Stabilization of a Pan-Tilt System Using a Polytopic Quasi-LPV Model and LQR Control

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***Abstract*—Linear parameter varying (LPV) models are widely used in control applications of the nonlinear MIMO dynamic systems. LPV models depend on the time varying parameters. This paper develops a polytopic quasi-LPV model for a nonlinear pan-tilt robotic system. A Linear Quadratic Regulator (LQR) that utilizes Linear Matrix Inequalities (LMIs) with well tuned weighting matrices is synthesized based on the developed LPV model. The number of time varying parameters in the developed polytopic LPV model is 4 so the number of vertices becomes 16. The desired controller is generated by the interpolation of LMIs at each vertex. The performance of the optimal LQR controller is evaluated by using the designed feedback gain matrix to stabilize the nonlinear pan-tilt system. Simulations performed on the nonlinear model of the pan-tilt system demonstrate success of the proposed LPV control approach.**

I. Introduction

Linear parameter varying (LPV) models are linear state space systems whose matrices depend on a time varying external parameter vector [1]. The entries of the parameter vector are the scheduling variables that represent the varying operating conditions of the system. LPV models are called as quasi-LPV when the scheduling variables contain the measurable system inputs, outputs or states instead of only exogenous signals.

Linear time-invariant (LTI) models are not sufficient when the nonlinear robotic systems are used in large workspaces [2]. Shamma and Athans [3] first developed LPV models for gain-scheduled controllers. Since then LPV models have attracted more researchers.

In literature, different LPV modelling approaches exist [4]-[5]. Jacobian linearization [6] is the simplest approach to obtain LPV models. This method is based on the first order linear approximations with respect to a set of equilibrium points. State transformation [7] is also a popular technique to derive a LPV model. The goal is to eliminate all nonlinear terms in the scheduling parameters. This method performs a coordinate change in the nonlinear equations of the system and provides quasi-LPV model of the system.

Marcos and Balas [8] developed a novel approach for the derivation of quasi-LPV models. This approach is called as function substitution because it is based on the substitution of a decomposition function by (scheduling parameter-dependent) functions linear in the scheduling vector. The decomposition function is the combination of all the terms of the nonlinear system that are not affine with respect to the nonscheduling states and control inputs. These terms are not function of the scheduling vector alone.

Today, well-known linear optimal controllers [9] are applied to nonlinear systems represented by LPV models. Therefore, the key feature of LPV models is to provide the use of linear optimal control methods to nonlinear MIMO dynamic systems. LPV models can be used to synthesize linear optimal robust controllers such as the linear quadratic regulator (LQR). This controller deals with the optimization of a cost function or performance index [10]. The states and the control inputs are weighted based on their importance to seek for appropriate transient and steady state behaviours. The LQR controller has been generally derived by solving an algebraic Riccati equation. When a set of Lyapunov inequalities is solved, it is difficult to find a common Lyapunov matrix analytically. This can be solved numerically by convex programming algorithms involving LMIs [11]. While the algebraic solution can only be applied to one plant, the numerical procedure can take into account multiple plants. Thus, the LQR deals with uncertain systems at different operation points.

Different linear optimal control strategies have been also synthesized on LPV models. Namerikawa [12] et al. and Apkarian [13] et al. developed H_∞ control of a robot manipulator using LPV models. Wu and Packard [14] also developed an LQG control design based on LPV plants with use of a quadratic integral cost function for the performance objective. Yu et al. [15] combined the gain scheduling theory with H_∞ controller for the LPV model of the robotic manipulator.

Many researchers synthesize the LPV controller for the stabilization purposes. Seghal and Tiwari [16] designed the LQR controller to maintain the triple inverted pendulum on a cart around its unstable equilibrium position using single control input. Similarly, Kumar and Jerome [17] described the method for stabilizing and trajectory tracking of Self Erecting Single Inverted Pendulum (SESIP) using the LQR. Castiello et al. presented a stabilization nonlinear control algorithm for a mini rotorcraft with four rotors and compared the results with LQR controller [18].

In this paper, a polytopic quasi-LPV model of the pan-tilt system with 4 dimensional time varying parameter vector is derived. The advantage of the developed LPV model is to allow linear optimal controllers to be used on the nonlinear pan-tilt system. The developed LPV model is used to synthesize an LQR controller. A robust optimization toolbox, YALMIP [19] is utilized for the controller synthesis. Since the parameter vector is designed as 4 dimensional, the desired LQR controller is synthesized by interpolating LMIs at $2^4 = 16$ vertices. The designed controller is employed for the stabilization of the non-linear pan-tilt system.

