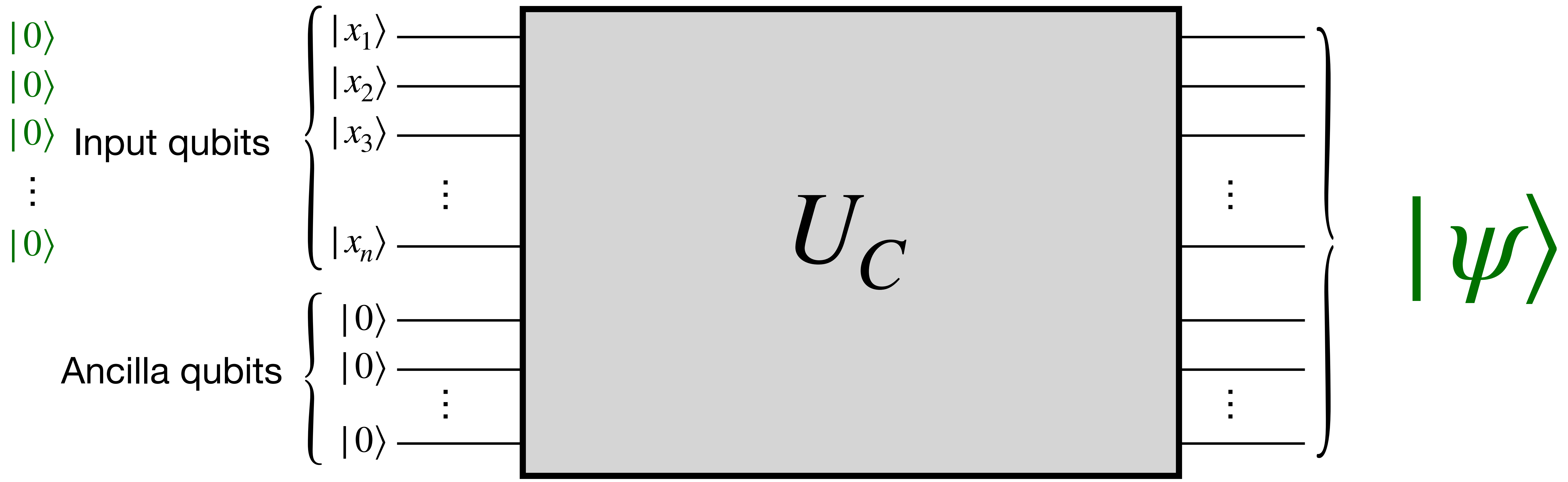

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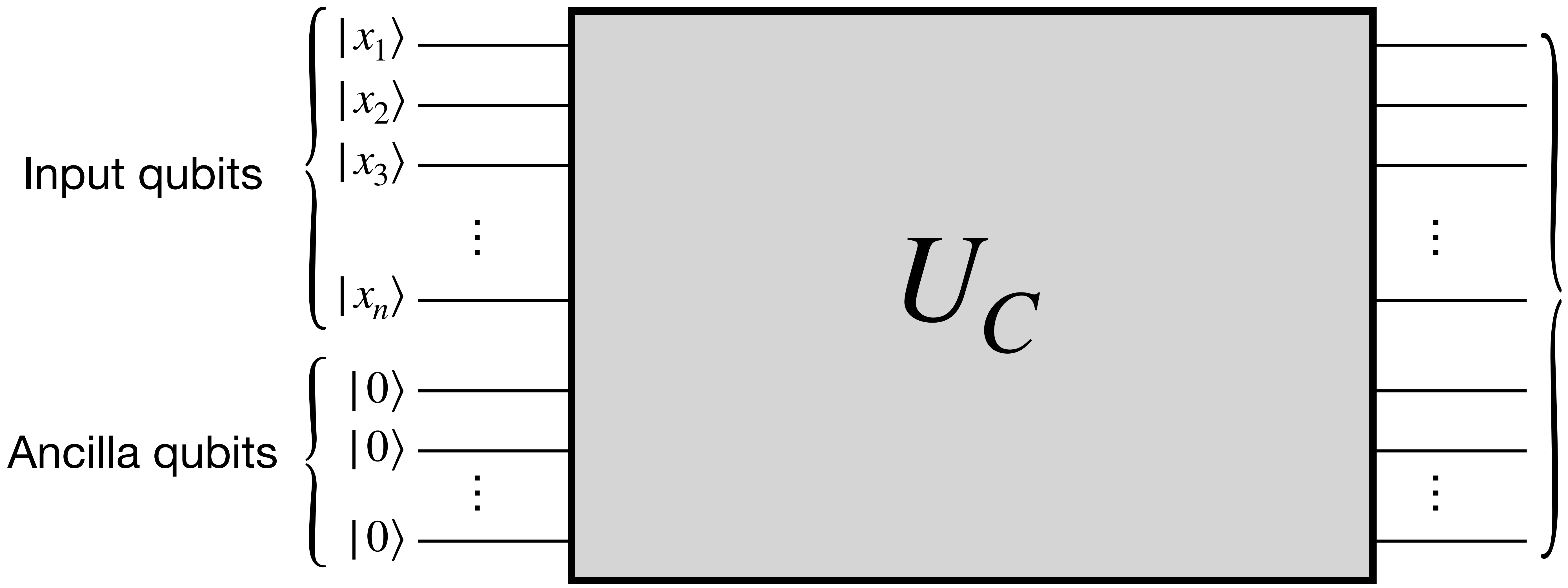

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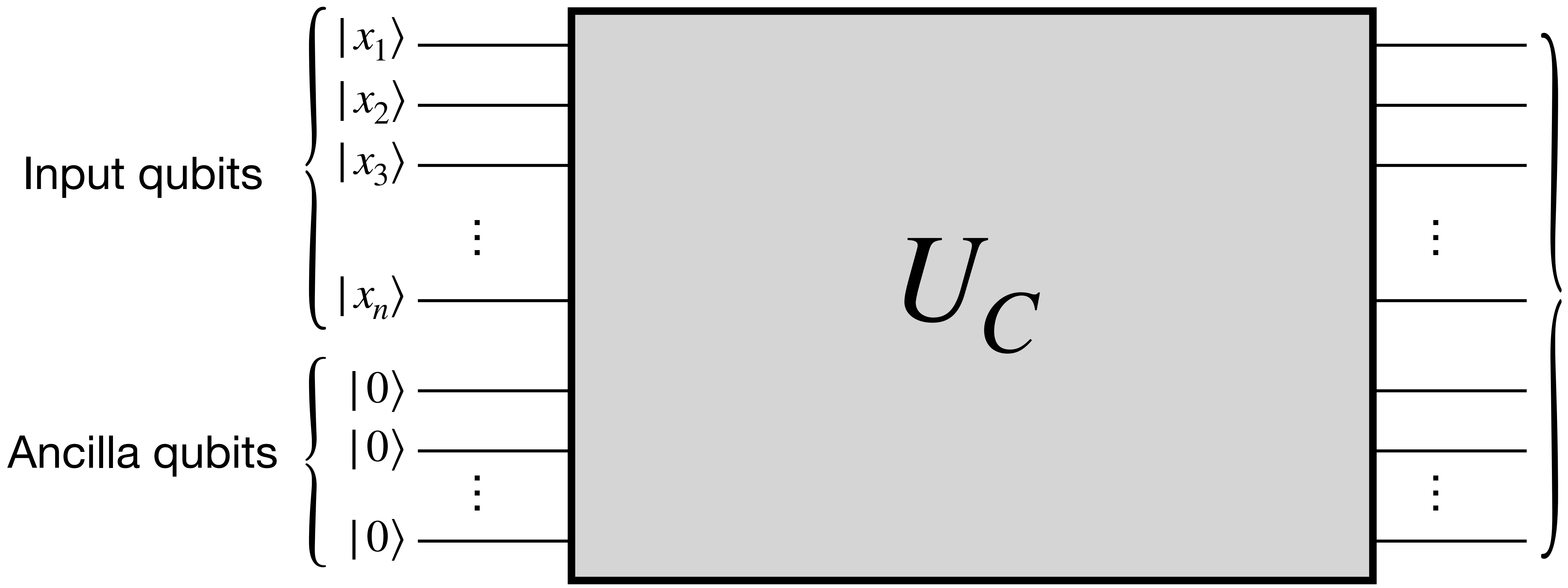

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To properly define the corresponding MCSP, one needs to handle "error probability" and "distance" between quantum objects.

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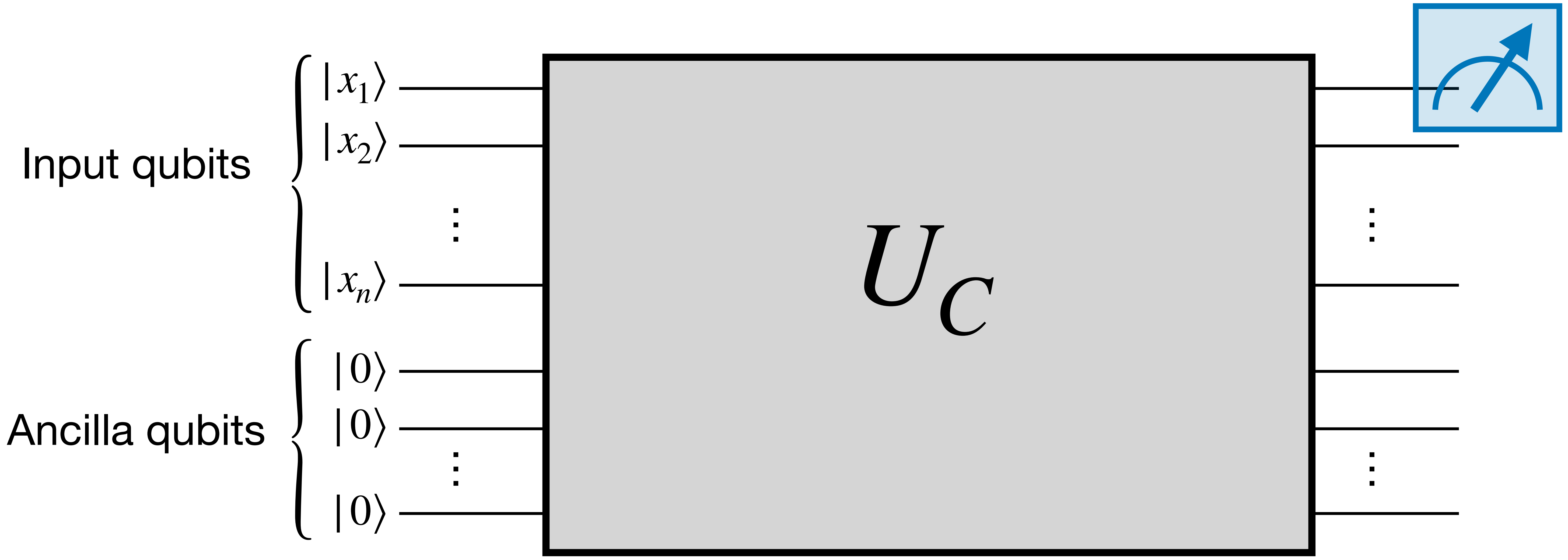

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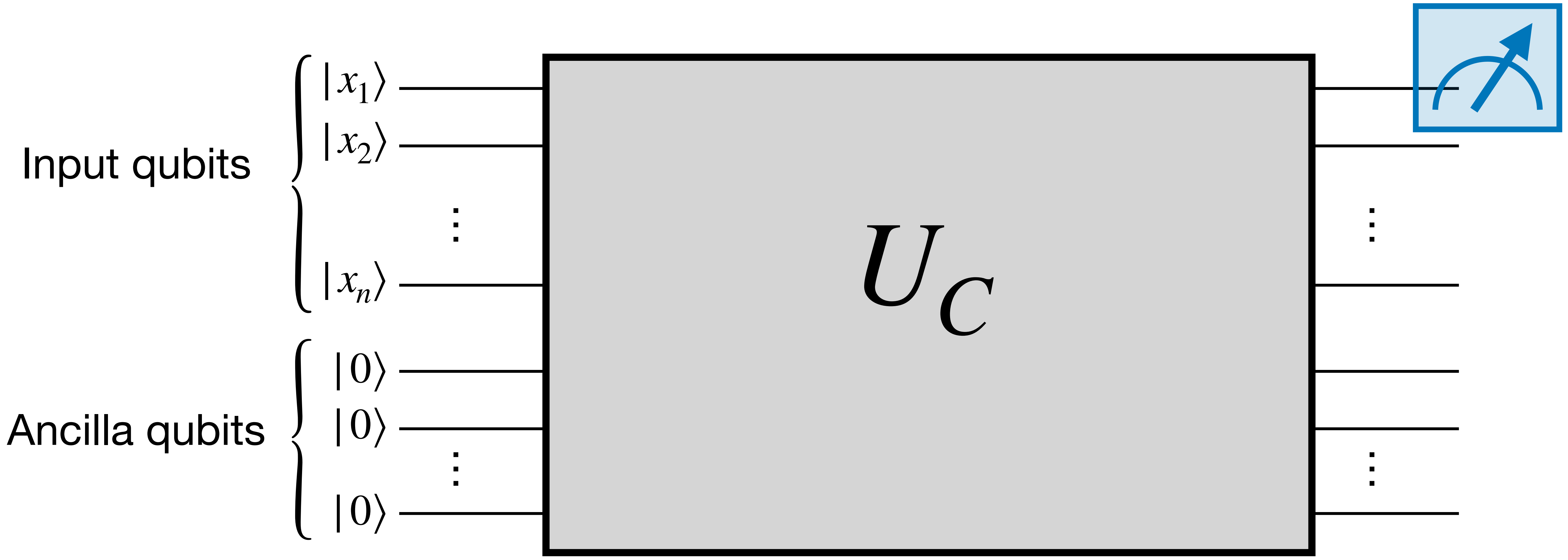

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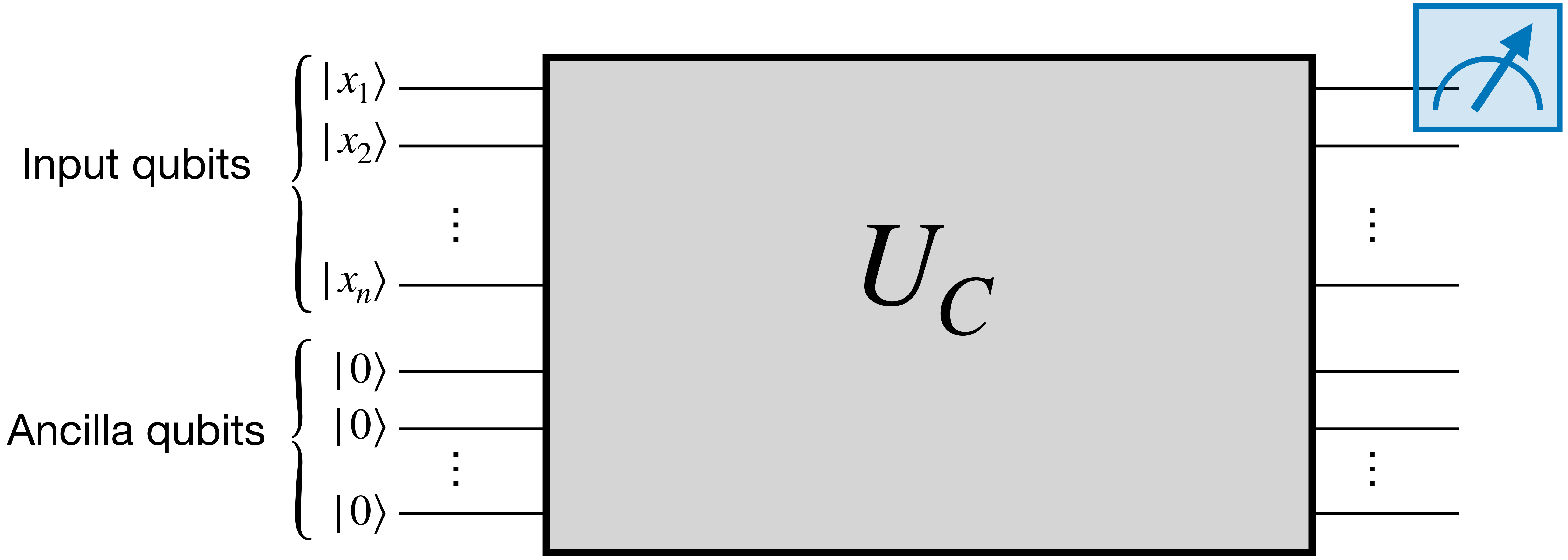

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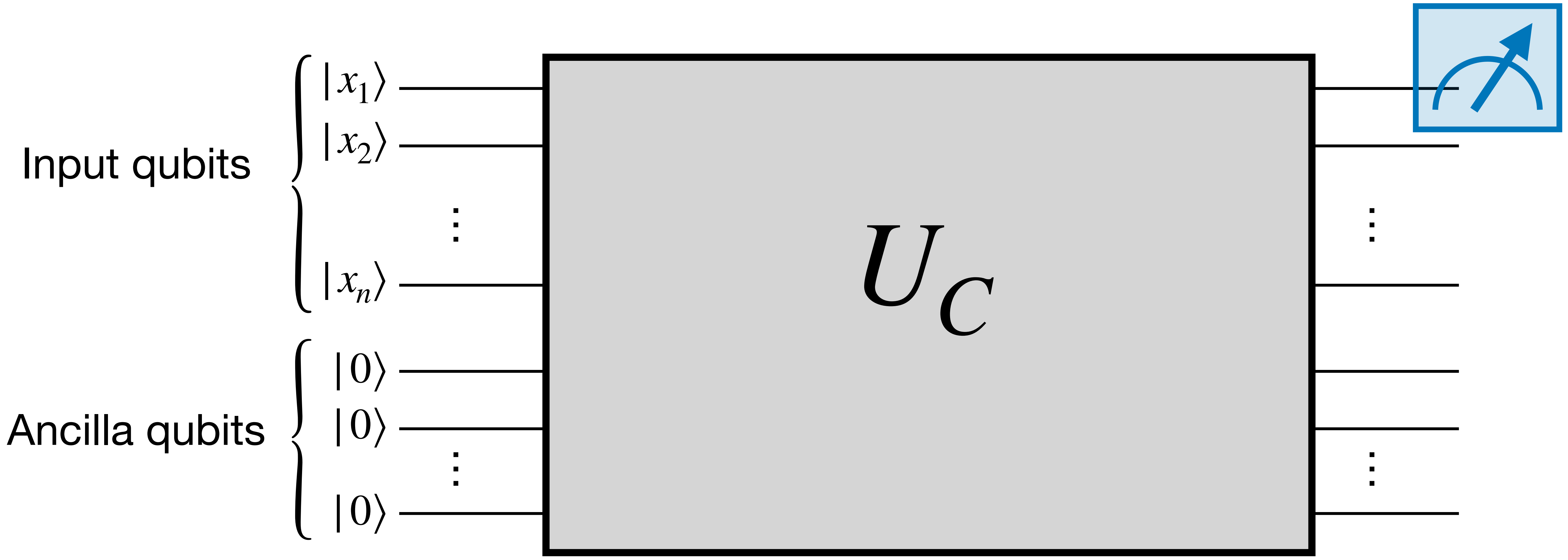

