An interactive evolutionary multiobjective optimization method based on the WASF-GA algorithm

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Solving a Multiobjective Optimization (MOP) problem

Two points of view:

- ✓ Multiple Criteria Decision Making (MCDM): helping the decision maker (DM) to find his/her most preferred solution.
- ✓ Evolutionary Multiobjective Optimization (EMO): generating a set of well-distributed Pareto optimal solutions approximating the whole (unknown) Pareto front.

The Weighting Achievement Scalarizing Function Genetic Algorithm (WASF-GA)

For a multiobjective optimization problem:

$$\begin{array}{ll} \text{minimize} & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\} \\ \text{subject to} & \mathbf{x} \in S. \end{array}$$

the DM gives a reference point $\mathbf{q} = (q_1, \ldots, q_k)$.

Where are the probably most interesting nondominated solutions for this $\mathbf{q}$?

$\Longrightarrow$ Region of interest of the Pareto front from $\mathbf{q}$.

How can we generate these nondominated solutions?

$\Longrightarrow$ WASF-GA is based on:

- An achievement scalarizing function (ASF).
- The classification of the individuals into several fronts at each generation.

Ruiz, A.B., Saborido, R., Luque, M. (2014). A Preference-based Evolutionary Algorithm for Multiobjective Optimization: The Weighting Achievement Scalarizing Function Genetic Algorithm, Journal of Global Optimization, in press.

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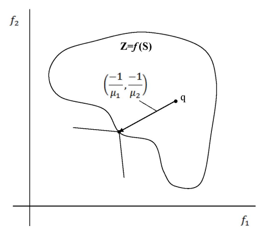
Wierzbicki's achievement scalarizing function

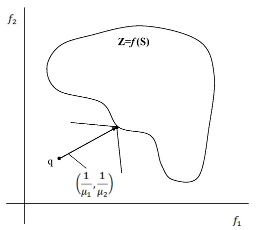
General Formulation

$$s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \mu) = \max_{i=1,\dots,k} \{ \mu_i (f_i(\mathbf{x}) - q_i) \} + \rho \sum_{i=1}^{k} (f_i(\mathbf{x}) - q_i),$$

where $\mu = (\mu_1, \dots, \mu_k)$ is a vector of positive weights ($\mu_i \in (0,1)$ for every $i = 1, \dots, k$) and $\rho > 0$ is the so-called augmentation coefficient.



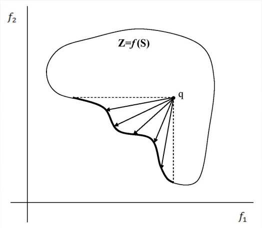
Achievable reference point



Unachievable reference point

Wierzbicki's achievement scalarizing function

Any Pareto optimal solution in the Region of interest from q can be obtained by minimizing $s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \mu)$ over S and varying μ in the weight vector space $(0,1) \times \ldots^{k)} \times (0,1)$.



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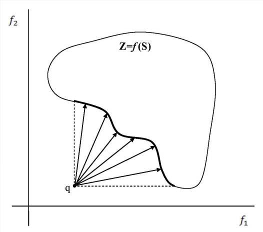

Unachievable reference point

Classification of the individuals into several fronts

Let W be a set of N_μ vectors of weights as evenly distributed as possible in $(0,1) \times \ldots^{k)} \times (0,1)$:

$$W = \{\mu^j = (\mu_1^j, \ldots, \mu_k^j), \mu_i^j \in (0,1) \text{ for every } i = 1, \ldots, k, j = 1, \ldots, N_\mu\}$$

- Problems with 2 objectives $\Rightarrow$ Generating N_μ evenly distributed weight vectors is easy.
- Problems with $k \geq 3$ objectives $\Rightarrow$ We will generate a sample of N_μ weight vectors which represent $(0,1) \times \ldots^{k)} \times (0,1)$ as evenly as possible.

The classification of the individuals into the different fronts is done according to the values that every individual takes on the ASF for the N_μ weight vectors in W.

