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Cell Division Behaviour in a Heterogeneous Swarm Environment

Adam Erskine, J. Michael Herrmann

The University of Edinburgh, School of Informatics, IPAB, 10 Crichton Street, Edinburgh EH8 9AB, U.K.
a.erskine@sms.ed.ac.uk

Abstract

We present a system of virtual particles that interact using simple kinetic rules. It is known that heterogeneous mixtures of particles are producing particularly interesting behaviours. Here we present a two-species swarm in which a behaviour emerges that resembles cell division. We show that the dividing behaviour exists across a narrow but finite band of parameters and for a wide range of population sizes. In a two dimensional environment the swarm's characteristics and dynamism manifests differently from those observable in a three dimensional environment. In further experiments we show that repeated divisions can occur if the system is extended by a biased equilibrium process to control the split of populations. We propose this repeated division behaviour provides a simple model for cell division mechanisms, which relates to discussions of the origin of life and is of interest for the formation of morphological structure and to swarm robotics.

Introduction

We investigate emergent behaviours found arising from the interactions within a heterogeneous swarm. The interactions are in the manner of that originally described by Craig Reynolds (Reynolds, 1987). He introduced a simple algorithm showing that such a swarm could manifest flocking behaviours. Each particle is influenced only by other particles in its local neighbourhood. Each update of the model represents a discrete time step. On each update every particle is drawn toward the centre of mass of its neighbours, aligns its velocity with its neighbours and is pushed away from any particles too close. Reynold's swarms were homogeneous.

Sayama (2009) extended this approach allowing multiple swarms to interact. Each swarm may have different sets of parameters. A set of parameters may be thought of as defining a species. By mixing two or more species of swarms unusual structures and dynamic behaviours have been seen (Sayama, 2010, 2012b,a). Many swarms could be identified that have a distinct biological look to them: cells, amoebas, diatoms abound. It is tempting to see the dynamics of the so-called *swarm chemistry* as a simple model for the real life counterparts of these forms.

We extend the heterogeneous swarm algorithm to include both growth and biased equilibrium mechanisms. Our explorations have found a set of species that show cell division like behaviour. Density and entropy measures allow us to make broad categorizations of behaviours. Single homogeneous swarm show limited behaviours, but more complex emergent behaviours are apparent with just two interacting species. Our investigations explore the robustness of this behaviour under parametric variation. Specifically we studied:

- How cell division is affected by the total size of the swarm and the populations of each subspecies.
- The differences in the behaviour exhibited in 2D and 3D environments.
- How cell division is affected by variation of several of each swarm's defining parameters.

Structure and form abound in and between biological organisms. Much of this comes about via self organization. One benefit of this is that its resultant emergent forms are, in some sense, available for free. Structure emerges from interactions without the need for it to be explicitly coded. An understanding of these rules and their application allow us the possibility of reusing this free structure in robotic systems. Self-organization of structures, self repair or growth without explicit command and control is beneficial. This approach may provide a model that allow us look at the automatic creation of morphological artefacts and dynamic behaviours. The tendency of many swarms to mirror biological forms, albeit superficially, raises the question of whether they can also be a model of biological processes.

Theories on the origin of life often invoke mechanisms to assure that proto-replicators are held in close association: within rock fissures; agglomeration at thermal vents; within the wind blown organic foams formed in the sea. Self-organized structures offer options for such discussions. A similar argument is made (Hutton, 2002) with reference to artificial chemistry. However this model is limited to the organizational dynamics arising from its kinetic interactions.

Single cell division and the dynamics of small multicellular groups contain the ebb and flow of chemical gradients, protein interactions and gene expressions. Whilst much is known, the precise chemo-mechanical details are still there for investigation. We propose that the dynamics of our cell division swarms may offer a simple model that allows some of these investigations. In order to allow this we require that a robust repeating cell division like mechanism be implemented. Thus we also look at modifications made to enable the observed cell division behaviour to repeat.