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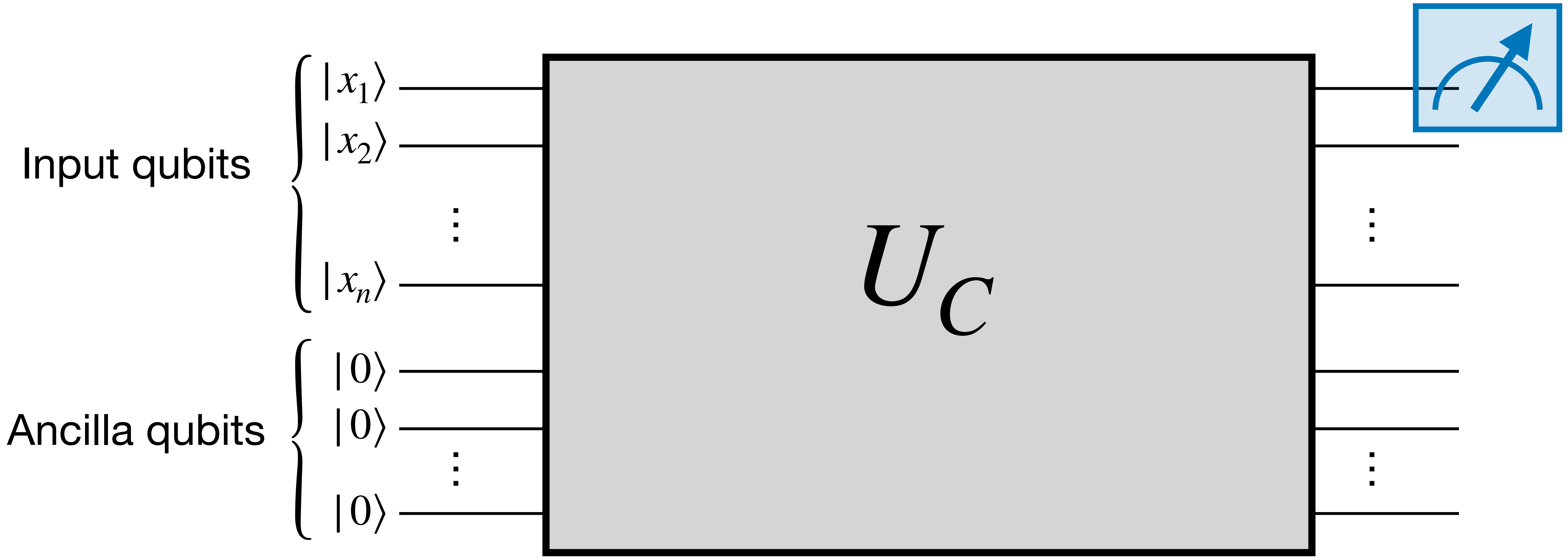

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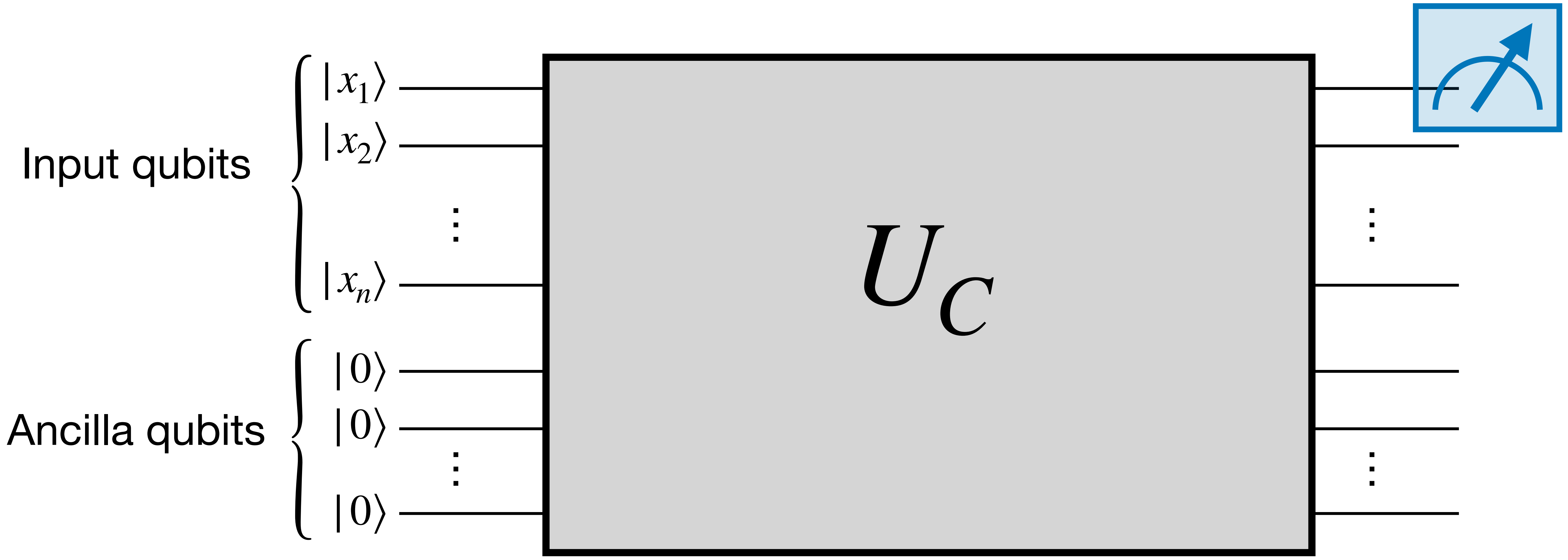

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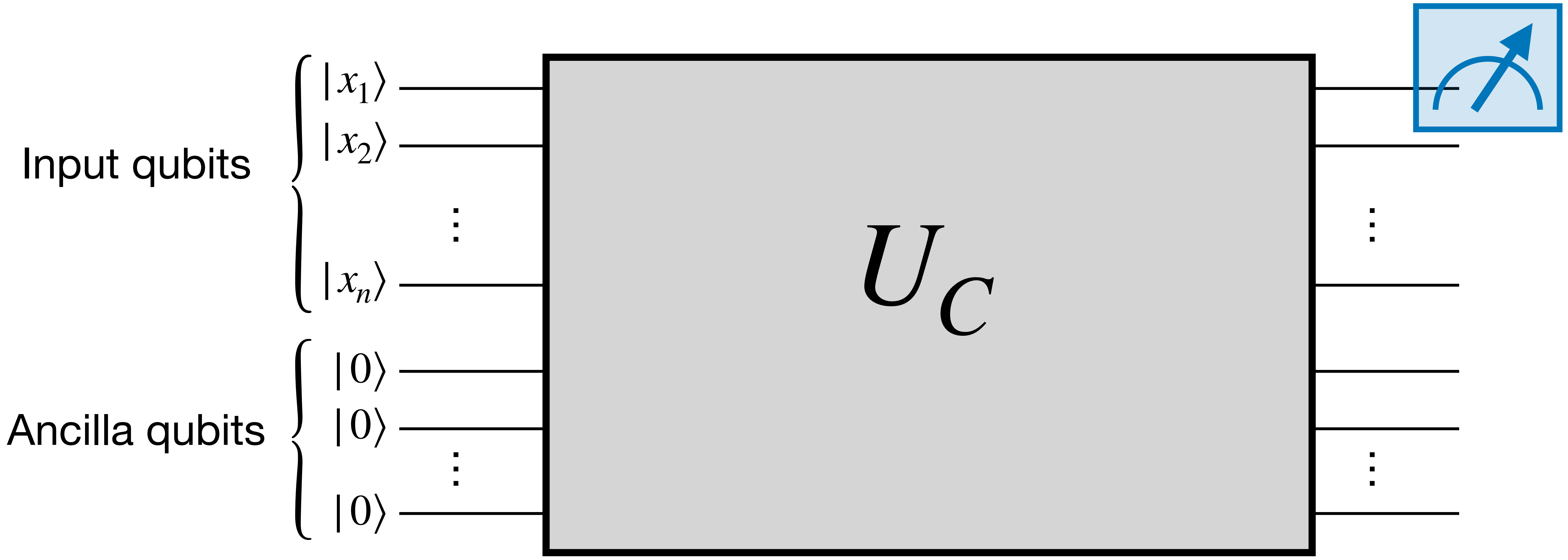

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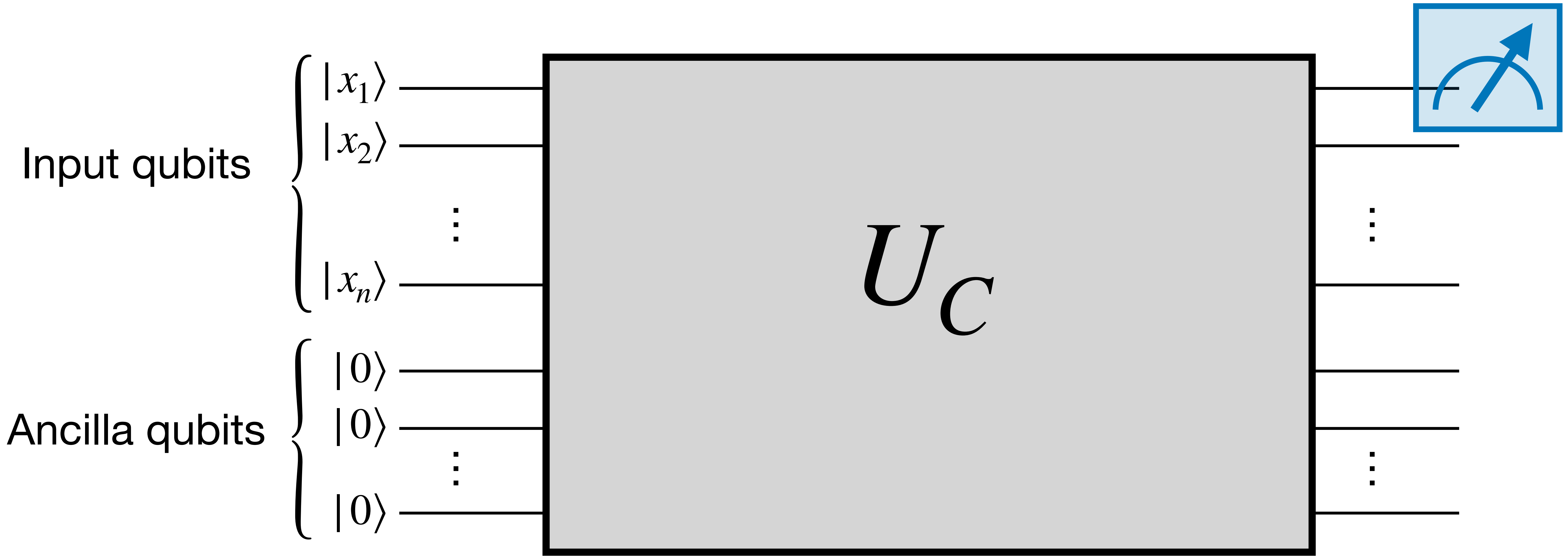

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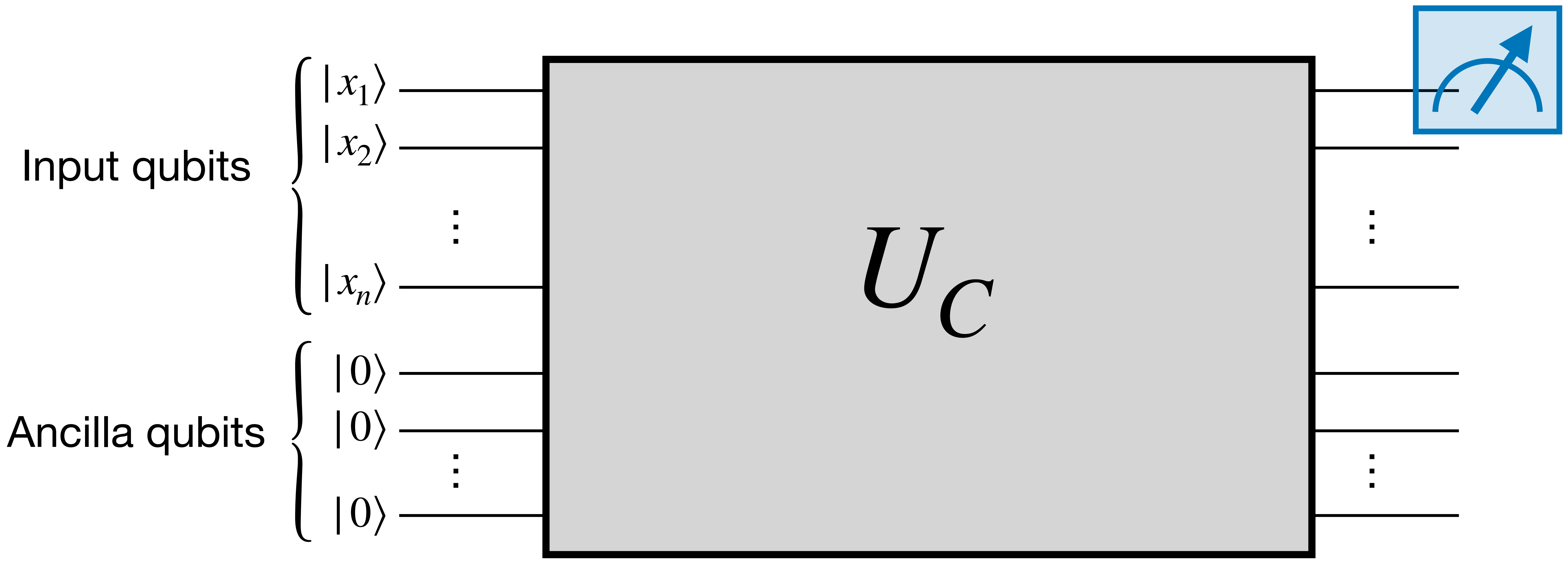

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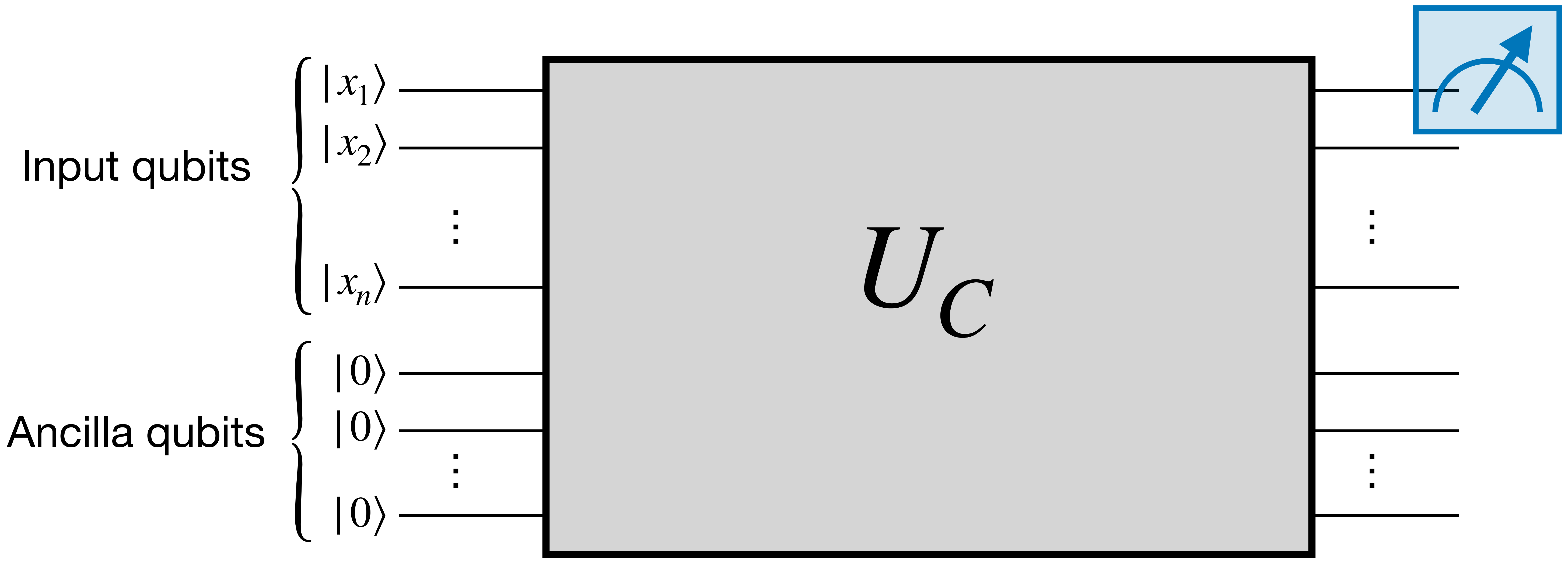

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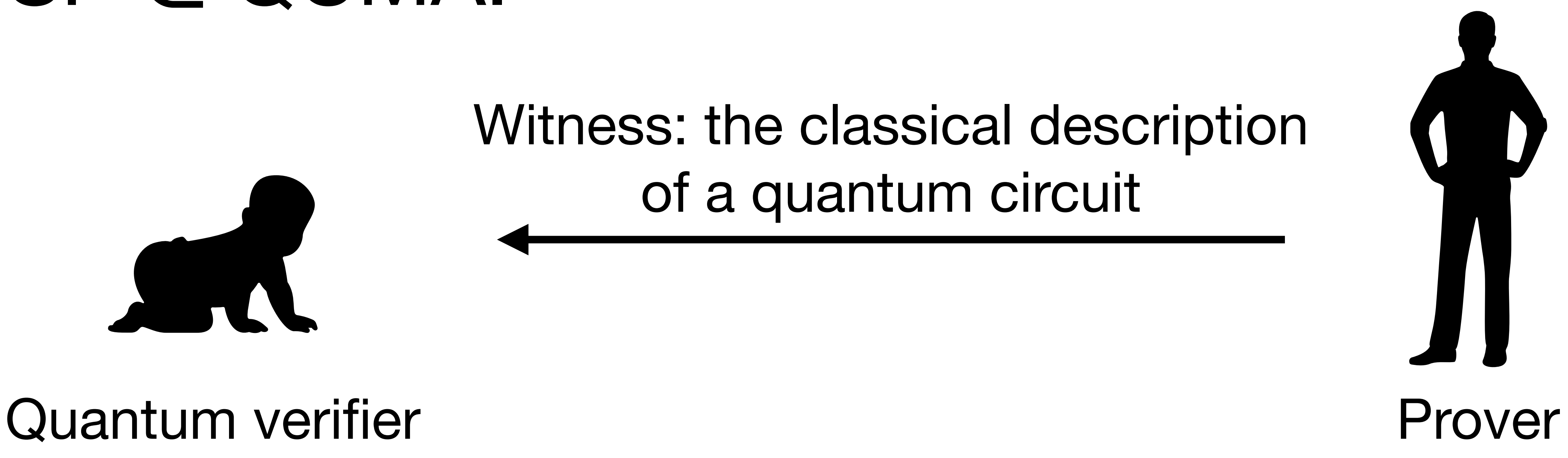