The remainder of this paper is organized as follows: Section II presents the nonlinear model of the pan-tilt system. In Section III, a polytopic quasi-LPV model is derived for the pan-tilt system. Section IV implements the LQR controller on the developed LPV model. Section V presents the simulation results of the LQR controller. Finally, Section VI concludes the paper with some remarks.

II. Nonlinear Modeling of the Pan-Tilt Platform

The 2 DOF pan-tilt platform which is given in Figure 1 is considered in this study. The nonlinear model of the pan-tilt system based on the Euler-Lagrange formulation is as follows [20]:

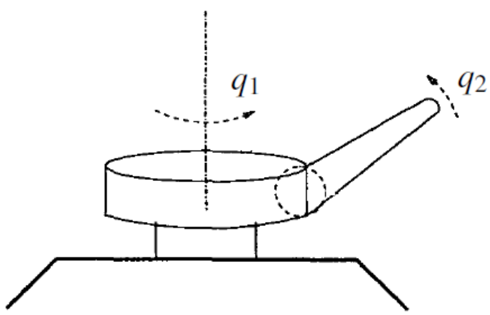


Fig. 1. Pan-tilt mechanism [20]

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{1}$$

where

$$q = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T, \quad \tau = \begin{bmatrix} \tau_1 & \tau_2 \end{bmatrix}^T$$

$$D(q) = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, \quad C(q,\dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$G(q) = \begin{bmatrix} 0 & 0.5 g m_2 l_2 \cos(q_2) \end{bmatrix}^T$$

$$D_{11} = \frac{1}{2} m_1 l_1^2 + m_2 l_1^2 + m_2 l_1 l_2 \cos(q_2) + \frac{1}{3} m_2 l_2^2 \cos^2(q_2)$$

$$D_{22} = \frac{1}{3} m_2 l_2^2, \quad D_{12} = D_{21} = 0$$

$$C_{11} = -m_2 l_1 l_2 \dot{q}_2 \sin(q_2), \quad C_{12} = -\frac{1}{3} m_2 l_2^2 \dot{q}_1 \sin(2q_2)$$

$$C_{21} = \dot{q}_1 \left[\frac{1}{2} m_2 l_1 l_2 \sin(q_2) + \frac{1}{6} m_2 l_2^2 \sin(2q_2) \right], \quad C_{22} = 0 \tag{2}$$

where $D(q)$ is the mass-inertia matrix, $C(q,\dot{q})\dot{q}$ defines centrifugal and Coriolis terms, $G(q)$ is the gravity vector, τ is the control input vector, and q, $\dot{q}$ and $\ddot{q}$ are the vectors of joint angles, velocities and accelerations, respectively. m_1 and m_2 are the masses of pan and tilt mechanisms, l_1 is the radius, l_2 is the length. In the light of (2), (1) can be rewritten as:

$$\tau_1 = \left[a + bcos(q_2) + ccos^2(q_2) \right] \ddot{q}_1 - \left[bsin(q_2) + csin(2q_2) \right] \dot{q}_1 \dot{q}_2$$

$$\tau_2 = c\ddot{q}_2 + \left[bsin(q_2) + csin(2q_2) \right] \frac{\dot{q}_1^2}{2} + dcos(q_2) \tag{3}$$

where a, b, c and d represent dynamic and kinematic parameters:

$$a = \frac{1}{2} m_1 l_1^2 + m_2 l_1^2, \quad b = m_2 l_1 l_2$$
$$c = \frac{1}{3} m_2 l_2^2, \quad d = \frac{1}{2} m_2 g l_2 \tag{4}$$

III. Derivation of the Quasi-LPV Model

Consider a LPV model in the state-space form

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t)$$
$$y(t) = C(\theta(t))x(t) + D(\theta(t))u(t) \tag{5}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$. The mappings $A(.)$, $B(.)$, $C(.)$ and $D(.)$ are continuous functions of the time-varying parameter vector $\theta(t) \in \mathbb{R}^l$. This model can be represented as a linear input-output map:

$$P(\theta) = \begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} \tag{6}$$