Background

D'arcy Thompson detailed many roles that physical processes might play in the morphological development of creatures and their artefacts (Thompson, 1917). He saw that the forms that soap bubbles took as their surface energies pulled and found equilibria bore resemblances to biological forms. He believed that this was not mere coincidence. It has been shown that this idea is indeed true — at least in part. Honeycomb, its hexagonal packing and shape of end caps, are both found in bubble foams but are not derived from a bubble formation mechanism (Ball, 2011). However the packing of the four cones in the ommatidia of a fly's compound eye may be due a mechanism of simple squeezing together like bubbles. Ball also documents work that notes that the spicule structures of sponges appears to form via a mechanism whereby a bubble array is created and then inorganic compounds are allowed to permeate the interstices of the bubble matrix. The creature is leveraging the free structure from what Ball refers to as a fossilized foam. The processes at all scales of life are complex when compared to the simple mechanisms that our model uses. And yet simple processes may shed light on the forms that life can take. Finite subdivision rules have been used to model cell division previously.

Reynolds' flocking algorithm have been subject to numerous variations, adding in: assumed fear, or leadership roles, or desire to stay close to roost sites etc. It has been shown (Feder, 2007) that in starlings it is the number of neighbours (not radius), that is important, and that the influence of neighbours was spatially anisotropic. Nearest neighbour interactions combined with an energy minimization argument has been used to generate line and vee formation flocks (Klotsman and Tal, 2011). These homogeneous swarm algorithms have been further extended by combining multiple 'species'. We should mention here again the studies of Sayama in particular on the relationship between 2D and 3D species (Sayama, 2012b). An evolutionary approach was adopted to discover interesting heterogeneous swarms (Sayama, 2010, 2012a).

Local interactions in biology have been much studied. Quorum sensing, the switching of behaviours due to local sensing, is seen in a large range of organisms from bacteria to honeybees (Miller and Bassler, 2001; Seeley et al., 2006). Various insects employ local microrules to drive artefact construction (Camazine et al., 2001). It has been shown (Schmickl and Hamann, 2011; Kengyel et al., 2009; Bodi et al., 2009) that bees, through local interactions, locate areas of a target temperature. Such biological inspirations have informed swarm robotic work. Review documents (Bayindir and Sahin, 2007; Mohan and Ponnambalam, 2009) highlight the extensive range of behaviours that may be implemented from swarm robotic interactions, including: pattern formation; aggregation; chain formation; self-assembly; coordinated movement; hole avoidance; foraging; self-deployment; grasping; pushing; caging.

Method

The basic heterogeneous swarm algorithm (Sayama, 2012b) gives each particle a set of parameters. Each particle's update of position and velocity is influenced only by its local particles within a specific neighbourhood radius. Each particle has a preferred normal speed, the maximum speed being bounded. Parameters c_1, c_2 and c_3 scale the influence of the neighbouring particles. The c_1 parameter is a measure of cohesion, the strength of pull toward the mean neighbour position. The c_2 parameter is a measure of alignment, the strength of pull toward mean neighbour velocity. The c_3 parameter is a measure of avoidance, the strength of push from close neighbours. On each update of the swarm each particle uses neighbouring particles to update its position and velocity.

$\mathcal{N}$ is the set of particles centred on particle *i* and being within particle i's neighbourhood radius. The average position of these is

$$\langle \mathbf{x} \rangle = \frac{1}{|\mathcal{N}|} \sum_{j \in \mathcal{N}} \mathbf{x}_j.$$

The average velocity of the particles within the neighbourhood radius of particle i is

$$\langle \mathbf{v} \rangle = \frac{1}{|\mathcal{N}|} \sum_{j \in \mathcal{N}} \mathbf{v}_j.$$

The acceleration of particle i is given by

$$\begin{aligned} \mathbf{a}_i &= -c_1 \left(\mathbf{x}_i - \langle \mathbf{x} \rangle\right) - c_2 \left(\mathbf{v}_i - \langle \mathbf{v} \rangle\right) \\ &\quad + c_3 \left(\sum_{j \in N} (\mathbf{x}_i - \mathbf{x}_j) / |(\mathbf{x}_i - \mathbf{x}_j)|^2 \right). \end{aligned}$$