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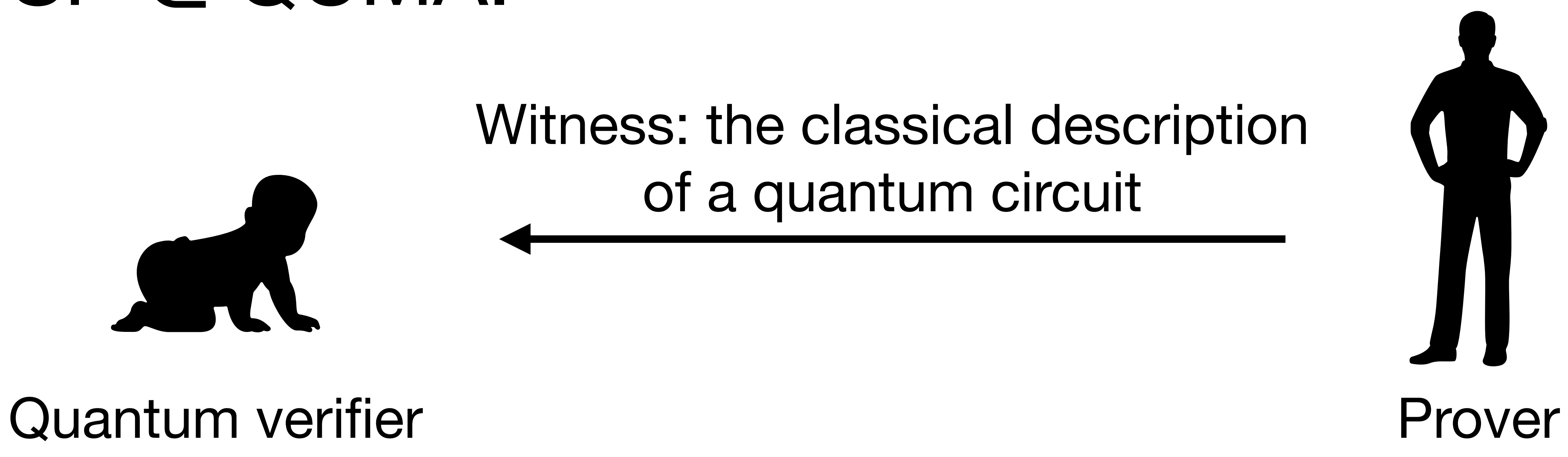

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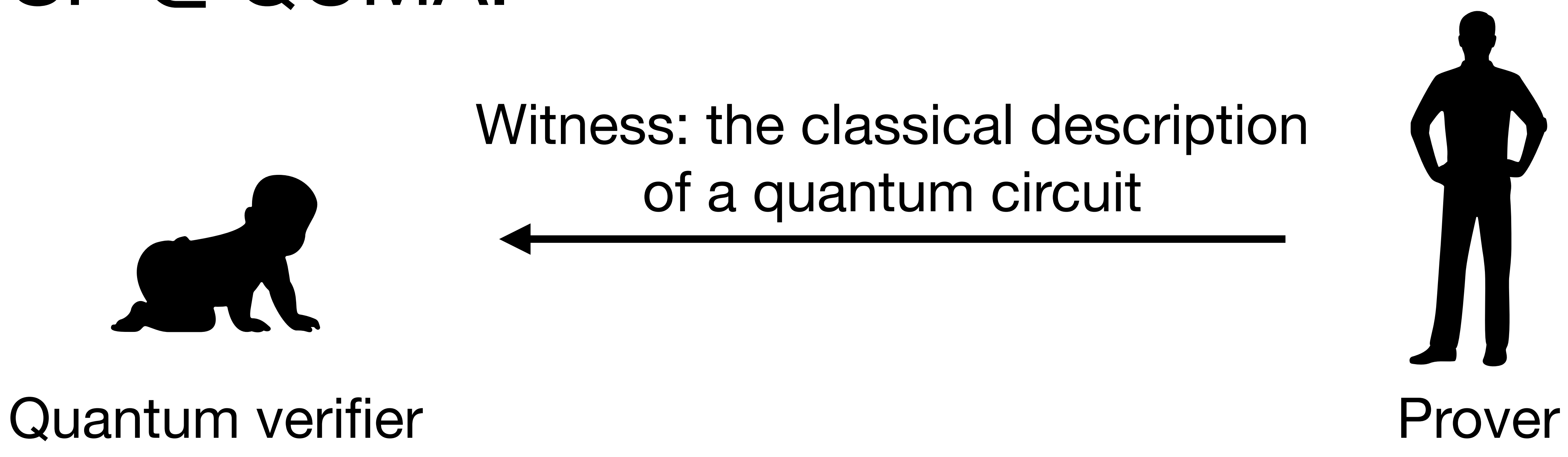

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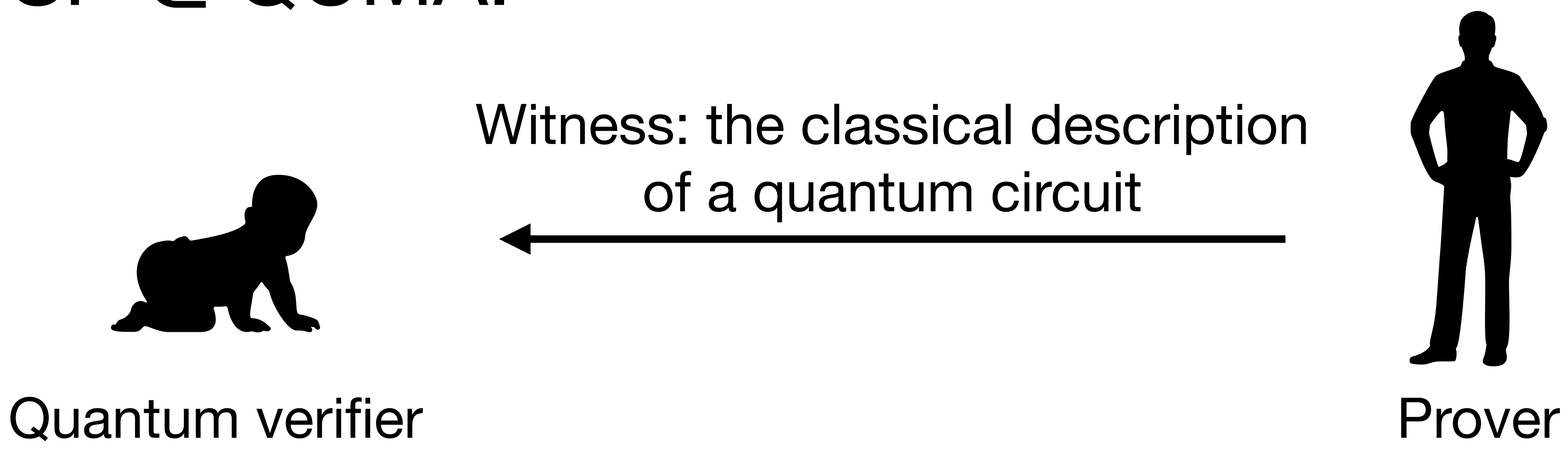

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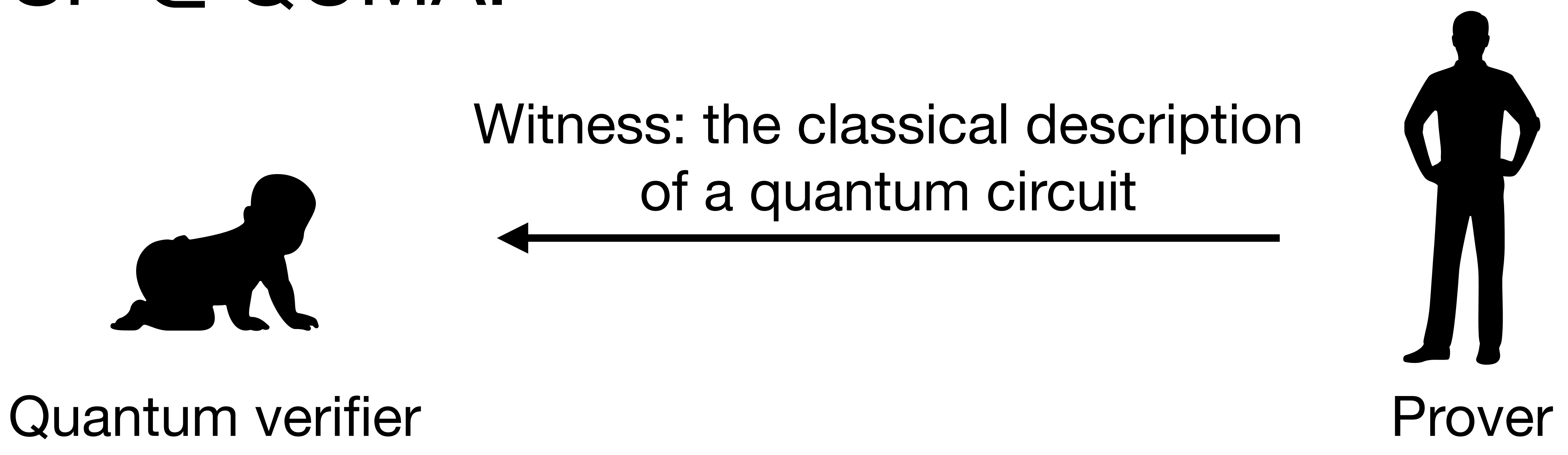

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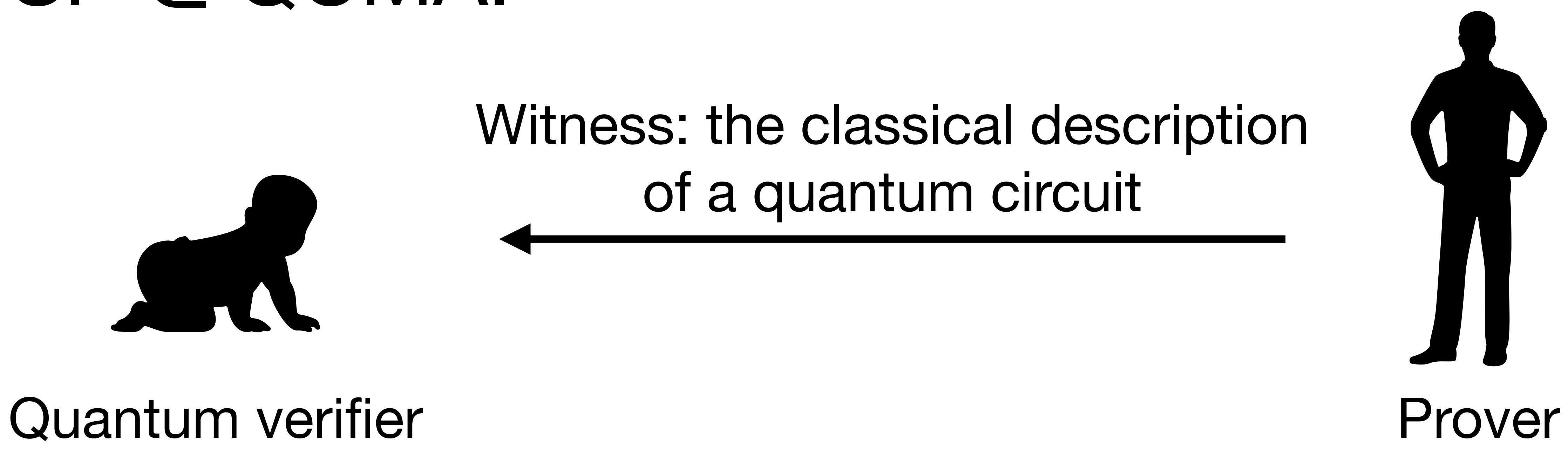

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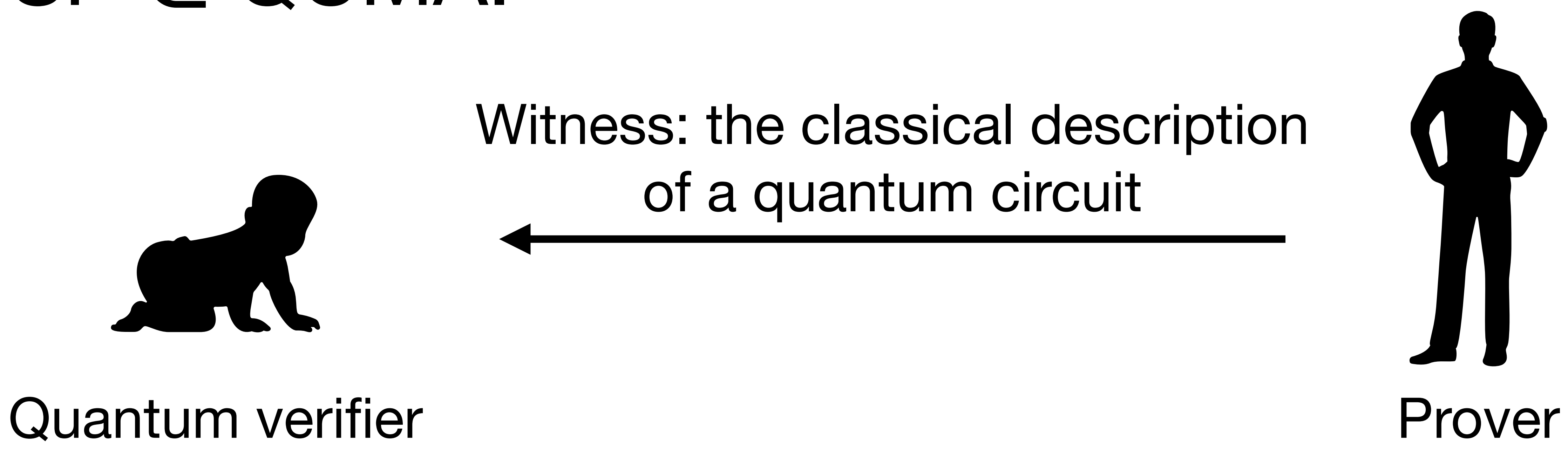

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Quantize classical results!

A Bird-Eye View on Our Results

A Bird-Eye View

	Results	Informal Theorem Index (Formal Theorem Index)
MQCSP (Def. 3.2)	MQCSP $\in$ QCMA	Theorem 1.4 (Theorem 3.9)
	MQCSP $\in$ BQP $\Rightarrow$ No qOWF	Theorem 1.4 (Theorem 4.8)
	SZK $\leq$ MQCSP	Theorem 1.4 (Theorem 3.13)
	multiMQCSP is NP-hard under a natural gate set	Theorem 1.4 (Theorem 3.14)
	$i\mathcal{O}$ + MQCSP $\in$ BQP $\Rightarrow$ NP $\subseteq$ coRQP	Theorem 1.4 (Theorem 4.10)
	PAC learning for BQP/poly $\Leftrightarrow$ MQCSP $\in$ BPP	Theorem 1.5 (Theorem 4.12)
	BQP learning $\Leftrightarrow$ MQCSP $\in$ BQP	Theorem 1.6 (Theorem 4.14)
	MQCSP $\in$ BQP $\Rightarrow$ BQE $\not\subset$ BQC$[n^k]$, $\forall k \in \mathbb{N}_+$	Theorem 1.7 (Theorem 4.19)
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	MQCSP $\in$ BQP $\Rightarrow$ Hardness amplification	Theorem 1.8 (Theorem 4.20)
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	QETH $\Rightarrow$ quantum hardness of MQCSP*	Theorem 1.10 (Theorem 4.27)
UMCSP (Def. 5.1)	UMCSP $\in$ QCMA	Theorem 1.11 (Theorem 5.5)
	Search-to-decision reduction for UMCSP	Theorem 1.12 (Theorem 5.16)
	gap-MQCSP $\leq$ UMCSP	Theorem 1.12 (Theorem 5.23)
	UMCSP $\in$ BQP $\Rightarrow$ No pseudorandom unitaries and no qOWF	(Theorem 5.24, Corollary 5.25)
	$i\mathcal{O}$ + UMCSP $\in$ BQP $\Rightarrow$ NP $\subseteq$ coRQP	(Corollary 5.26)
	UMCSP $\in$ BQP $\Rightarrow$ Hardness amplification for BQP	(Corollary 5.27)
	UMCSP $\in$ BQP $\Rightarrow$ BQE $\not\subset$ BQP$[n^k]$, $\forall k \in \mathbb{N}$	(Corollary 5.28)
SMCSP (Def. 5.2)	SMCSP can be verified via QCMA	Theorem 1.11 (Theorem 5.9)
	Search-to-decision reduction for SMCSP	Theorem 1.12 (Theorem 5.18)
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A Bird-Eye View

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- Cryptography.

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Mostly quantize classical results!

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	multiMQCSP is NP-hard under a natural gate set	Theorem 1.4 (Theorem 3.14)
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	PAC learning for BQP/poly $\Leftrightarrow$ MQCSP $\in$ BPP	Theorem 1.5 (Theorem 4.12)
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	MQCSP $\in$ BQP $\Rightarrow$ BQE $\not\subset$ BQC$[n^k]$, $\forall k \in \mathbb{N}_+$	Theorem 1.7 (Theorem 4.19)
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UMCSP (Def. 5.1)	UMCSP $\in$ QCMA	Theorem 1.11 (Theorem 5.5)
	Search-to-decision reduction for UMCSP	Theorem 1.12 (Theorem 5.16)
	gap-MQCSP $\leq$ UMCSP	Theorem 1.12 (Theorem 5.23)
	UMCSP $\in$ BQP $\Rightarrow$ No pseudorandom unitaries and no qOWF	(Theorem 5.24, Corollary 5.25)
	$i\mathcal{O}$ + UMCSP $\in$ BQP $\Rightarrow$ NP $\subseteq$ coRQP	(Corollary 5.26)
	UMCSP $\in$ BQP $\Rightarrow$ Hardness amplification for BQP	(Corollary 5.27)
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SMCSP (Def. 5.2)	SMCSP can be verified via QCMA	Theorem 1.11 (Theorem 5.9)
	Search-to-decision reduction for SMCSP	Theorem 1.12 (Theorem 5.18)
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	SMCSP $\in$ BQP $\Rightarrow$ No pseudorandom states and no qOWF	Theorem 1.13 (Theorem 5.30)
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Mostly quantize classical results!

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A Bird-Eye View

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    - Although we can use Solovay-Kitaev theorem to generalize other gate sets, this causes overhead in circuit complexity.