The parameter vector $\theta(t)$ depends on measurable quantities as follows:

$$\theta(t) = f(\upsilon(t)) \tag{7}$$

where $\upsilon(t) \in \mathbb{R}^k$ represents *scheduling signals* and $f : \mathbb{R}^k \to \mathbb{R}^l$ is a continuous mapping. A compact set can be defined as $\mathscr{P}_\theta \subset R^l : \theta \in \mathscr{P}_\theta, \forall t > 0$ [21]. If it is assumed to be a polytope, then $\mathscr{P}_\theta$ can be represented as the convex hull,

$$\mathscr{P}_\theta := Co\{\theta_{\upsilon_1}, \theta_{\upsilon_2}, ..., \theta_{\upsilon_L}\} \tag{8}$$

where $L = 2^l$ are the total number of vertices. If the state space model depends affinely on the parameters, then the LPV model is called as *parameter-affine*. Thus, $P(\theta)$ in (6) becomes:

$$P(\theta) = \sum_{i=0}^{l} \theta_i P_i = P_0 + \theta_1 P_1 + ... + \theta_l P_l \tag{9}$$

LPV system is called as a *polytopic* model as depicted in (10) if the system can be represented by a linear combination of LTI models at the vertices. This can be achieved by when (9) holds and θ can be expressed as a convex combination of L vertices θ_{υ_i}.

$$P(\theta) = Co\{P(\theta_{\upsilon_1}), P(\theta_{\upsilon_2}), ..., P(\theta_{\upsilon_L})\} = \sum_{i=1}^{L} \alpha_i P(\theta_{\upsilon_i}) \tag{10}$$

where $\sum_{i=1}^{L} \alpha_i = 1$, and $\alpha_i \geq 0$ are the convex coordinates. To obtain the quasi-LPV model of the pan-tilt system, $\upsilon(t)$ is selected as the state vector of the system:

$$\upsilon(t) = x(t) = \begin{bmatrix} q_1 & q_2 & \dot{q}_1 & \dot{q}_2 \end{bmatrix}^T \tag{11}$$

where q and $\dot{q}$ represent the joint angles and velocities. We derive the polytopic quasi-LPV model of the pan-tilt system (1) by employing the ideas in [15]. From (3) and (4), $\ddot{q}_1$ and $\ddot{q}_2$ are calculated as:

$$\ddot{q}_1 = \frac{\tau_1 + [bsin(q_2)\dot{q}_1 + csin(2q_2)\dot{q}_1]\dot{q}_2}{a + bcos(q_2) + ccos^2(q_2)} \tag{12}$$

$$\ddot{q}_2 = \frac{\tau_2 - [bsin(q_2) + csin(2q_2)]\frac{\dot{q}_1^2}{2} - dcos(q_2)}{c} \tag{13}$$

If we let $h = a + bcos(q_2) + ccos^2(q_2)$, then the system matrices which depend on the time varying parameters are computed as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \theta_1 \\ 0 & \theta_2 & \theta_3 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \theta_4 & 0 \\ 0 & \frac{1}{c} \end{bmatrix}$$

$$C = \begin{bmatrix} I_{2\times 2} & 0_{2\times 2} \end{bmatrix} \quad D = 0_{2\times 2} \tag{14}$$

where

$$\theta_1 = \frac{(bsin(q_2) + csin(2q_2))\dot{q}_1}{h}$$

$$\theta_2 = -\frac{d}{c}\frac{cos(q_2)}{q_2}$$

$$\theta_3 = -\frac{1}{2c}(bsin(q_2) + csin(2q_2))\dot{q}_1$$

$$\theta_4 = \frac{1}{h} \tag{15}$$

where the parameter vector is $\theta(t) \in R^4$, I and 0 are the identity and zero matrices. $u(t)$ implies the controlled input torques and $y(t)$ is the vector of joint positions. Therefore, $n = 4$, $n_u = n_y = 2$.

IV. LQR Synthesis based on the Developed LPV Model

The goal is to stabilize the nonlinear pan-tilt system (1) by using Linear Quadratic Regulator (LQR) on the proposed quasi-LPV model (14)-(15) as shown in Figure 2.