The dynamics are further modified by the c_4 parameter which is a probability of ignoring the neighbours' effects. The particle's velocity is updated using the acceleration $\mathbf{a}_i$.

$$\mathbf{v}_i' \leftarrow \mathbf{v}_i + \mathbf{a}_i.$$

The magnitude of a particle's velocity has an upper bound. This is one of the swarm's parameters. Similarly each swarm has a parameter that is the preferred magnitude of the particles velocity. If a particle is not travelling at this preferred velocity, $\mathbf{v_n}$, then parameter c_5 is then used to nudge the velocity back to its toward its preferred velocity using

$$\mathbf{v}_i \leftarrow c_5 \left(\mathbf{v_n}/|\mathbf{v}_i'|.\mathbf{v}_i'\right) + (1 - c_5)\, \mathbf{v}_i'.$$

Finally each particle's position is updated using

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{v}_i.$$

Quantification

The eight parameters (c_1 through c_5, neighbourhood radius, speed and maximum speed) define a large parameter space. To search this space we require automated means to detect behaviours of interest.

In our swarms we can calculate the average density of particles. This density measure differentiates single blobs from both dispersed swarms and multiple blobs: single blobs show a higher density. We note that this may not always be true: a large hollow single blob may be less dense than multiple blobs that are close together. A second measure, a spatial entropy, allowed differentiation between multiple blobs and dispersed swarms. It has been suggested (Bonabeau et al., 1999) that a spatial entropy can be defined as

$$H = -\sum_k P(k) \log P(k),$$

where $P(k)$ is the fraction of particles found in patch k. H decreases as clusters form. We used patches that are always cubes of side 0.1 times the maximum extent of the swarm i.e. the minimal cube containing the swarm is split into 1000 patches. Two similar treatments are made in (Batty, 1974) and (Wolfram, 1984).

We also use the Kullback-Leibler divergence from an evenly distributed population as a measure. This is defined by

$$D_{KL} = \sum_k P(k) \log \left(\frac{P(k)}{Q(k)} \right),$$

where P is the distribution of the particle positions and Q is the distribution of an evenly dispersed swarm. Note that since Q is evenly distributed, we have simply $D_{KL} = \log\left(\frac{1}{Q(x)}\right) - H$. For the cell division like behaviour D_{KL} thus increases when the swarm has divided into separate clumps.

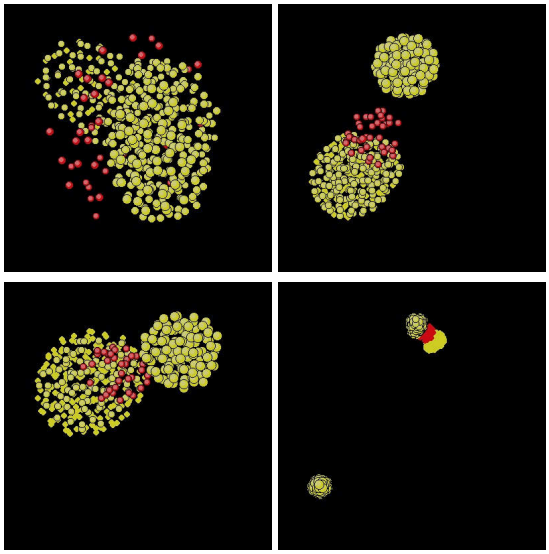


Fig. 1: Typical evolution of cell division in our swarm. Top left shows red particles as a toroid about the yellows. Top right shows the yellow swarm divided in two with a separate red swarm. Bottom left has the reds rejoining the larger yellow blob. Finally bottom right, the process repeating and is shown at a larger scale.

Results

Single species characterization

A homogeneous swarm appears to exhibit behaviours drawn from a fairly limited palette of possible behaviours. We note four behaviours: full dispersal, blob or sphere, multiple blobs, and one we call a point swarm (all particles collapse toward a single point). In full dispersal the particles separate and move apart, there is little or no tendency to aggregate. In a blob the particles form a sphere (or approximate sphere) or shell of a sphere. Multiple blobs are simply a multiple version of the last form. Point swarms are seen for swarm parameters where the avoidance value is at or near zero. This results in all particles collapsing to a single point. This state tends to not show as a sphere or a point. Instead the particles, which all exist in a tiny spatial volume, show as an irregular clump of particles that jump about. Particles have discretised speeds so at each update a particle tends towards the average position of the clump, but the step size is larger than the size of the clump, thus the particles are unable to actually occupy a single point.