Special Properties in the Quantum Setting

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With a focus on quantum states

SMCSP

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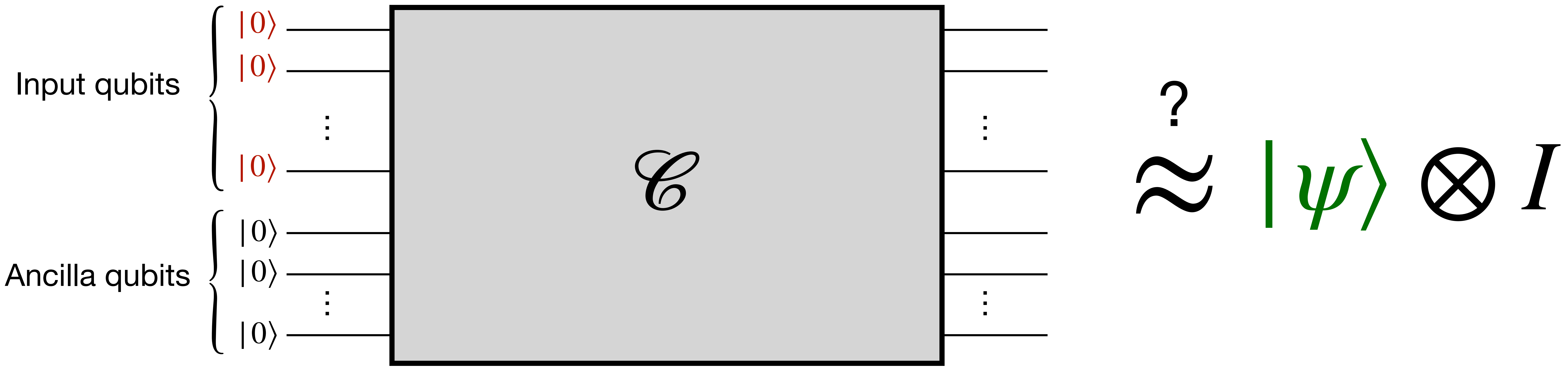

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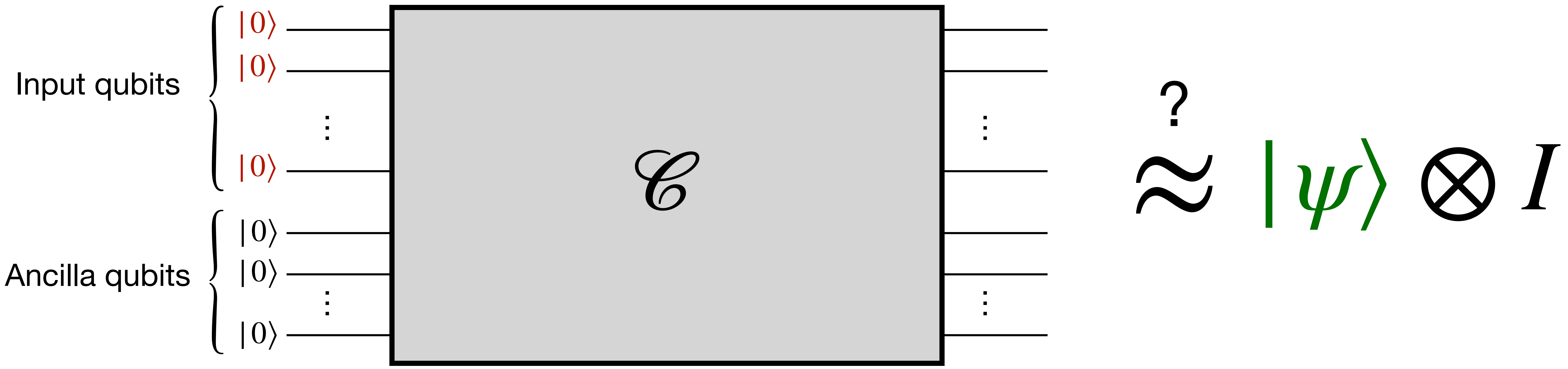

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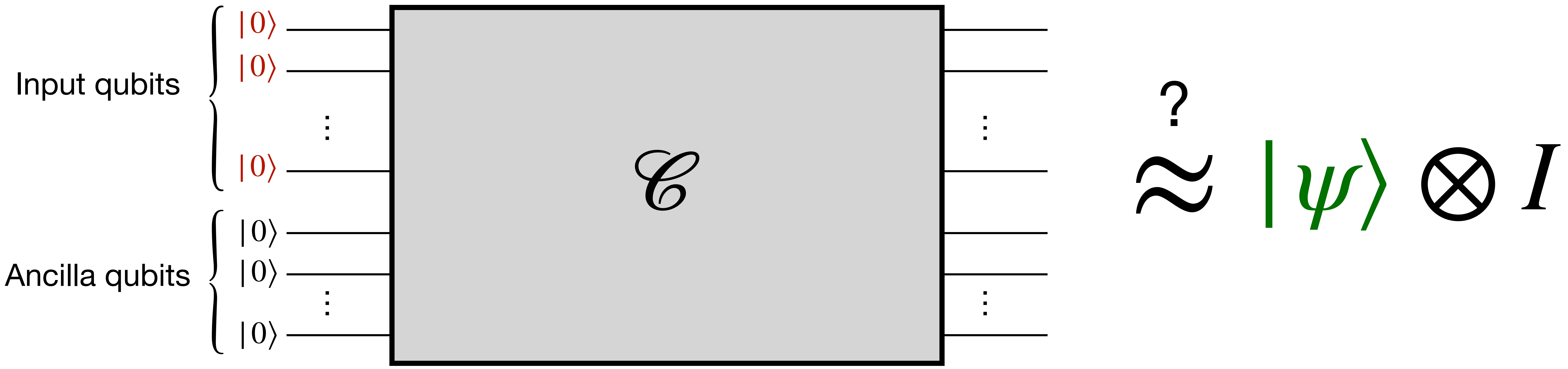

- **Remark 1:** Can define a version with "classical description" of $|\psi\rangle$ as the input.
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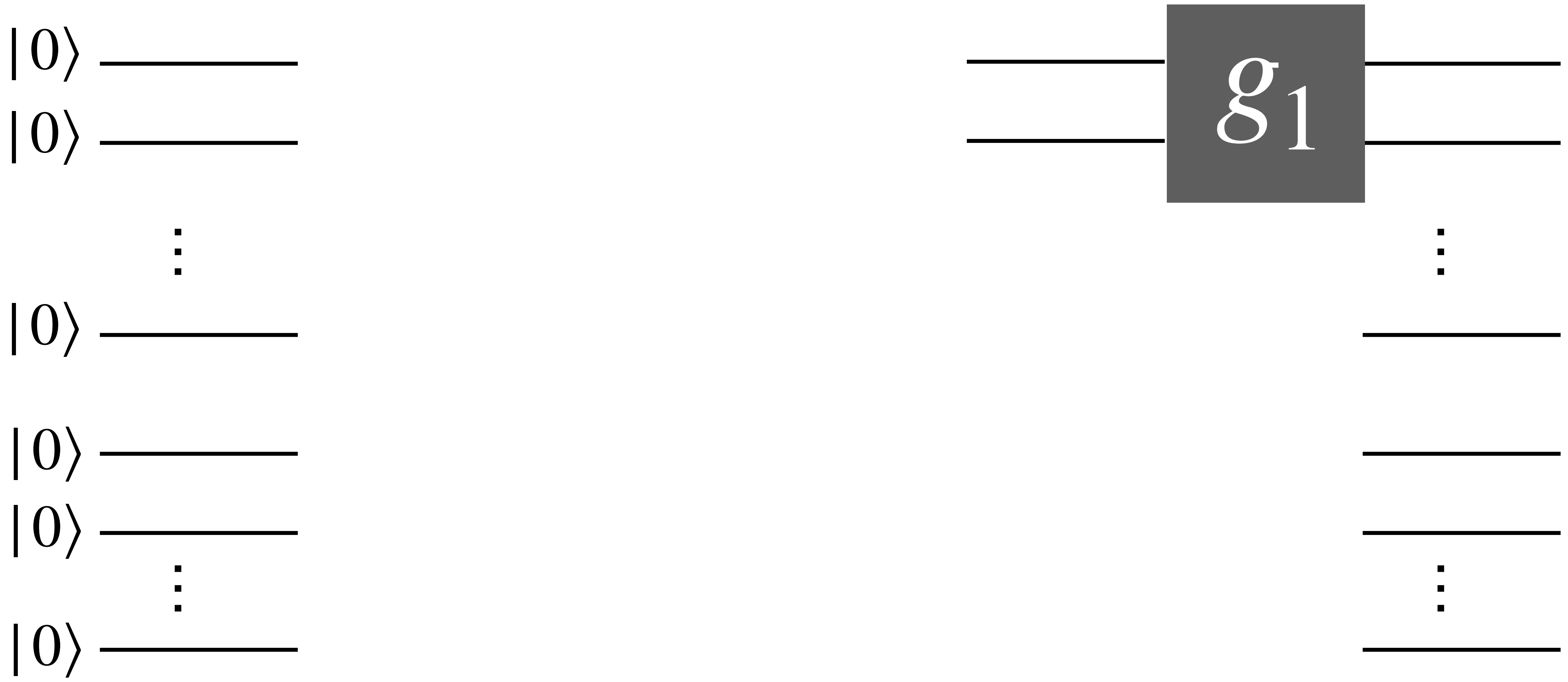

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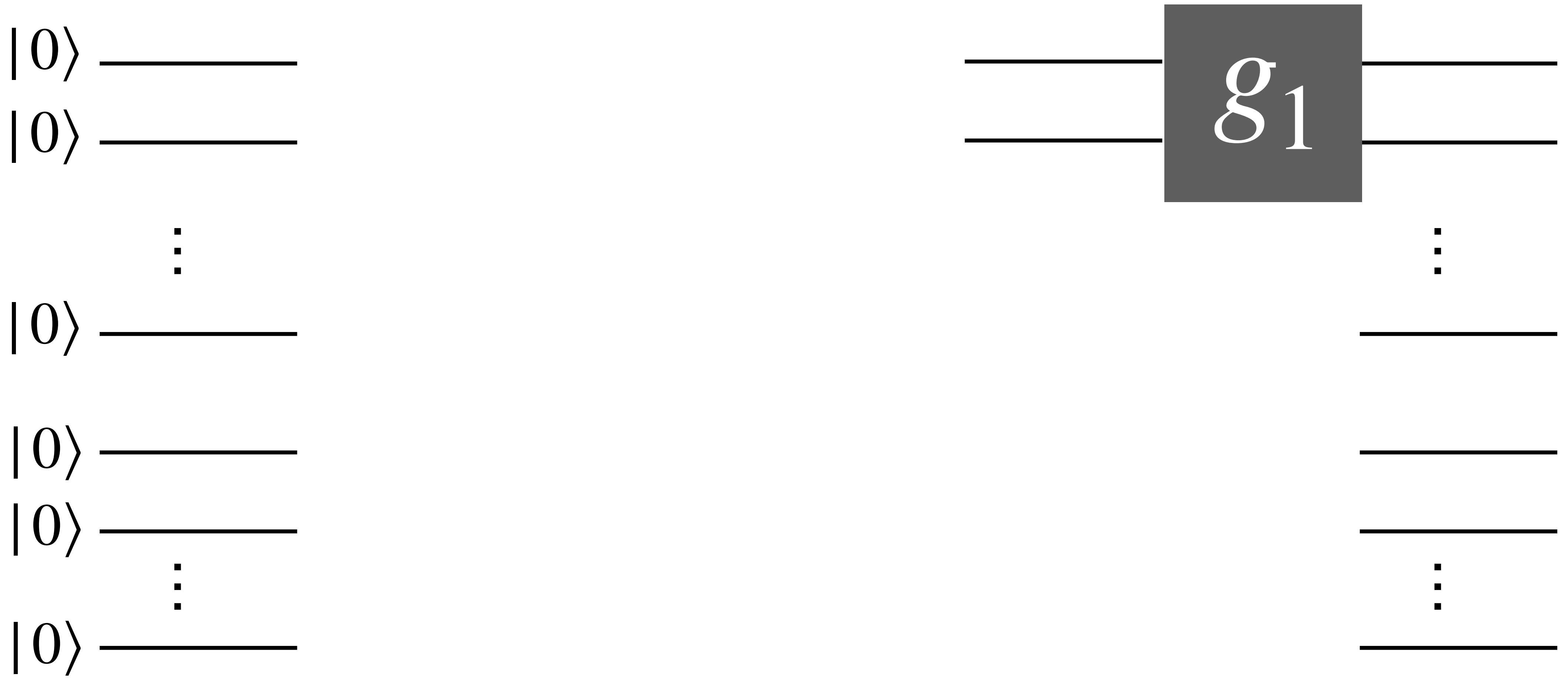

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$$(g_1^{-1}|\psi\rangle, s-1)$$

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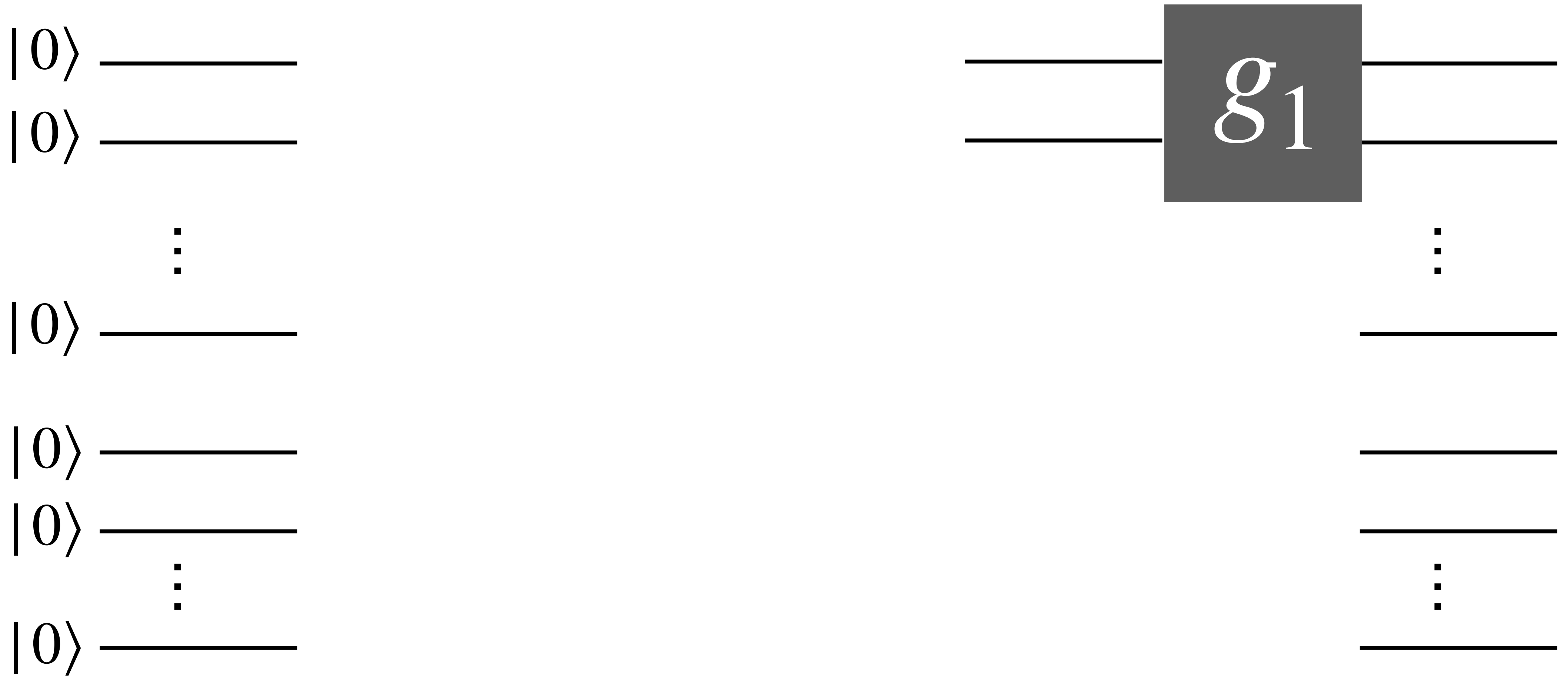

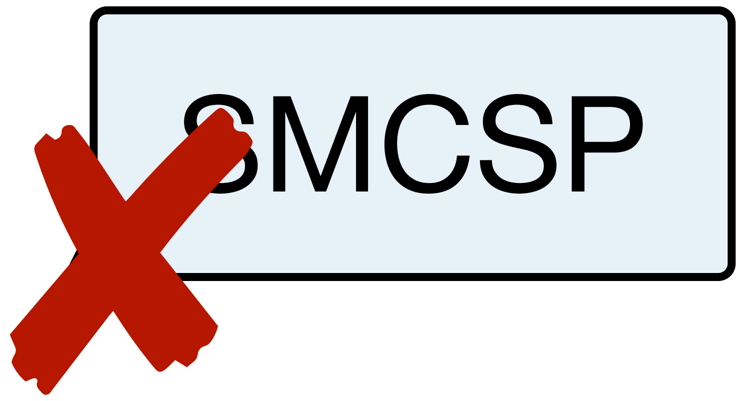

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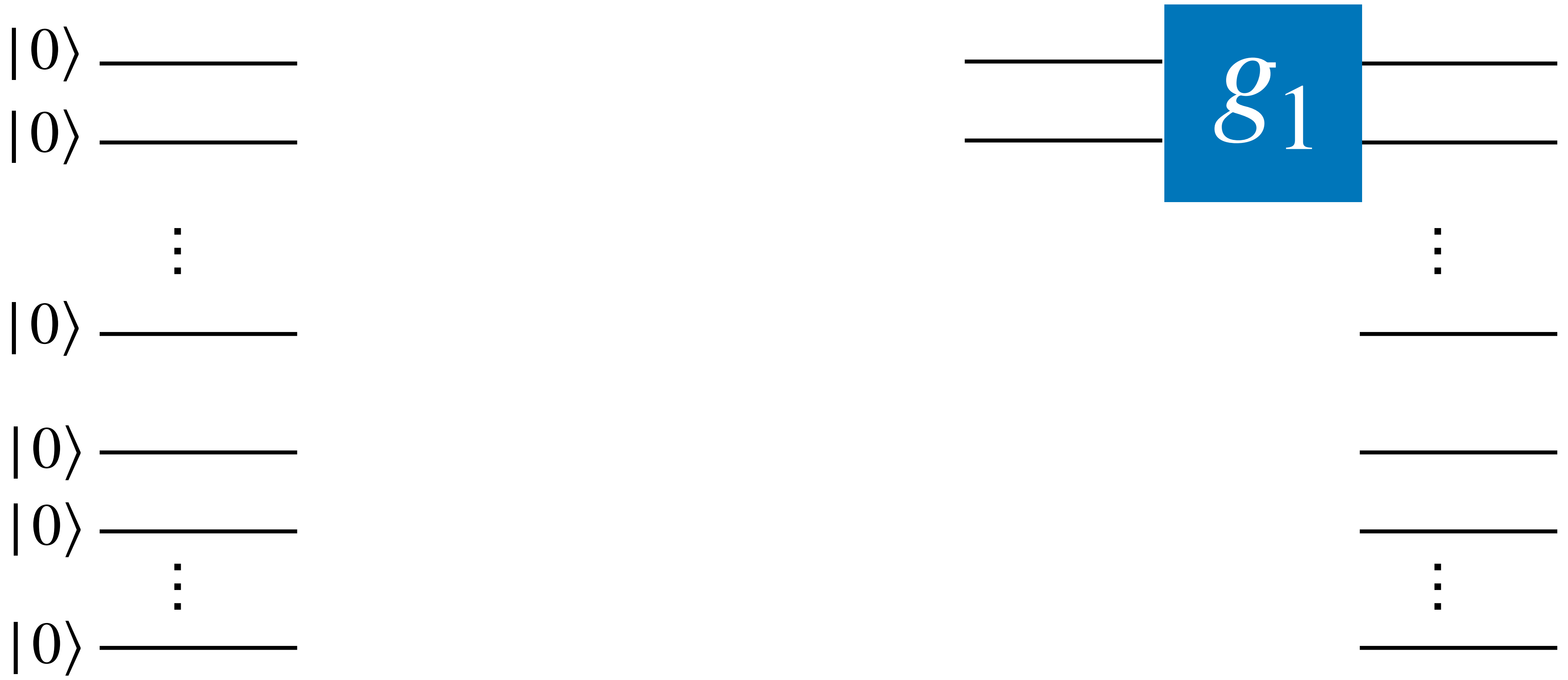