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Properties

- At each generation, solutions with the best values of the ASF in the N_μ weight vectors and with $\mathbf{q}$ as reference point are emphasized.
- Each front is formed by N_μ solutions ($N_\mu \leq N$).
- Nondominated solutions are preferred over dominated ones:

  If $\mathbf{x}$ dominates $\bar{\mathbf{x}} \Rightarrow s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \mu) < s(\mathbf{q}, \mathbf{f}(\bar{\mathbf{x}}), \mu)$, for every weight vector $\mu \Rightarrow \mathbf{x}$ belongs to a lower level front than $\bar{\mathbf{x}}$.
- Output: N_μ solutions (first front of the last generation), which approximate the region of interest from $\mathbf{q}$.

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Motivation

- An interactive method can very useful to solve a multiobjective optimization problem.
- There are many interactive MCDM methods but however only few interactive EMO algorithms in the literature.
- Many multiobjective optimization problems cannot solve by means of MCDM techniques.
- An interactive method based on EMO algorithms is able to solve many kinds of multiobjective optimization problems.
- The WASF-GA's features allow us to build an interactive method in an easy way.

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Main ideas

- It is based on the WASF-GA algorithm.
- Given a reference point $\mathbf{q}$, the DM decides how many solutions (N_S) wants to obtain and to see for these reference values. For default, N_S can be equal to $2k$.
- N_S weight vectors are generated, which are dispersed between them and evenly distributed as much as possible.
- N_S nondominated solutions are generated in the region of interest by the WASF-GA algorithm.
- At each iteration, Interactive WASF-GA can be very fast since that only few weight vectors are considered (N_S).
- The final population of one iteration is used as initial population in the following one.

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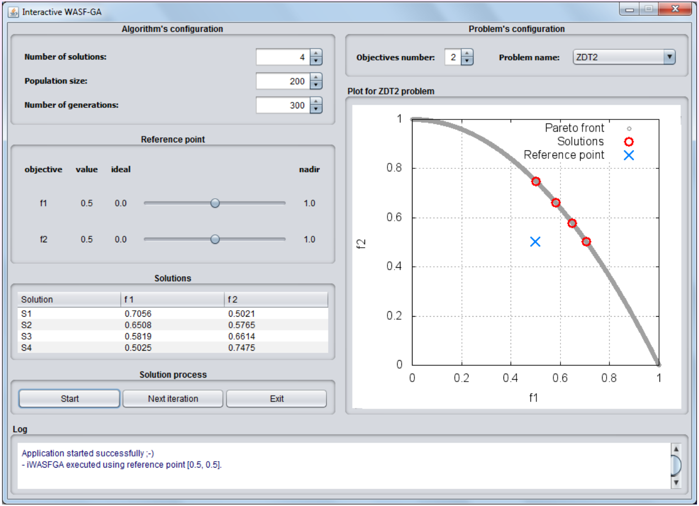
Interactive WASF-GA
Algorithm's configuration
Number of solutions: 4
Population size: 200
Number of generations: 300
Reference point
objective value ideal nadir
f1 0.5 0.0 1.0
f2 0.5 0.0 1.0
Solutions
Solution f1 f2
S1 0.7056 0.5021
S2 0.6508 0.5765
S3 0.5819 0.6614
S4 0.5025 0.7475
Solution process
Start
Next iteration
Exit
Problem's configuration
Objectives number: 2
Problem name: ZDT2
Plot for ZDT2 problem
Pareto front
Solutions
Reference point
f1
f2
Log
Application started successfully ;-)
- iWASFGA executed using reference point [0.5, 0.5].

Conclusions

- A new interactive EMO algorithm is proposed here.
- It is based on the WASF-GA algorithm.
- Given some reference levels by the DM, several nondominated solutions are generated in a region of interest.
- A number of weight vectors equals to the number of solutions to be shown to the DM must be considered.
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THANK YOU VERY MUCH FOR YOUR ATTENTION!