We find that the four single species states can be classified by the density and spatial entropy (or Kullback-Leibler divergence). By sweeping through the parameter space of the cohesion and avoidance parameters it was possible to find regions of each of the four swarm types. The spatial entropy and densities were measured for each. A visual check of the final state of the swarm was also made. The

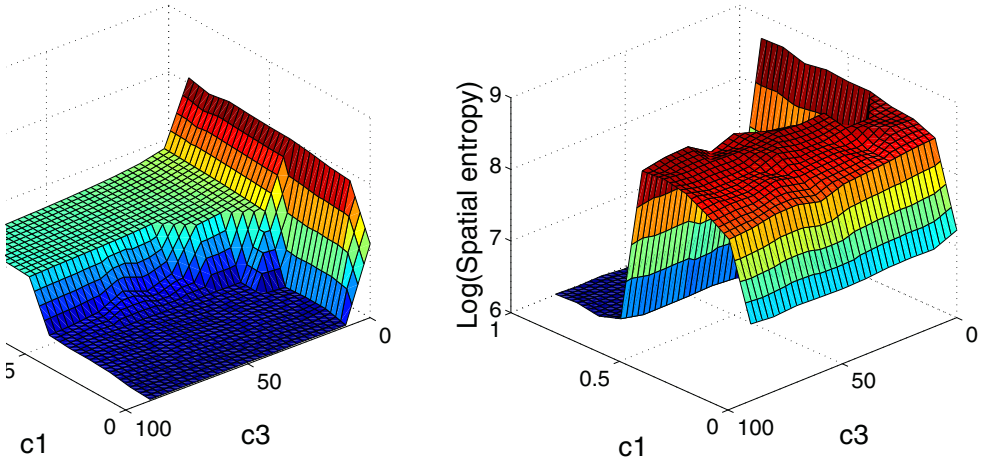


Fig. 2: Density and entropy measurements for a homogeneous swarm as a function of its cohesion (c_1) and avoidance (c_3) parameters. The logarithm of each measure is plotted in order to squash the vertical extent of the surface plots as the range of values extends over several decades.

measures were plotted against the parameters to generate the surface plots shown in Fig. 2.

Cell division behaviour species

We present two species of swarms that individually formed single blobs (multiple blobs if their populations were large enough), but in combination result in a cell division like behaviour. Typical stages of this are shown in Fig. 1. The values for the parameters used in these swarms are shown in Tab. 1.

spc	rad	spd	msp	c_1	c_2	c_3	c_4	c_5
1	20.5	1.94	20.7	1	1	18.6	0.05	1
2	300	15.58	37.08	1	0.05	9.11	0.47	0.61

Tab. 1: Parameter values for the cell division like behaviour swarms. Headings are: spc = swarm species, rad = neighbourhood radius, spd = normal speed, msp = maximum speed, c_1 = cohesion, c_2 = alignment, c_3 = avoidance, c_4 = whim, c_5 = speed control.

When displaying the particles the parameters c_1 through c_3 are used to define the displayed colour of the particles. Here, species 1 displays as a yellow colour and species 2 as a red colour. Clearly if we alter these parameter values the colours will alter. For descriptive convenience we choose to describe the two swarms as the yellow and the red swarms respectively. Typically the swarm population constructed of around ten yellow particles for every red particle. When this heterogeneous swarm is run the initially mixed species separate. A toroid of red particles forms about the yellows until a split occurs. The separate yellow blobs move apart, with the red particles forming a blob in between. At some point the reds are drawn into one of the yellow blobs and the process repeats. The repetition only occurs within a single blob of the yellows.