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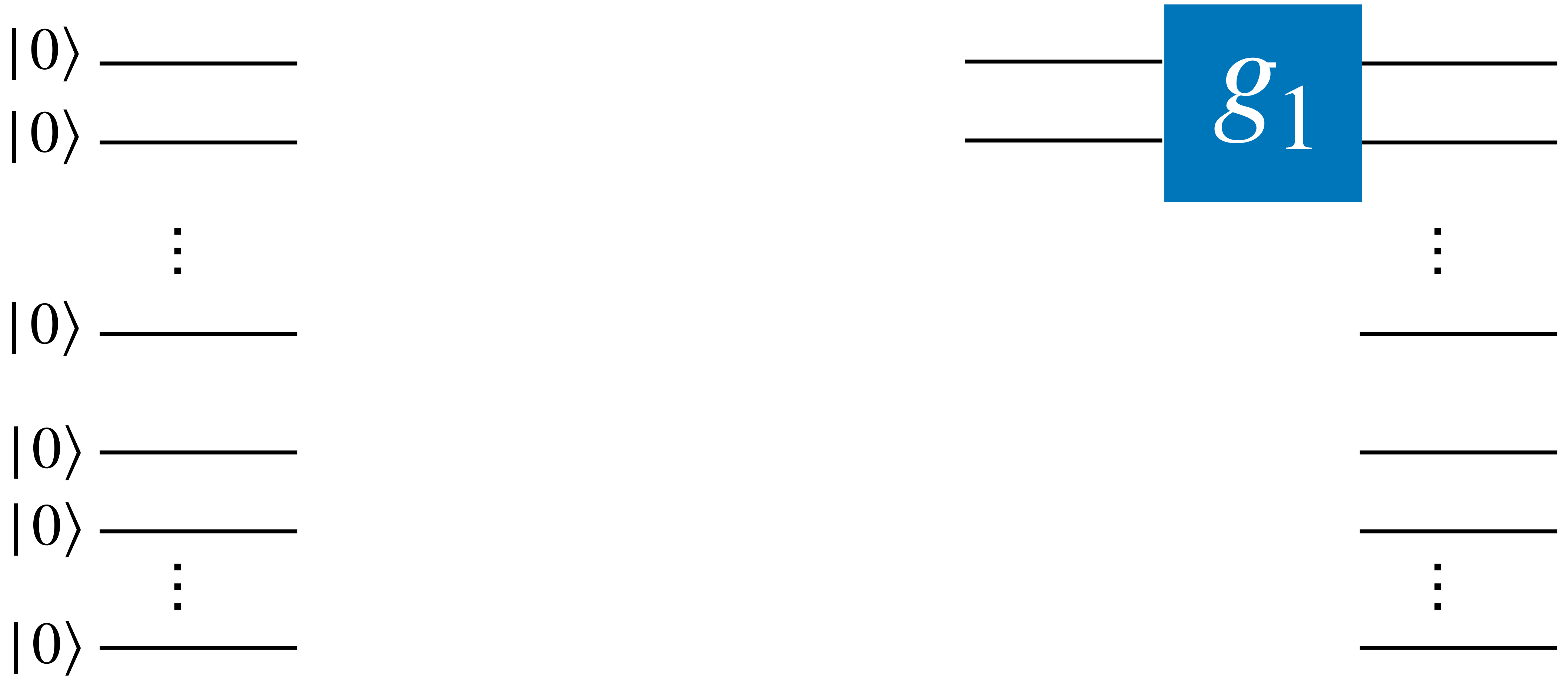

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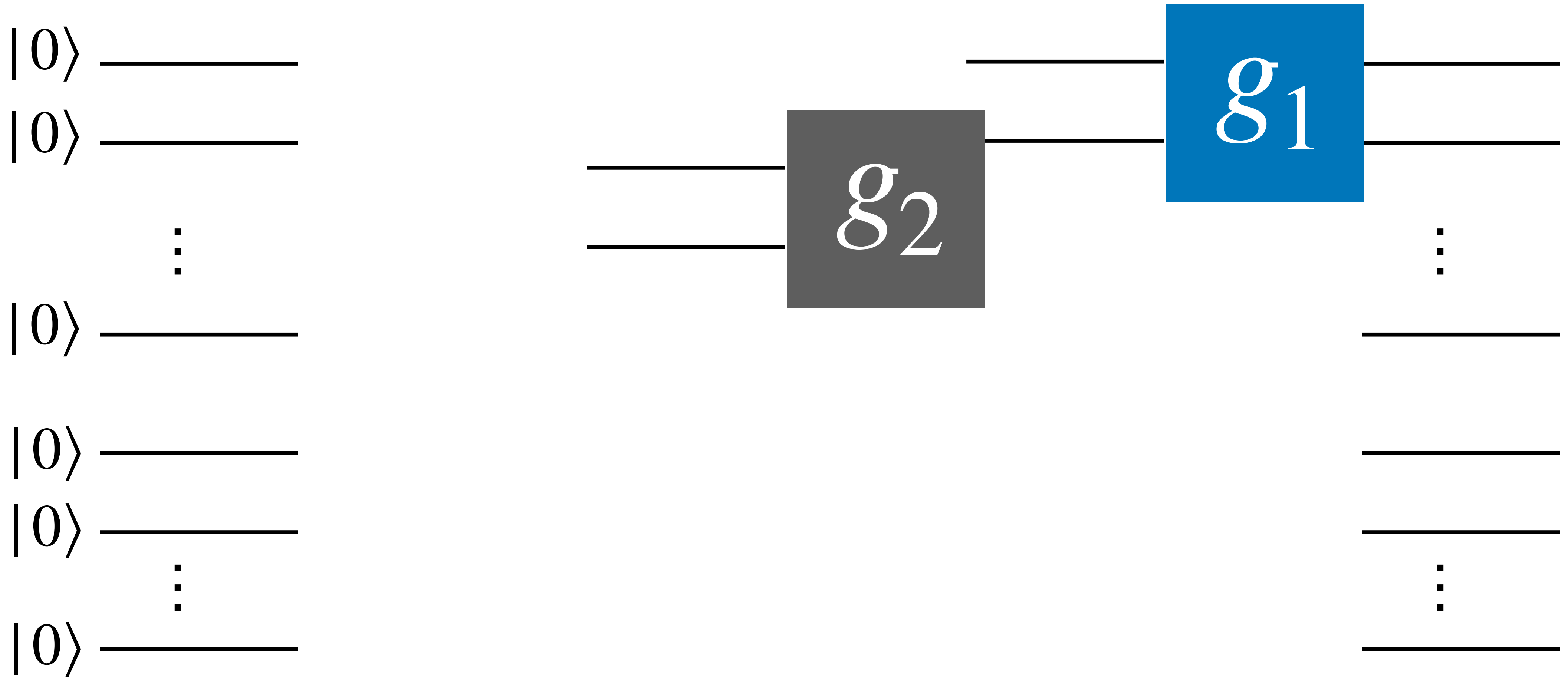

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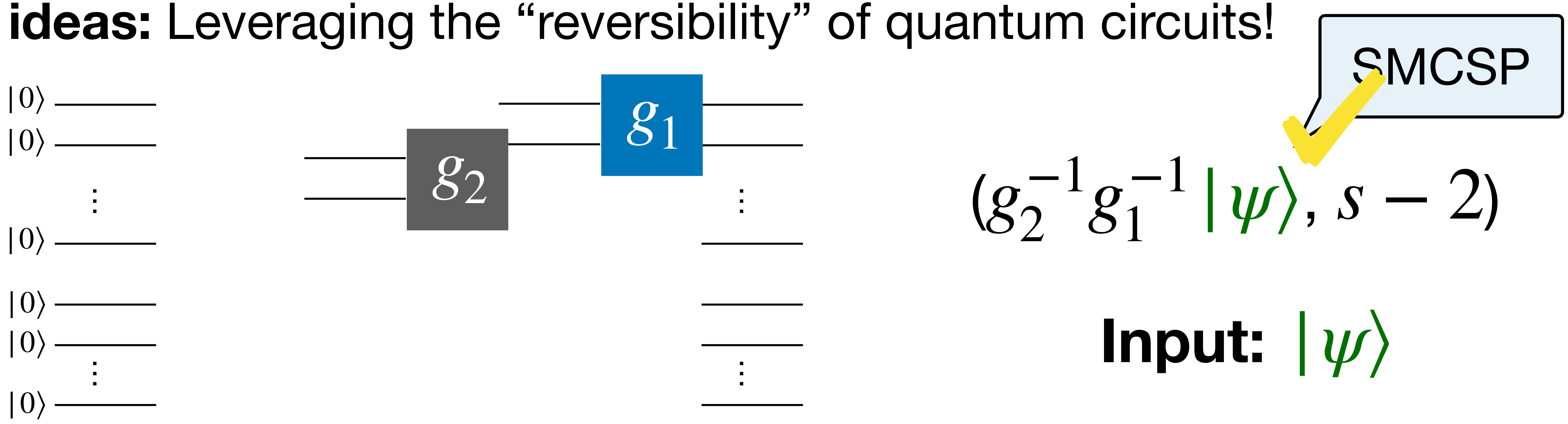

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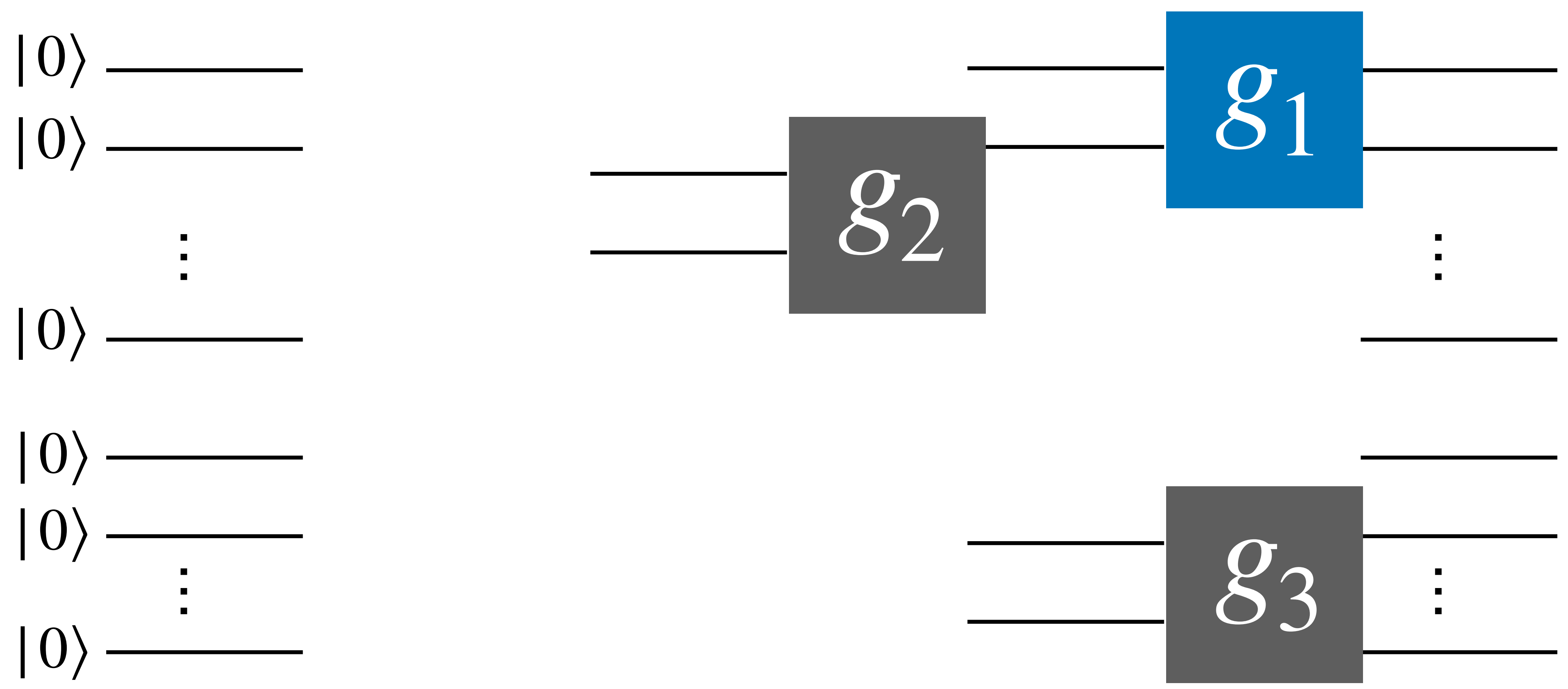

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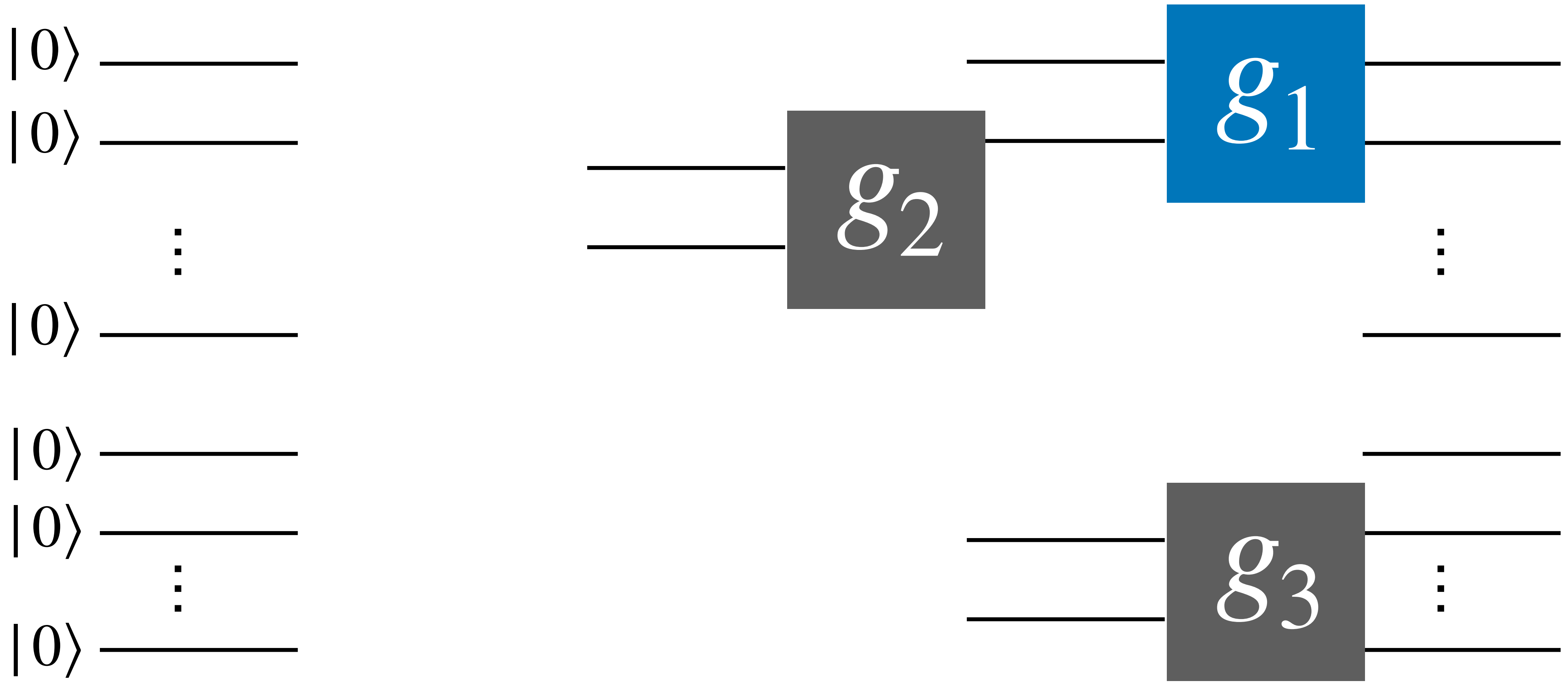

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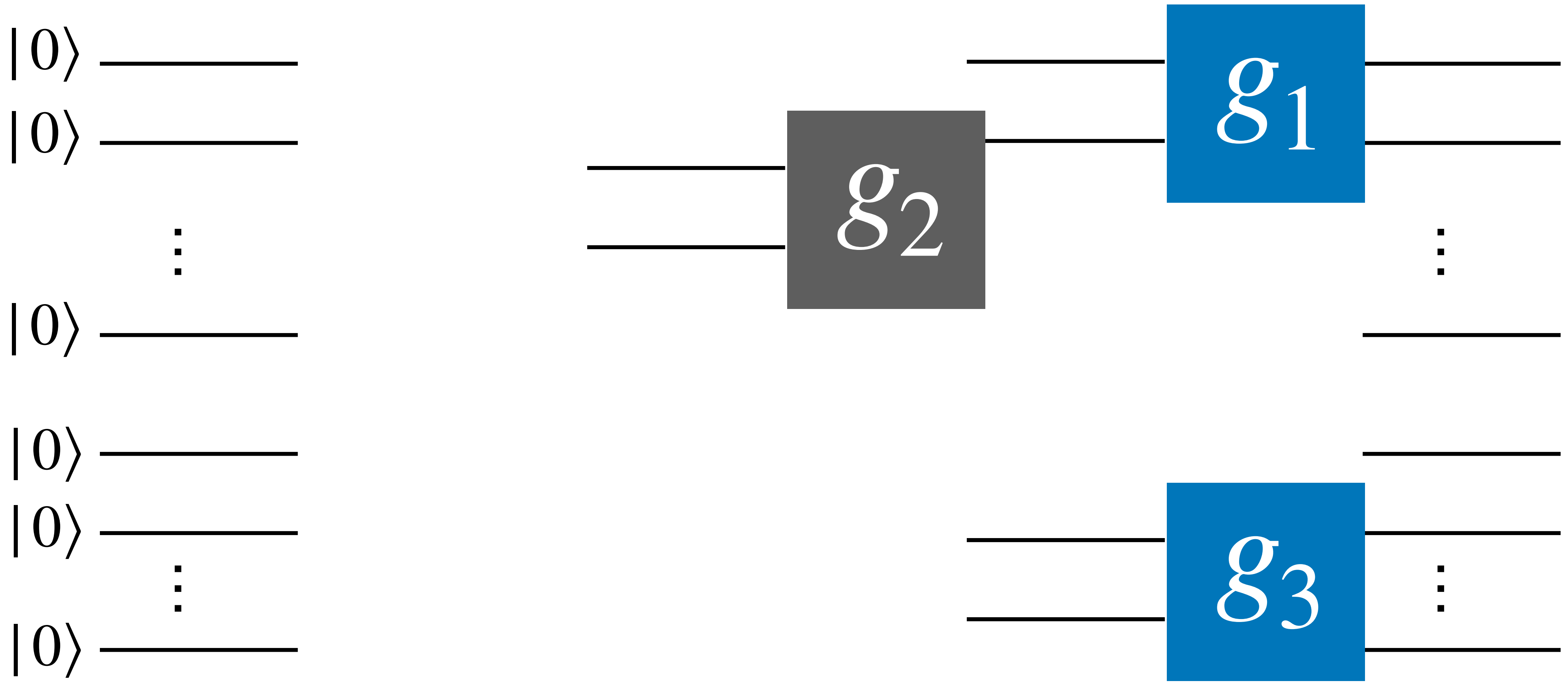

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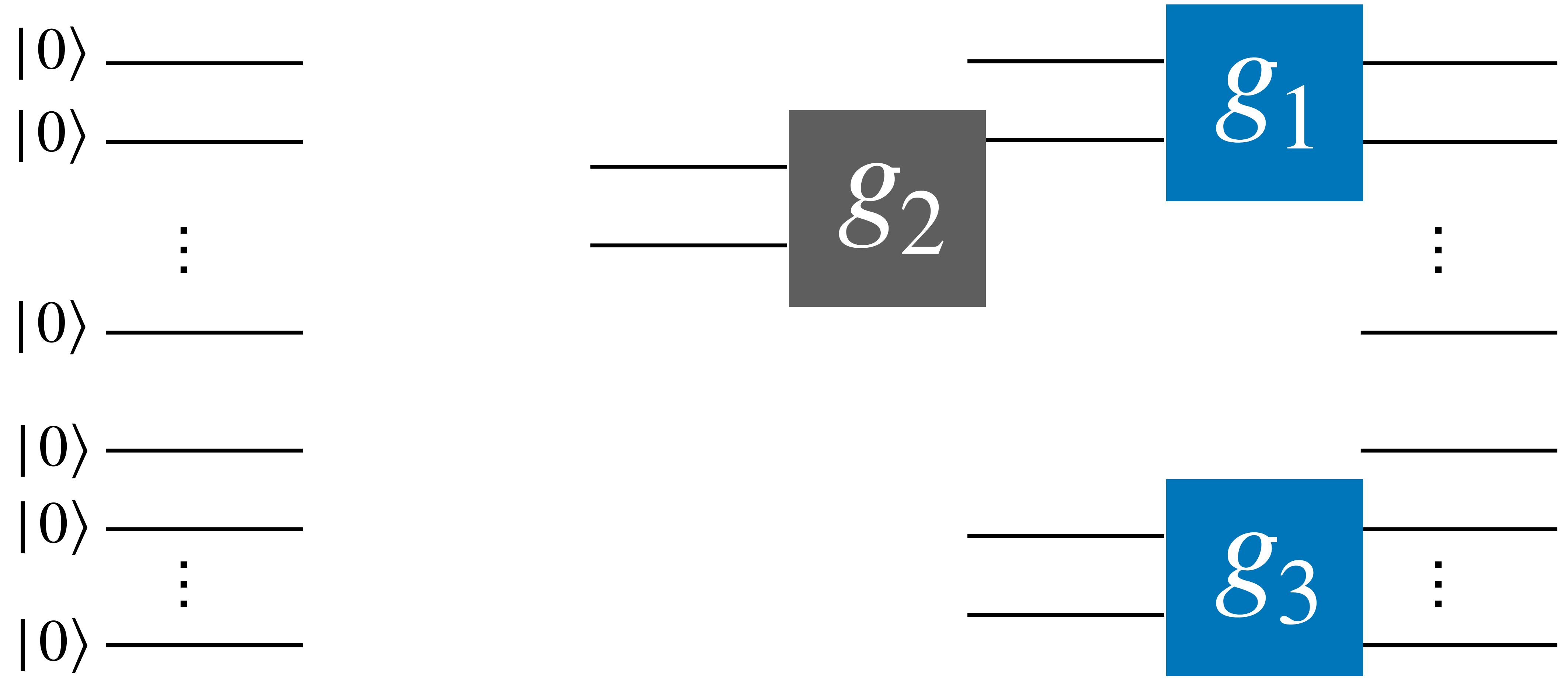

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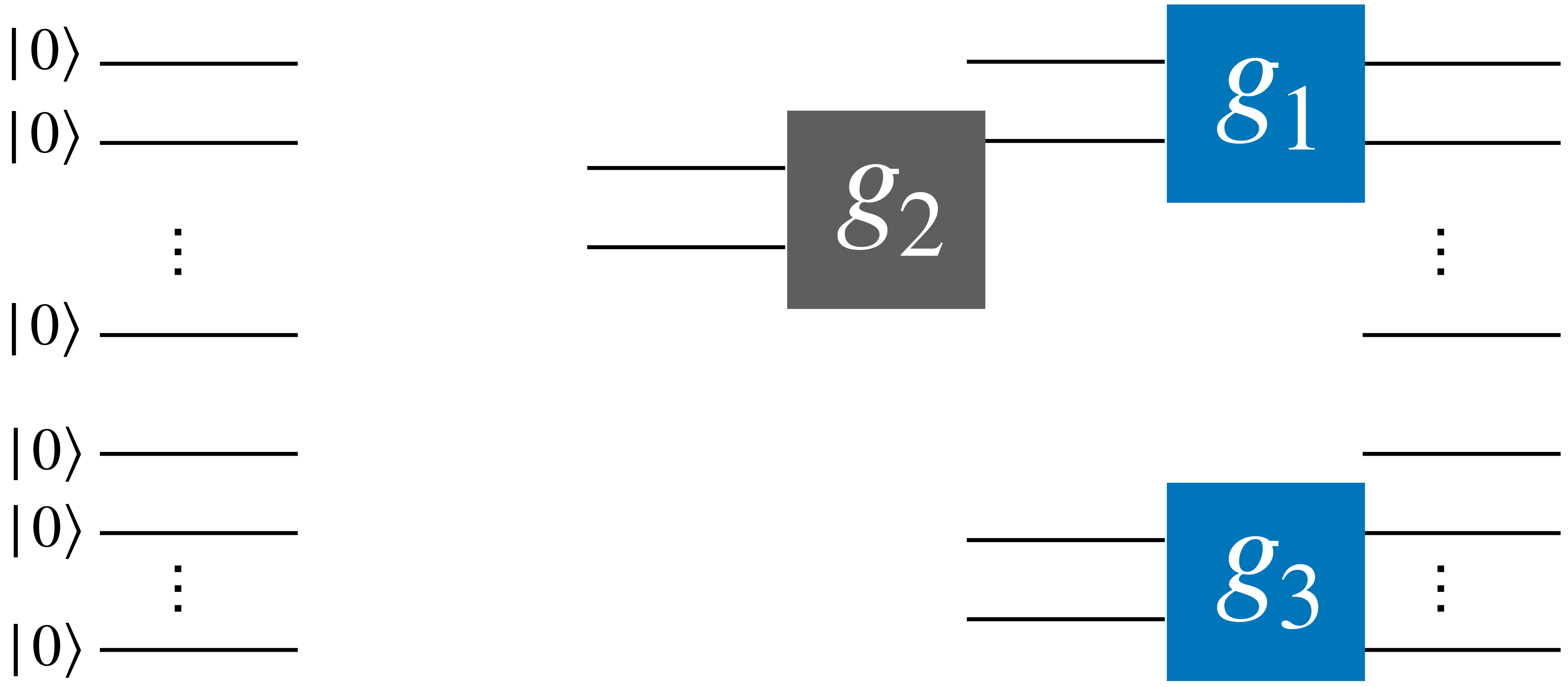

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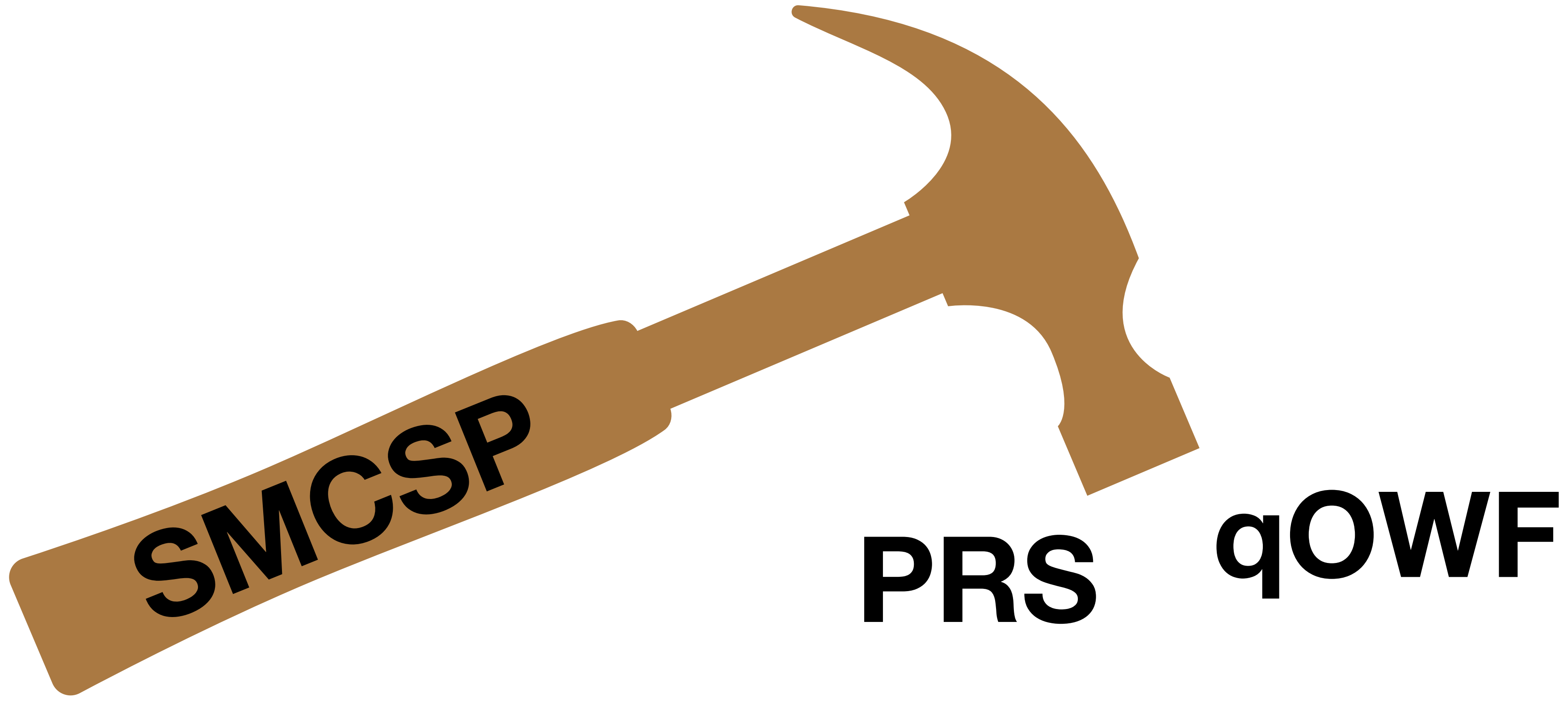

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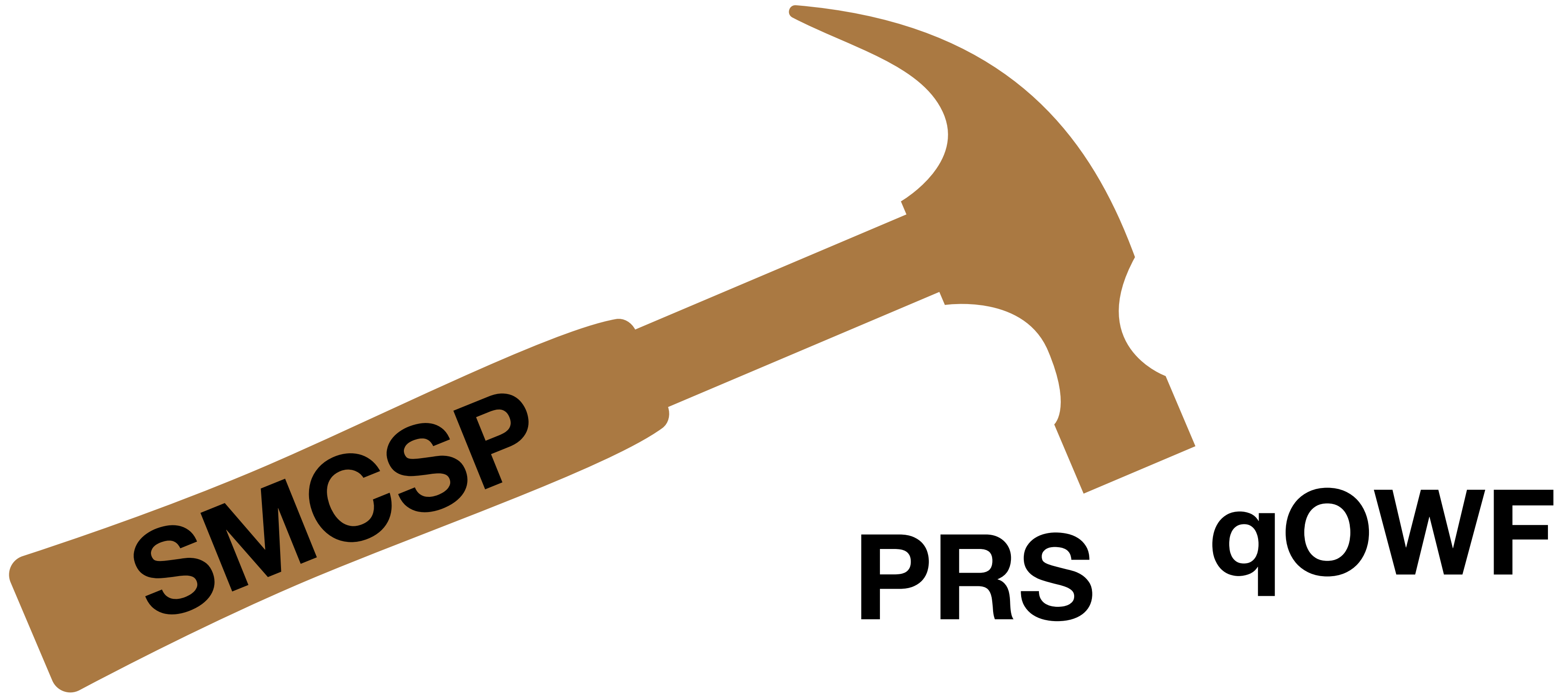

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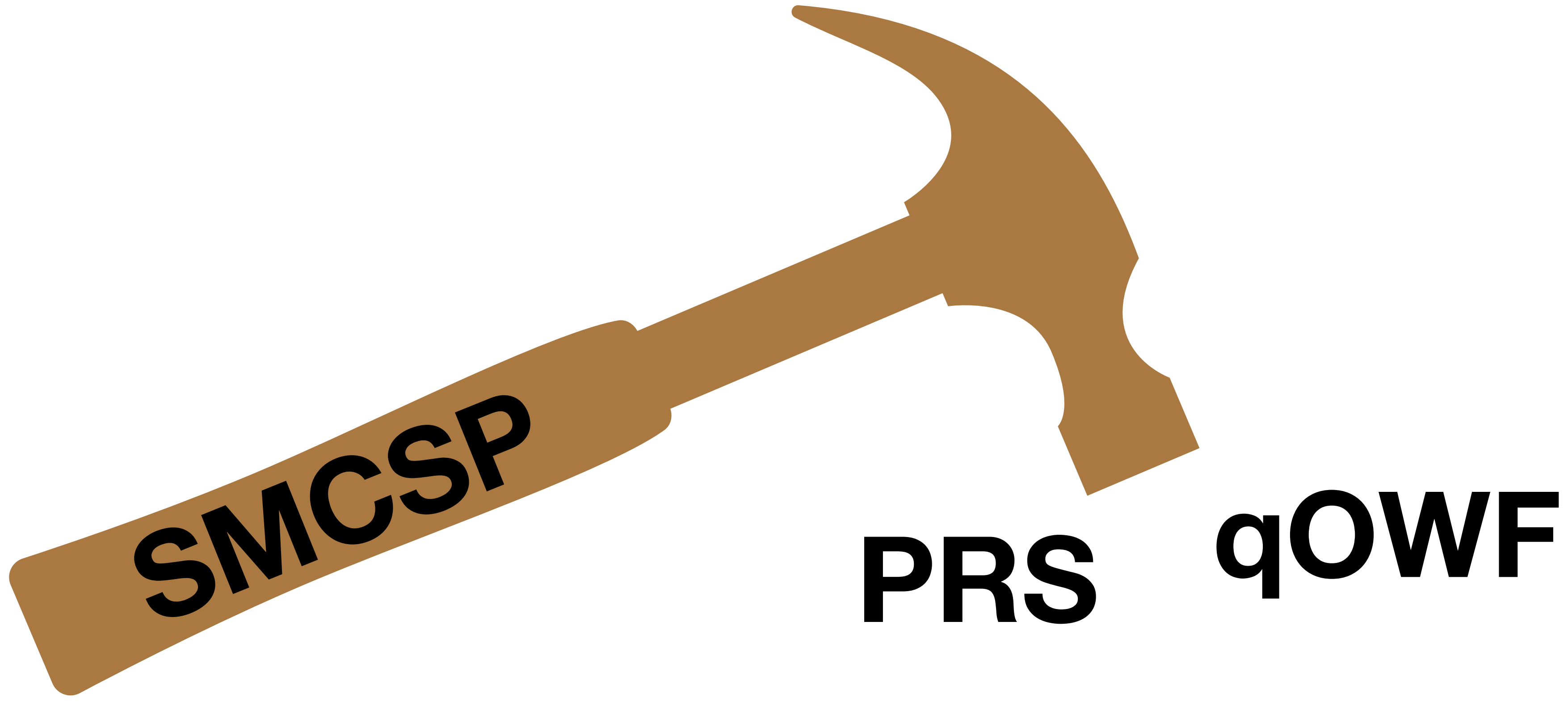

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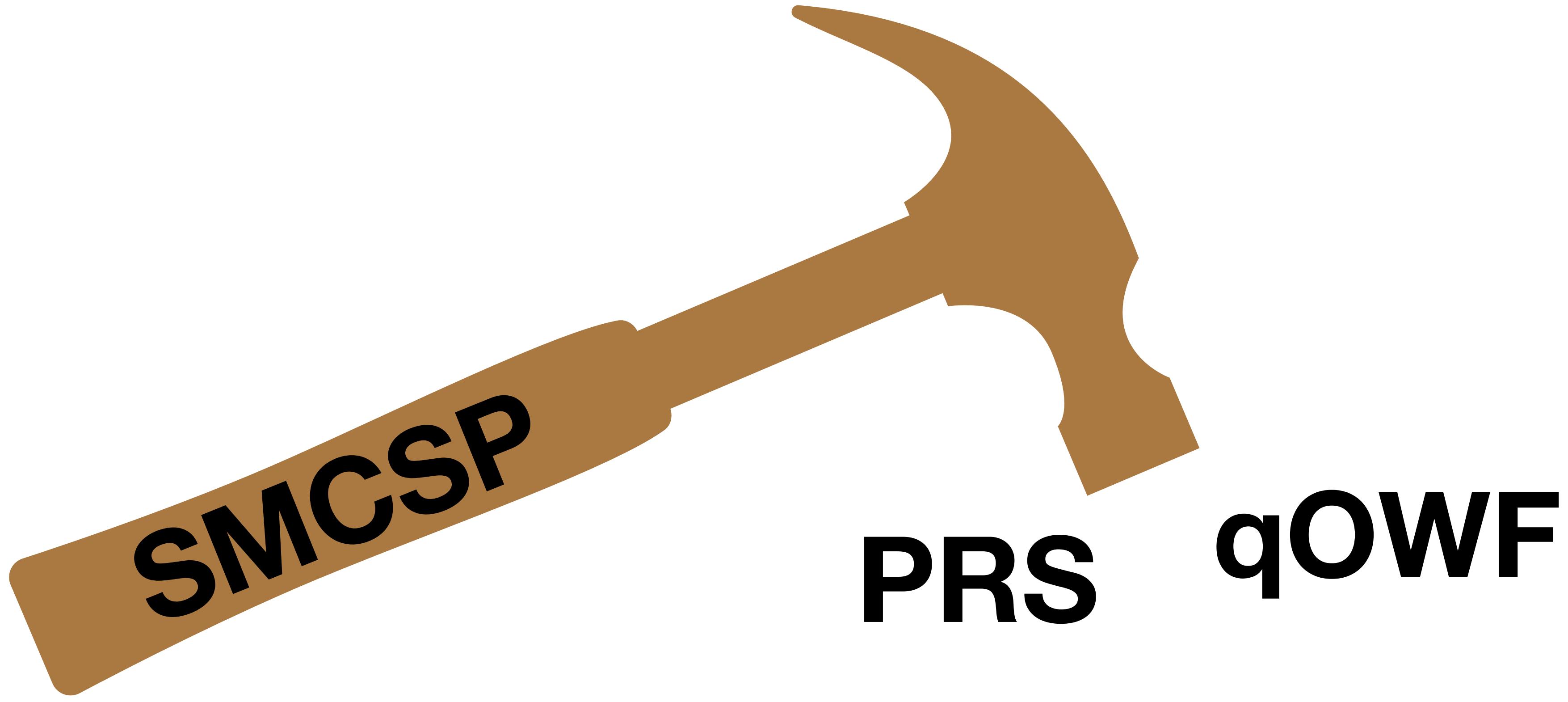