Comparison with 2D Swarm Chemistry

We explored the differences in 2D and 3D behaviour of our swarms. With no change to the swarm parameters cell division behaviour still occurred. Differences in the 2D version included: the red particles travel to the inside of a yellow circle of particles causing an inside out division to occur; the separated blobs do not travel apart; and the red particles do not get drawn back into one of the yellow blobs.

An outside in division was achieved via modification of both swarms' parameters, Fig. 3. As parameters have been changed, the particles no longer appear as red and yellow but as magenta and cyan respectively. Now the red particles form a ring around the yellow circle and squeeze it until division occurs. Again the separate parts do not travel apart.

It is possible that reintegration of the red particles with one of the yellow blobs would occur if the swarm was left to run. It is also possible that with further parameter modification a recipe may be found that results in the split parts separating.

Robustness under population dynamics

Yellow versus red populations. We varied the two species' populations to determine the limits on the cell division like behaviour. Each run lasted for 2000 time ticks. The density and entropy measures were captured at the end of each run. For confirmation the final state of the swarm was captured as an image. Yellow populations were varied over a range from 100 to 550 in steps of 50, and red population over the range 10 to 90 in steps of 10. Fig. 4

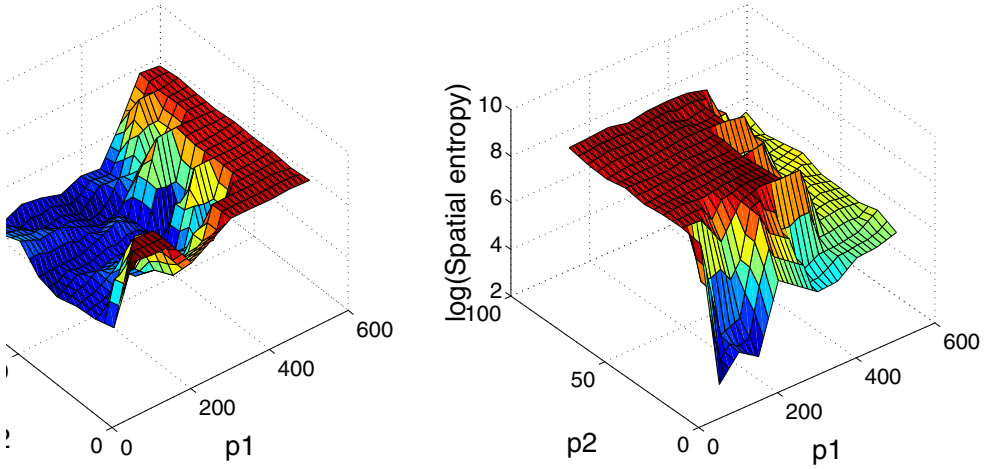


Fig. 4: Density and entropy measurements for a heterogeneous swarm as a function of its yellow and red populations (p_1 and p_2). The logarithm of each measure is plotted in order to squash the vertical extent of the surface plots as the range of values extends over several decades.

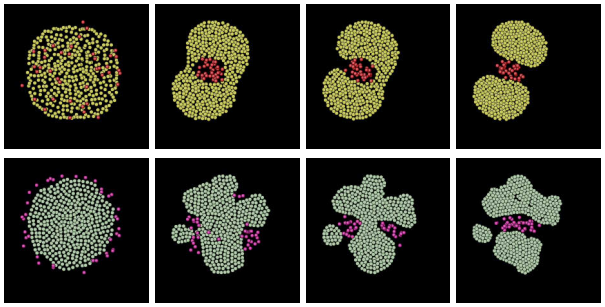


Fig. 3: 2D cell division. Upper row shows an 'inside out' division. Lower row shows and 'outside in' division.

shows the density and entropy measures as a surface plot for all combinations of these populations. Cell division is marked by low density (blue on left hand plot) and high entropy (red on right hand plot). We see that the cell division behaviour extends over a wide range of populations. Very low red or high yellow populations tend to never show cell division. The line between division and no division is noisy. We assume this is due to variability in starting position of particles and/or the arbitrary duration of each run. We explore both of these possibilities.