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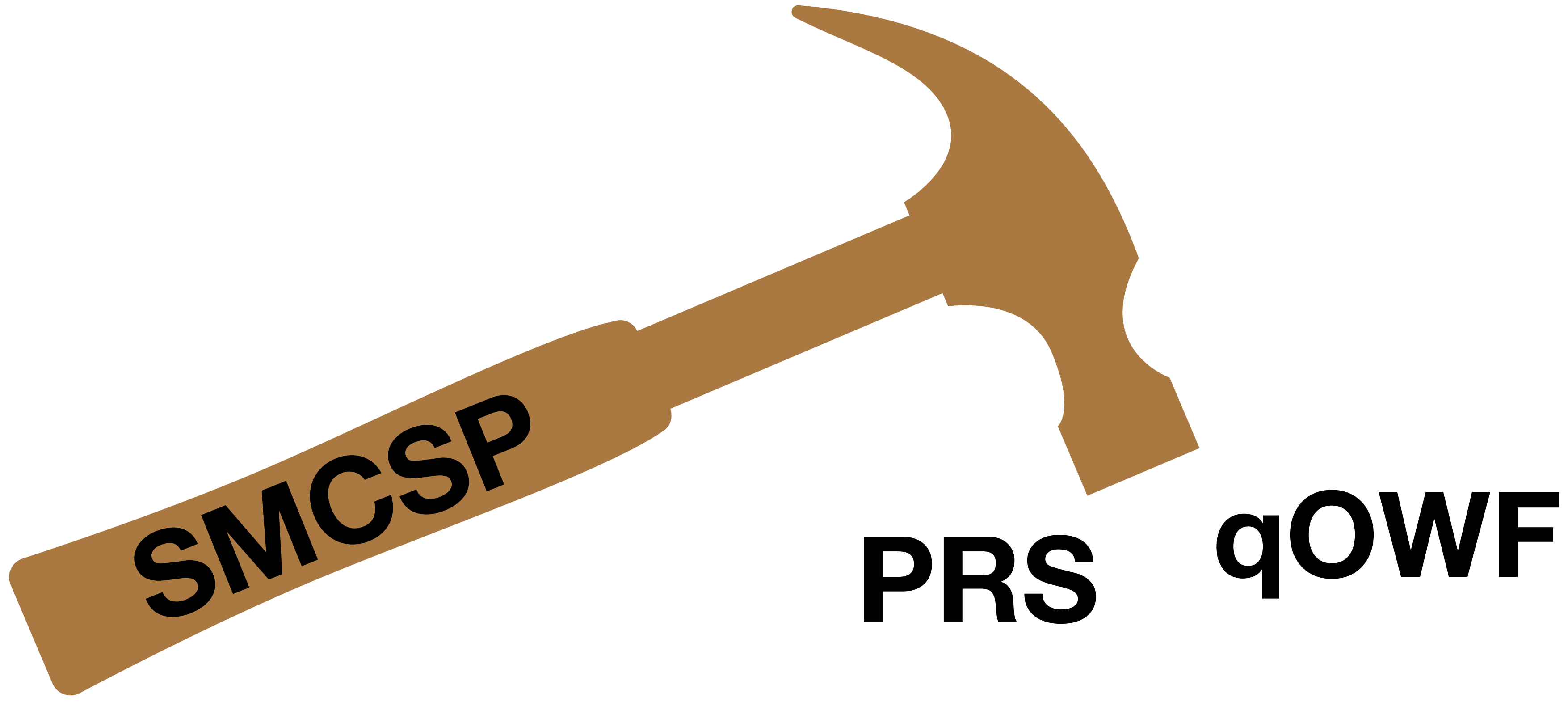

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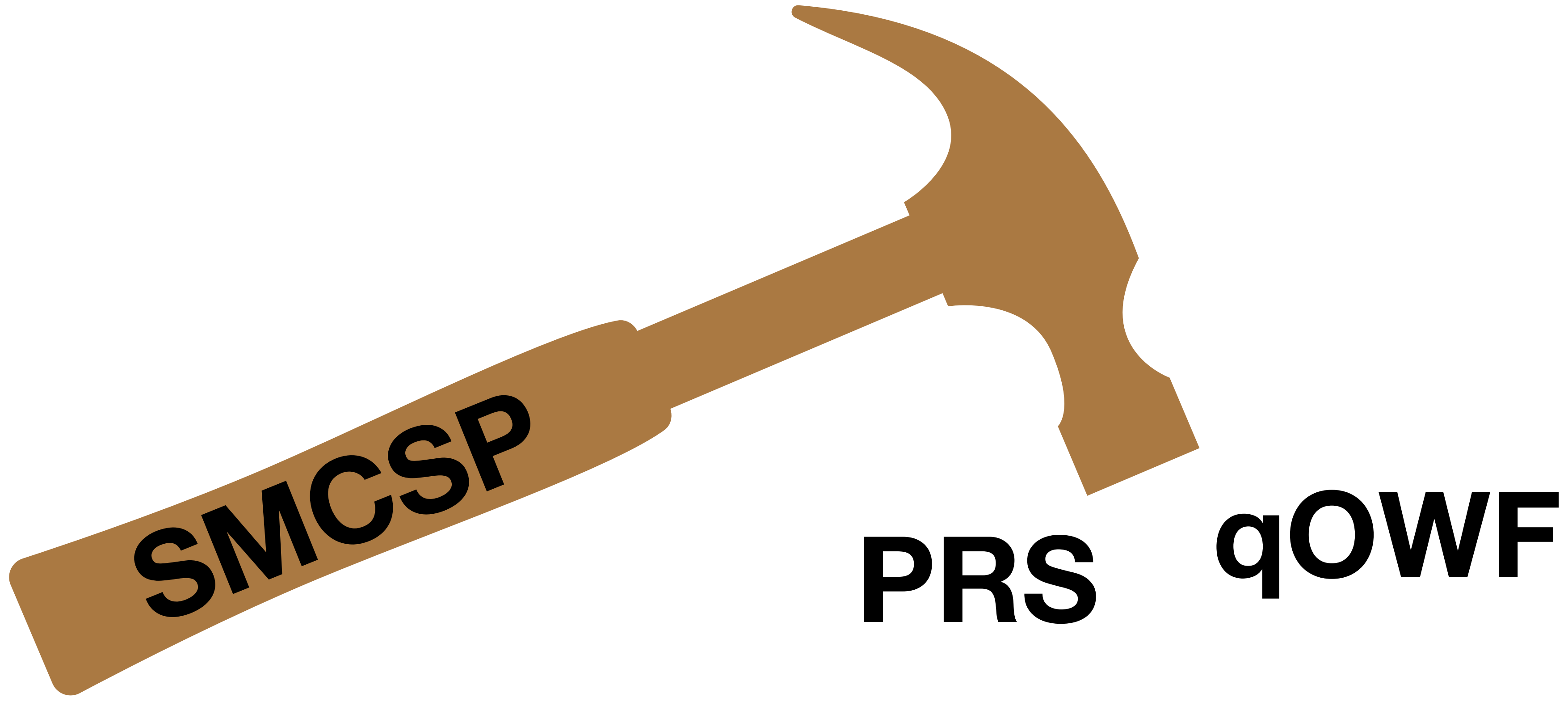

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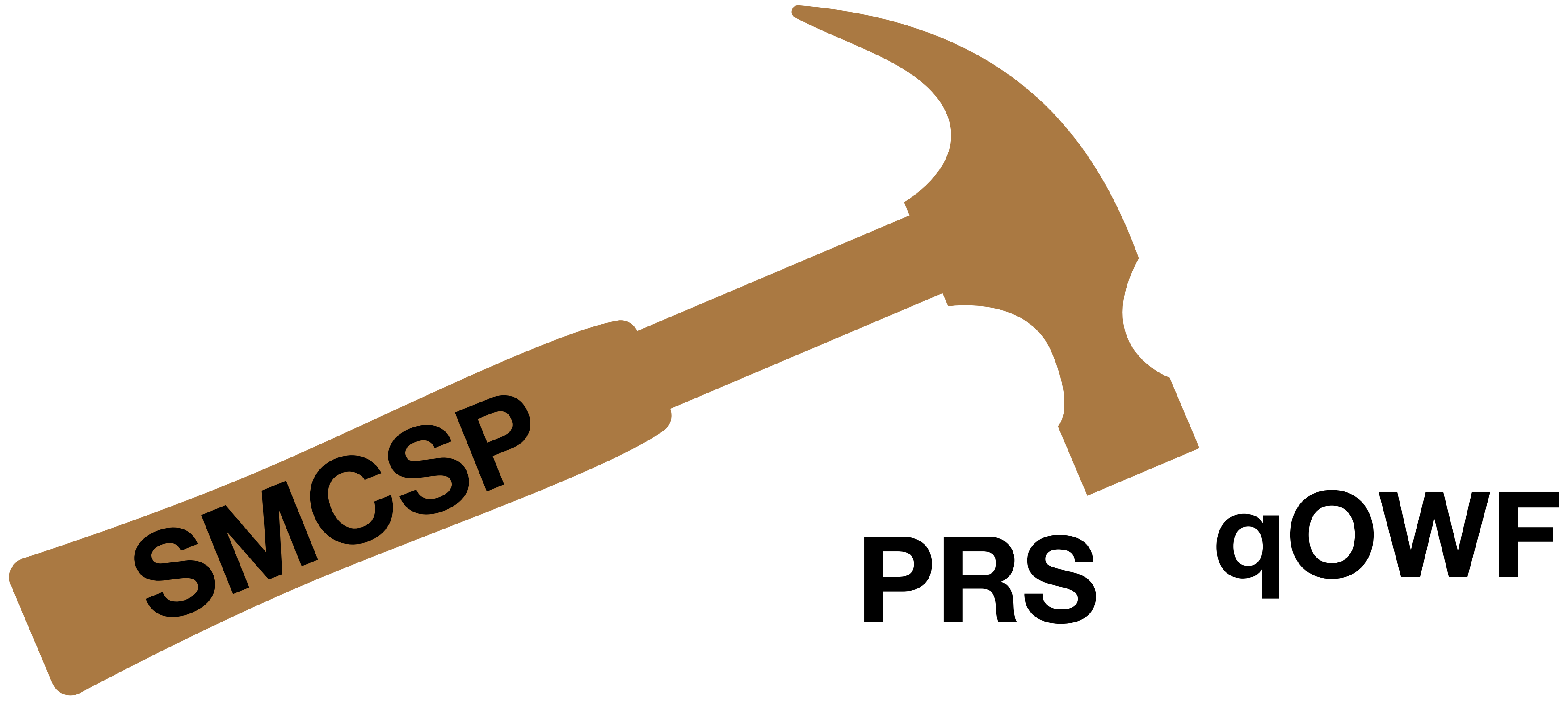

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→ **CFT-SMCSP**

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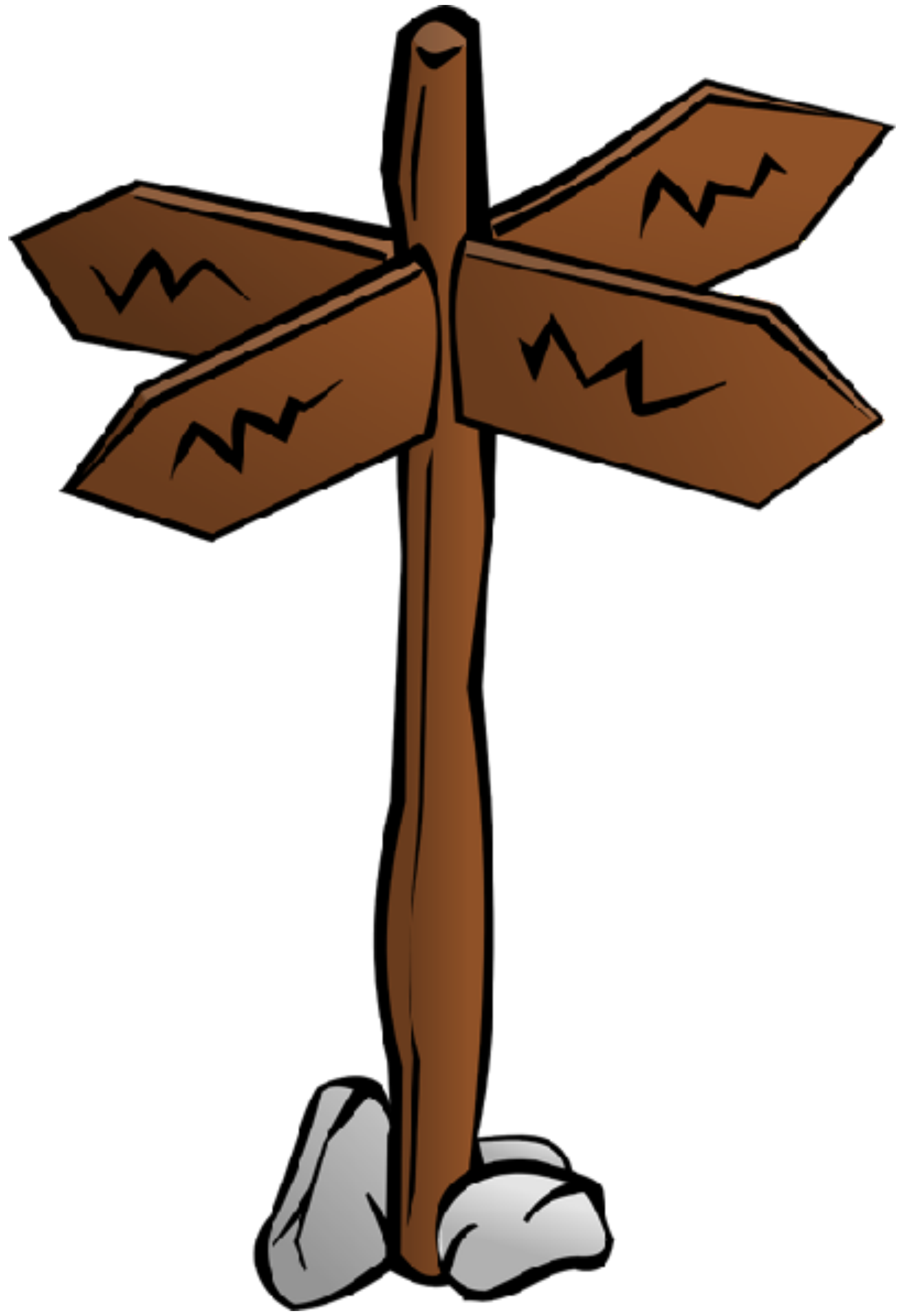
- Implications of quantum algorithms for (quantum) MCSPs.
- A quantum search-to-decision reduction for SMCSP.

Future Directions



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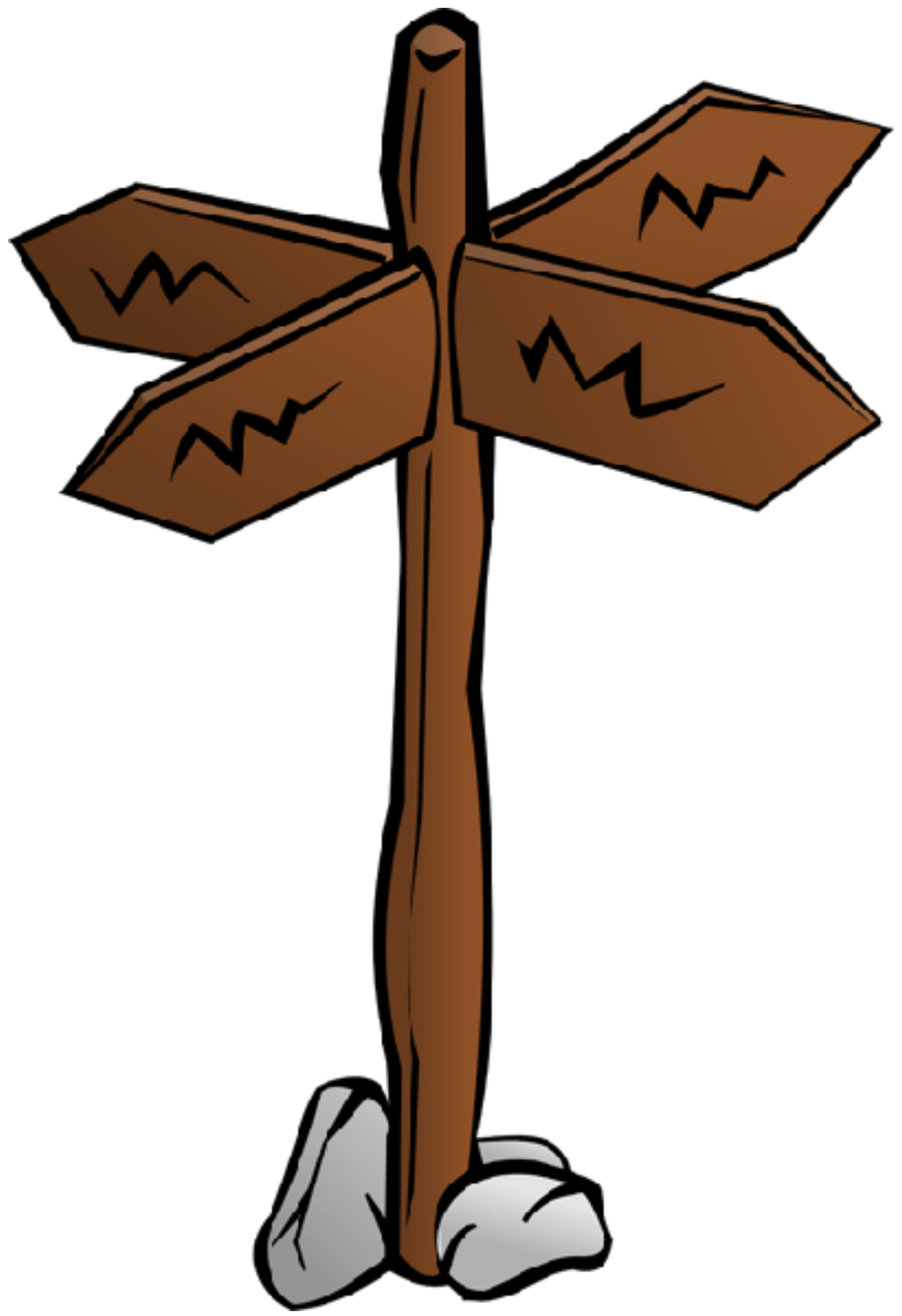

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- Is MQCSP, SMCSP, UMCSP in NP?
- It seems to be challenging to handle super-linear number of ancilla qubits.

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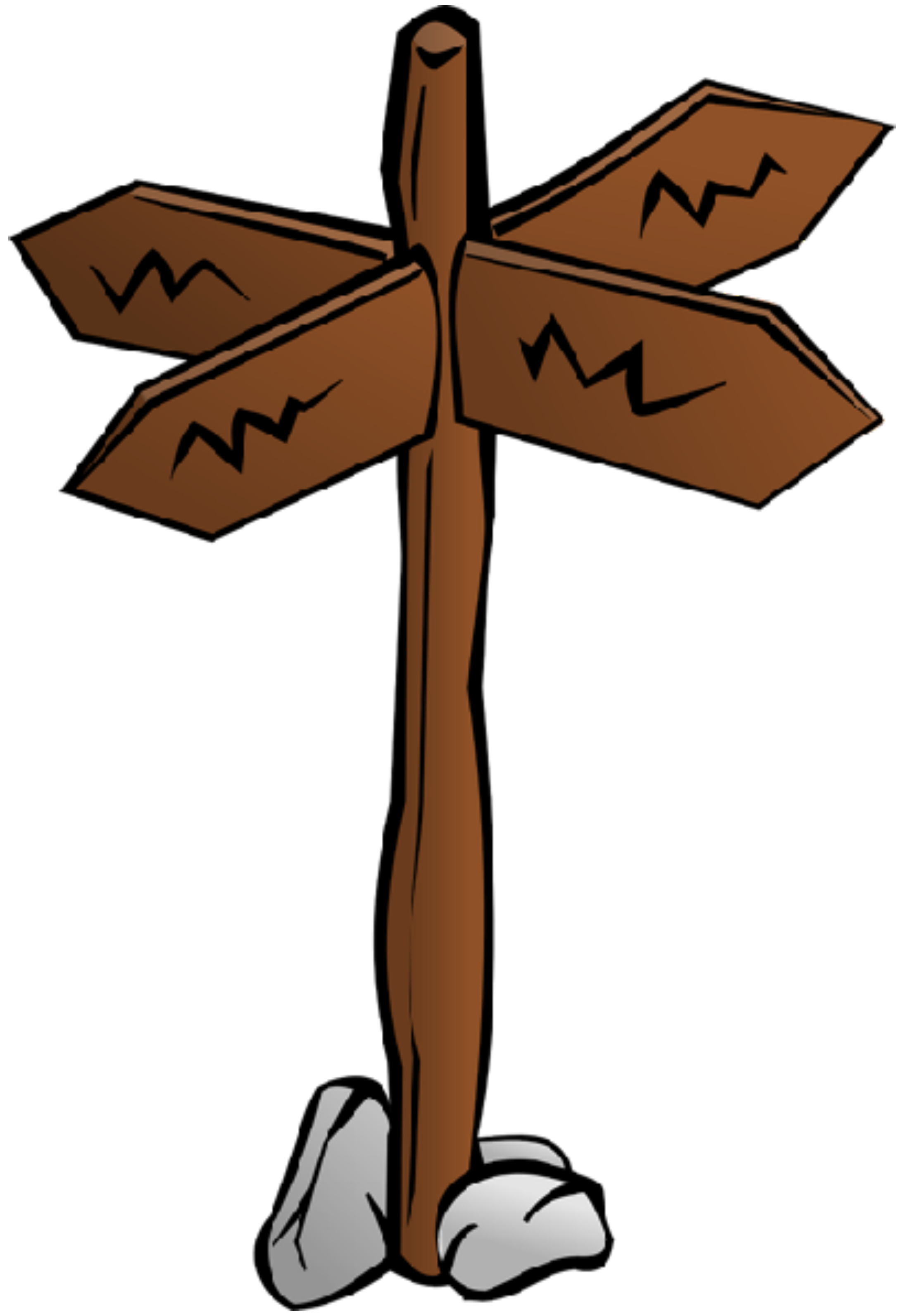

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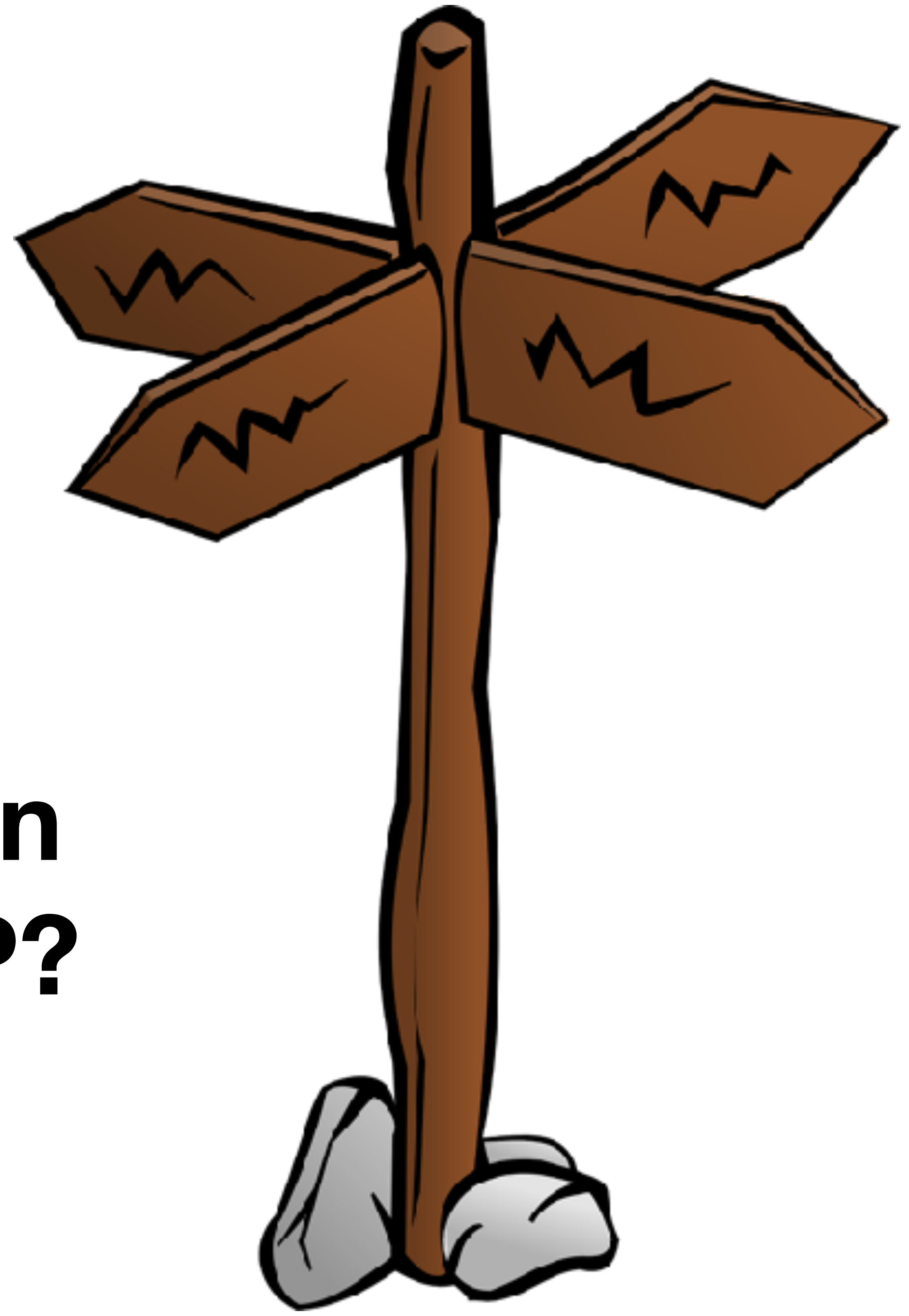

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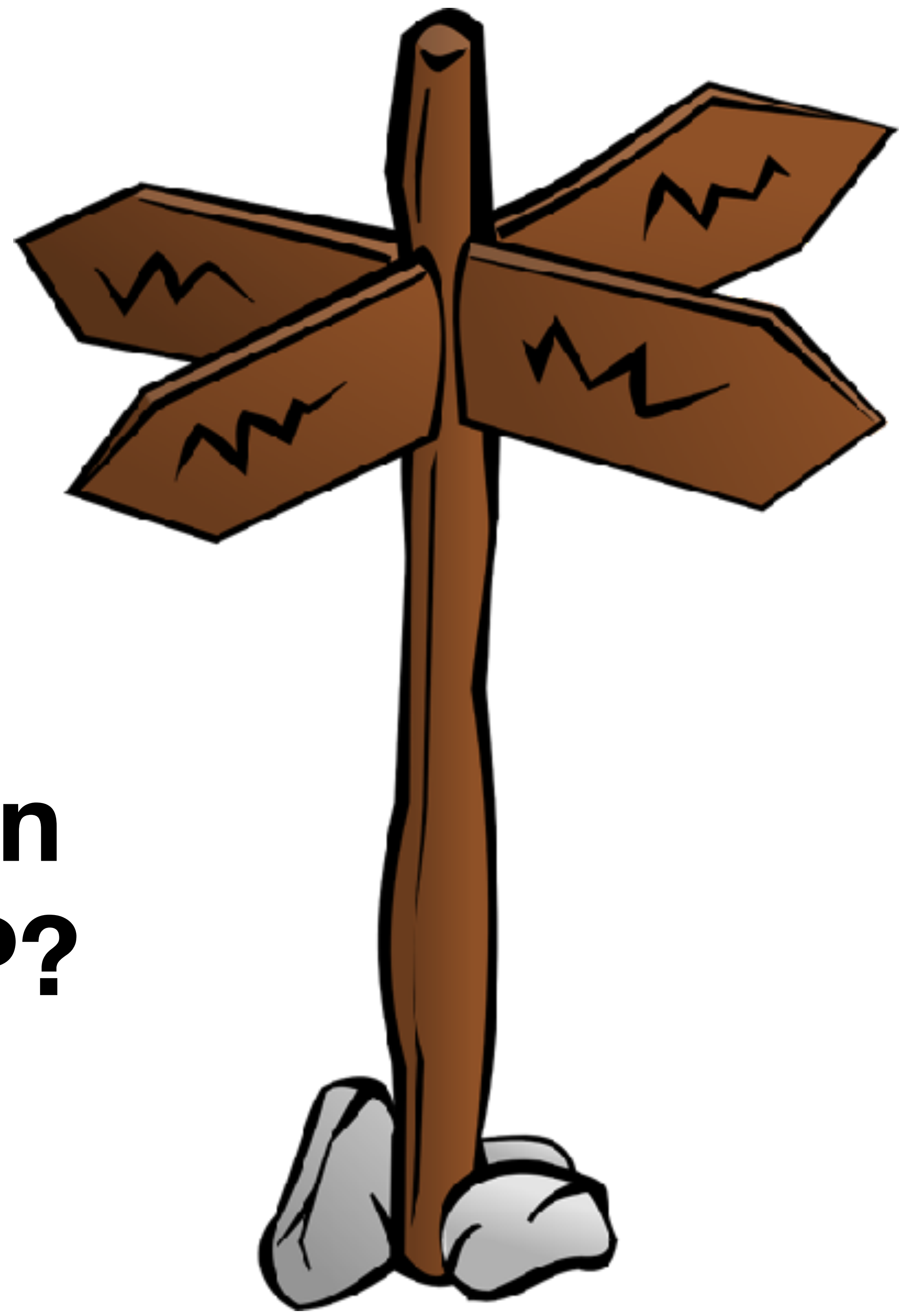

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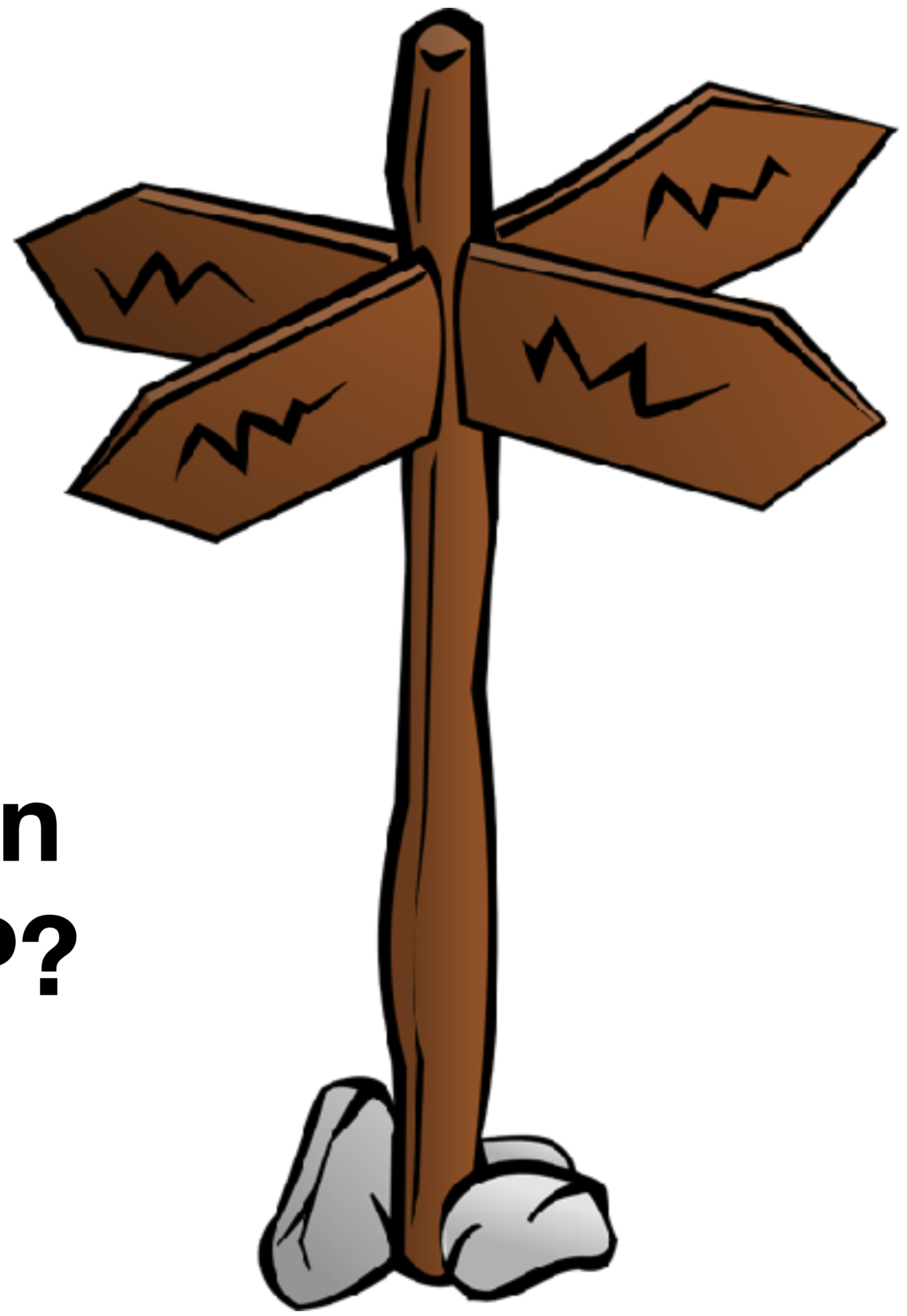

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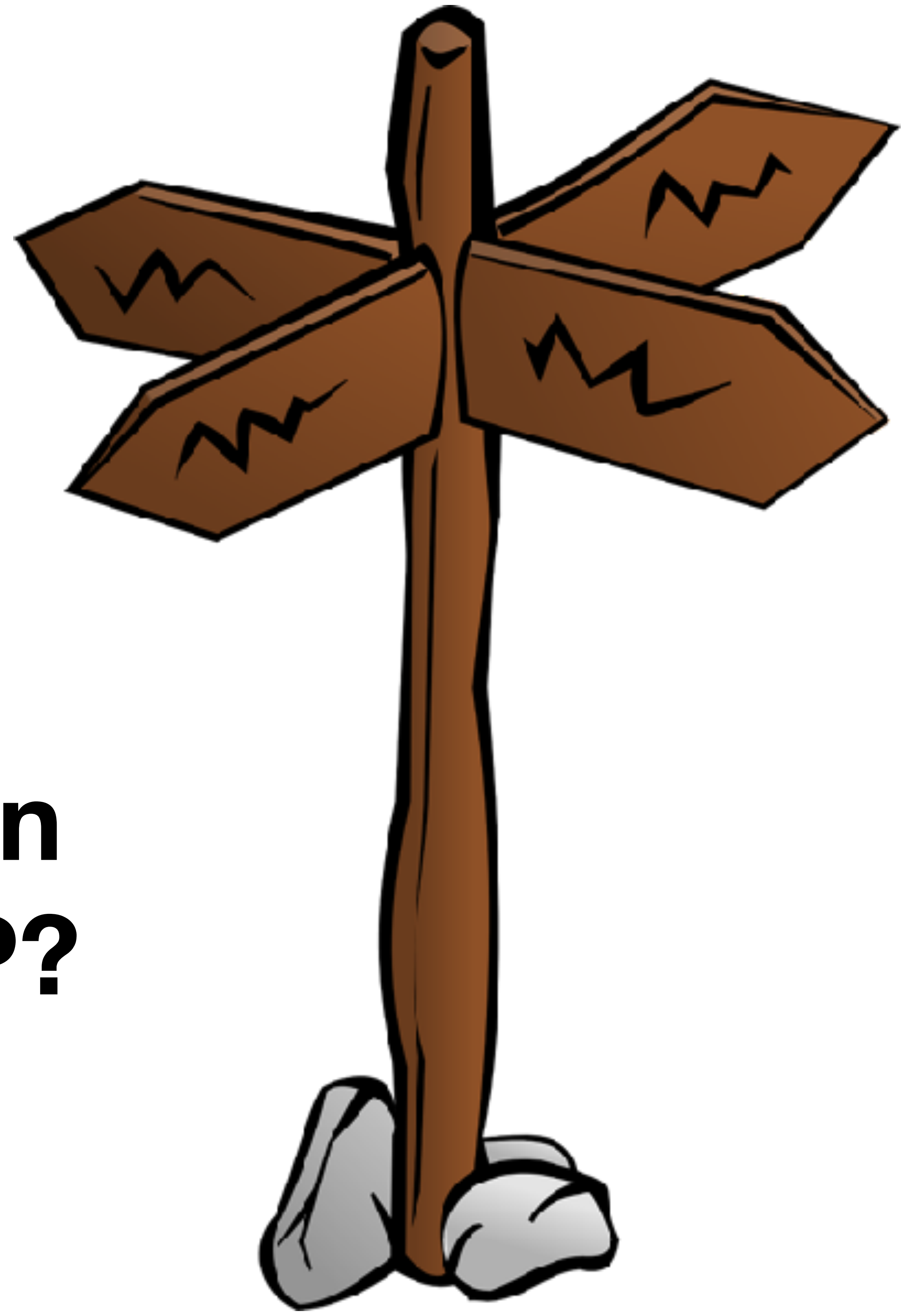

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Thanks for your attention 🙂