We fixed the red population at 50, and executed 5 runs for yellow populations varying from 300 to 600 in steps of 25. When the yellow population is below 375 division always occurred. For populations above 450 it never occurred. In the range between division may or may not occur. The difference between each run was the randomized initial positions of the particles in the swarms. The KL divergence and the density (averaged over the 5 runs) are summarized in Fig. 5. The density increases and the KL measure drops above a population of 350 coinciding with the onset of swarms that fail to divide. When division never occurs the values level off.

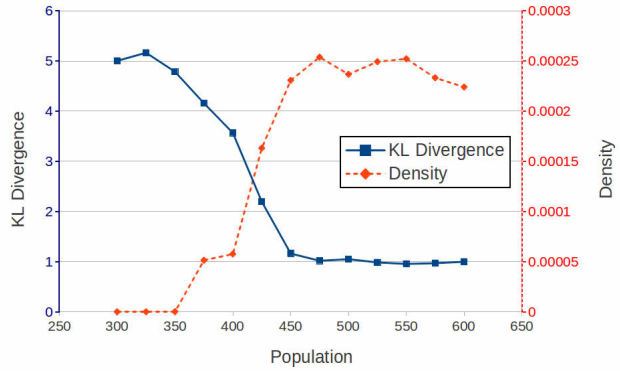


Fig. 5: Density and Kullback-Leibler divergence measures as function of yellow population after 2000 time ticks. For a fixed population of red particles (50), we vary the population of the the yellow swarm (from 300 to 600).

Effect of lengthening run time. We repeated the previous investigation but allowing the model to run now for 10000 steps. There is still no distinct population boundary between split/no split behaviour. Yellow populations less than 425 always result in division. Those greater than 475 never divide. Populations between these limits may divide. Fig. 6 confirms this observation in that the step up in density occurs at higher yellow populations. Executing the swarm for still

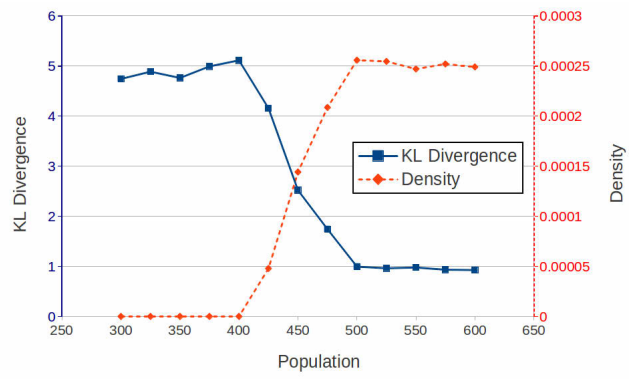


Fig. 6: Density and Kullback-Leibler divergence measures as function of yellow population after 10000 time ticks. For a fixed population of red particles (50), we vary the population of the yellow swarm (from 300 to 600).

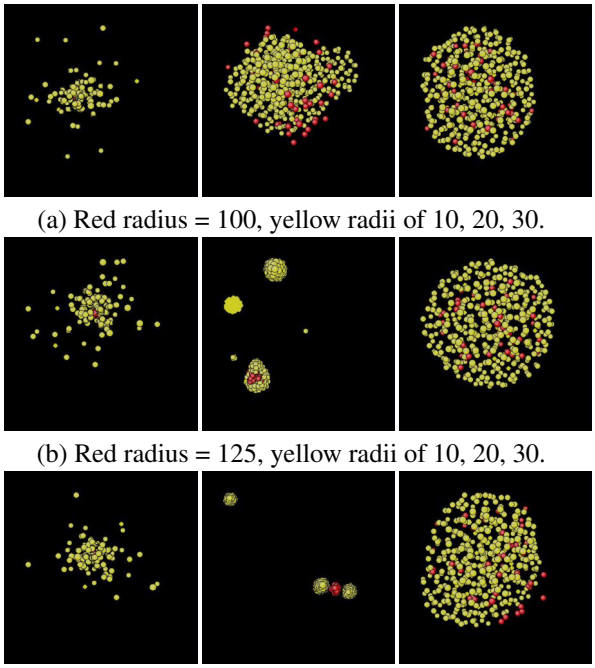
longer durations suggested that with relatively small red swarms the whole swarm may be unable to divide. However when the red population was increased (to 180) the swarm which appeared to be stable would occasionally eject a small blob of yellow particles. This suggests that such a swarm may slowly lose yellow particles until the remaining yellow blob is small enough to show the normal division behaviour.

Robustness under parameter variation

A full search of the parameter space is currently too onerous. Therefore we choose a simpler approach. We look to vary single parameters whilst keeping all other parameters unchanged. We vary the parameter being studied until the cell division behaviour disappears.

Variation of neighbourhood radius. Using a yellow:red population mix of 300:50 we varied, independently, the neighbourhood radii of each swarm. For cell division behaviour the red species was required to have a neighbourhood radius greater than 125 and for the yellow 'species' it needed to be within the range of about 13 to 25. Samples are shown in Fig. 7. Each swarm was run for 2000 time ticks. Yellow radii above 28 result either in a single cloud or have the red particles held within the yellows.

Variation of avoidance and cohesion parameters. We separately swept through combinations of avoidance and cohesion parameters. First we varied red avoidance between 5 and 40, yellow between 10 and 60. Then we varied the red and yellow cohesion values from 0.2 to 1.0. A number of different behaviours were noted. Several behaviours would not be distinguishable via the use of measurements alone, so each run was watched and categorized. A single run of each permutation was made. All runs lasted 2000 time ticks. Tab. 2 shows the results for avoidance variation and Tab. 3



(a) Red radius = 100, yellow radii of 10, 20, 30.

(b) Red radius = 125, yellow radii of 10, 20, 30.

(c) Red radius = 300, yellow radii of 10, 20, 30.

Fig. 7: Final states of each run. Examples of neighbourhood radius variation.

shows the results for cohesion variation.

Cell division behaviours exist over narrow ranges of both these parameters. Cell division behaviour of the sort we have been looking at is thus very sensitive to the values of both avoidance or cohesion parameters. As with the other parameter studies whether this is true for other population and parameter mixes is unknown. It appears that for small yellow avoidance values ($c_3 \leq 40$) the red avoidance value needs to be around half that of the yellow value for any division to occur. Given the parameter set of the swarms, it appears that larger yellow cohesion values are needed to stop the yellow swarm from disintegrating. Perhaps above this level (around 0.6) the yellow swarm requires a greater 'pull' from the reds to begin to divide. As with the other parameter studies whether this is true for other population and parameter mixes is unknown.

Repeated division

The cell division behaviour in the previous sections splits a clump of yellow particles in two. Only one of those clumps will subsequently divide again. This occurs as the red particles tend to only associate with the larger clump of yellow particles. In order for this division behaviour to be seen as a possible model for real world division we needed a mechanism that would allow any yellow clump to potentially divide. In (Sayama, 2012a) each particle is modelled as expressing one parameter set drawn from a group of parameter sets. This formulation allowed a

Avoidance values	Yellow=10	Yellow=20	Yellow=30	Yellow=40	Yellow=50	Yellow=60
Red=5	3D	2D	2D	2D	2D	Y
Red=10	0	3D	2D	2D	2D	2D
Red=15	0	0	0	3D	3D	2D
Red=20	0	0	0	0	3D	2D
Red=30	0	0	0	0	3D	Y
Red=40	0	0	0	0	3D	Y

Tab. 2: Division types as function of avoidance parameter, c_3, for a selection of the parameter variations tried. Categories are: '0' — No division seen, reds may form toroid round yellows. '3D' — Division seen, behaviour was characteristic of the standard 3D cell division. '2D' — Considered the same as 2D case. Inside out split but clumps are largely static after split. Reds may be drawn in. 'Y' — Yellows disintegrate into small clumps, reds form their own clump.

Cohesion values	Yellow=0.2	Yellow=0.4	Yellow=0.6	Yellow=0.8	Yellow=1.0
Red=0.2	Y	Y	Y	0	0
Red=0.4	Y	Y	3D	0	0
Red=0.6	Y	Y	0	0	0
Red=0.8	Y	Y	3D	0	0
Red=1.0	Y	2D	2D	3D	3D

Tab. 3: Division types as function of cohesion parameter, c_1. Categories are as per Tab. 2.

natural extension to evolutionary techniques to be applied. We choose a similar approach. Each particle expresses itself either as a red or a yellow particle. There is a small probability that any particle may change the behaviour it expresses. This is modelled as a biased equilibrium processes. Each yellow, on being chosen to pick a behaviour, will select changing to red with a 0.1 probability. Each red will select changing to yellow with a 0.9 probability. This ensures a rough 90:10 percent mix in the population, but allows any clump of yellow particles to develop a red population. This mechanism only works as a divided cell tends to move apart. If the parts remain close, either by artificial confinement or as would be the case in the 2D version, then any new reds in one clump tend to be immediately sucked into the clump with the larger red population.

This mechanism alone provides for each clump to continue to divide over time. However, as clumps do not tend to recombine the ultimate future for this approach is a dispersed swarm. We added a growth mechanism to allow clumps to increase in size. New particles would be created close to randomly chosen existing particles. This can be viewed as new particles being recruited from the environment. Fig. 8 shows some examples from a swarm that implements both the biased equilibrium and growth mechanisms. The swarm still tends to appear somewhat dispersed, however, there are still many clumps that continue to divide.

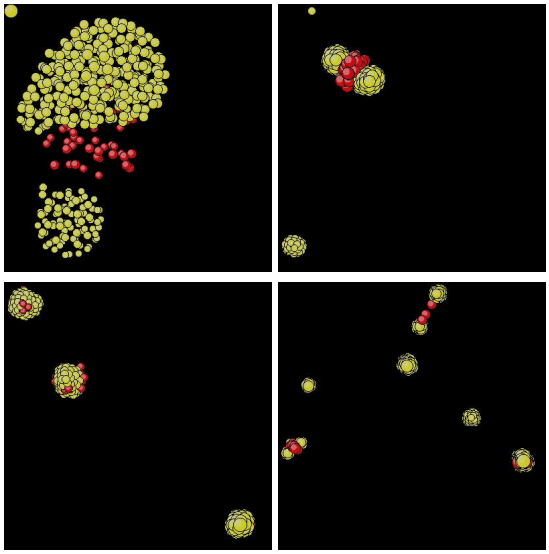


Fig. 8: Repeated division. Top left shows first division. Top right shows the second division. Bottom Left shows multiple blobs with red particles from the bias equilibrium process. Bottom right shows multiple divisions occurring.

Discussion

We presented a heterogeneous swarm that exhibits cell division like behaviour. Prior to dividing, the red particles form a toroid — but only because the yellows support it. There are configurations where this appears a long lived phenomenon. Division occurs for a wide range of swarm sizes, but there appears to be a size above which the yellow swarm tends to stability. We found some evidence that such a swarm may gradually lose yellow particles suggesting that cell division may reappear if the swarm runs for long enough. Balancing the growth and biased equilibrium can be hard and the population will tend to fragment. It would be appealing to improve the linkage between these mechanisms so that division would become more regularly periodic.

We observed differences in the emergent behaviour depending whether the swarm ran in a 2D or 3D environment. If the parameter values used in a 3D environment were used, unchanged, in a 2D environment

then we observed an 'inside out' division. This resulted in a relatively static set of divided clumps. By modifying the parameter values used we were able to recapture the 'outside in' division seen in 3D. This still failed to show the full dynamics seen in 3D. However, the fact that there are parameter mixes that show behaviour in 3D that matches that seen in 2D suggest the opposite may also be true.

The cell division behaviour was sensitive to the swarms' parameter recipes. Yellow neighbourhood radius needs to be in a narrow band. The red neighbourhood radius appears to have a lower limit, while much larger values seem to result in division behaviour. Cell division behaviour is seen only across a narrow band of both avoidance and cohesion parameters. On one side of the band no division is observed. On the other side either an 'inside out' division similar to that seen in 2D, or a spontaneous yellow disintegration that requires no interaction with the red particles, is observed.

The inclusion of a biased population equilibrium and growth mechanisms enabled the swarm to show ongoing cell division like behaviours.

Acknowledgements

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