We concentrate on LMI formulation of the LQR problem [22]. This method seeks to find an optimal controller that minimizes a cost function:

$$J = \int \left(x^T Q x + u^T R u\right) dt \tag{16}$$

where the cost function is parameterized by $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{n_u \times n_u}$ matrices that weight the state vector and the controller input, respectively. $Q > 0$ and $R > 0$ are symmetric positive definite matrices. The selection of the weighting matrices is critical for the controller performance. The LQR approach minimizes the value of the cost function (16) by constructing a linear state feedback law:

$$u = Kx \tag{17}$$

where $K \in \mathbb{R}^{n_u \times n}$ is the feedback gain matrix. The controller, K, is designed by solving the following semidefinite programming problem:

$$\min \operatorname{tr}(P) \tag{18}$$

subject to

$$(A+BK)^T P + P(A+BK) \leq -Q - K^T R K \tag{19}$$

where $P > 0$ is the Lyapunov matrix. (18)-(19) is a non-convex optimization problem. It can be converted into a convex problem by multiplying left and right side of (19) with P^{-1} and applying Schur Complement [23]:

$$\max \operatorname{tr}(Y) \tag{20}$$

subject to

$$\begin{bmatrix} -(AY+BL) - (AY+BL)^T & Y & L^T \\ Y & Q^{-1} & 0 \\ L & 0 & R^{-1} \end{bmatrix} \geq 0 \tag{21}$$

$$Y = P^{-1} > 0 \tag{22}$$

where L is introduced as $L = KY$ and Y is the inverse of the Lyapunov matrix, $Y = P^{-1}$. The feedback matrix can be recovered as:

$$K = LY^{-1} \tag{23}$$

We use the robust optimization toolbox YALMIP [19] to design the feedback controller. The designed controller will be applied to the nonlinear pan-tilt system for the stabilization.

V. Simulation Results

The physical constraints that are applied to the joints are as follow:

TABLE I
Physical Constraints

Parameter	**Minimum Value**	**Maximum Value**
q_1	$-160°$	$160°$
q_2	$0°$	$80°$
$\dot{q}_1$	$-120°/sec$	$120°/sec$
$\dot{q}_2$	$-30°/sec$	$30°/sec$

According to Table I, scheduling trajectories are designed as quintic polynomials in Figures 3 and 4. Since the position trajectories are designed as 5th degree polynomials, joint velocity trajectories are 4th degree polynomials.

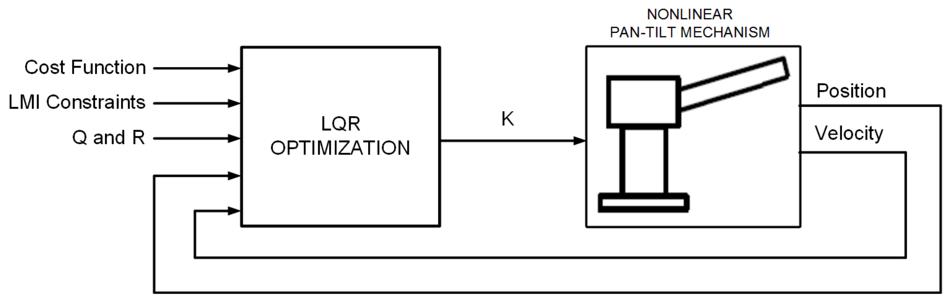


Fig. 2. Control block diagram

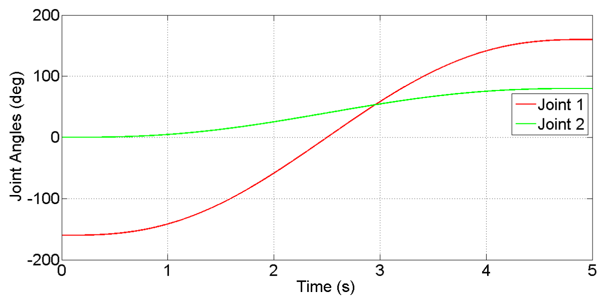


Fig. 3. Scheduling joint position signals

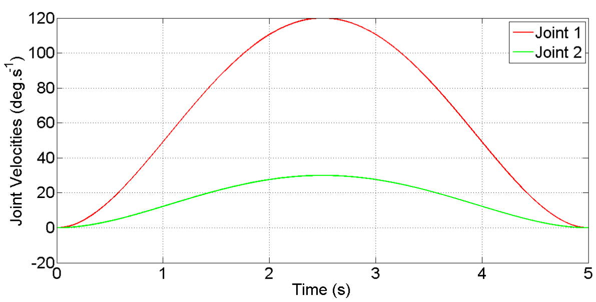


Fig. 4. Scheduling joint velocity signals

The parameter trajectories, θ_j, are generated by (15) in Figures 5-8. θ_1 depends on q_2 and $\dot{q}_1$. On the other hand, θ_2, θ_3 and θ_4 are the function of only q_2. Due to this dependency, the parameter values have the following upper and lower bounds:

TABLE II
PHYSICAL CONSTRAINTS

Parameter	**Upper Bound**	**Lower Bound**
θ_1 $(rad.sec^{-1})$	0.9978	-4.63×10^{-15}
θ_2 $(rad.sec^2)^{-1}$	-2.55	-14.7
θ_3 $(unitless)$	1.17×10^{-14}	-3.31
θ_4 $(kg.m^2)^{-1}$	1.19	0.71

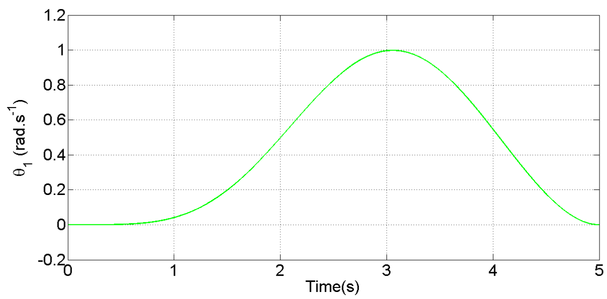


Fig. 5. Parameter trajectory: θ_1

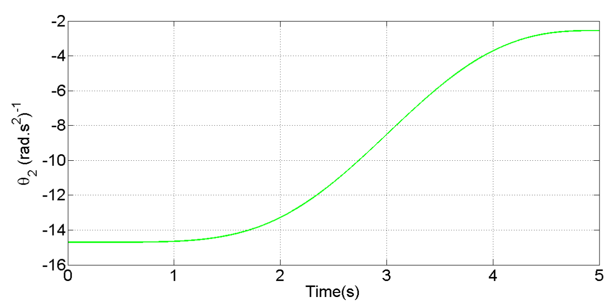


Fig. 6. Parameter trajectory: θ_2

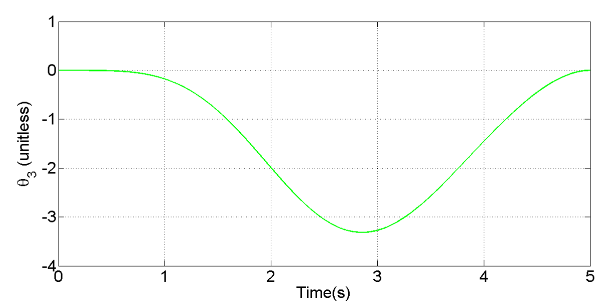


Fig. 7. Parameter trajectory: θ_3

LQR controller is synthesized based on the developed polytopic quasi-LPV model. The total number of vertices is $L = 2^4 = 16$. The desired state feedback controller is designed by interpolating LMIs at each vertex. The elements of the state feedback gain matrix, K, are determined using the weighting matrices, Q and R. The following weighting matrices are

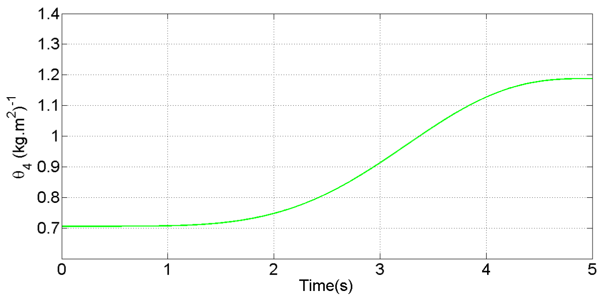


Fig. 8. Parameter trajectory: θ_4

chosen:

$$Q = \begin{bmatrix} 10^{-5} & 0 & 0 & 0 \\ 0 & 10^{-5} & 0 & 0 \\ 0 & 0 & 10^{-6} & 0 \\ 0 & 0 & 0 & 10^{-6} \end{bmatrix} \quad R = 10^{-9} I_{4\times 4}$$

The joint positions should be controlled more tightly than velocities. Therefore, more weighting is added to position states than the velocity states in Q matrix. R provides a limit for the magnitude of the control signal. The elements of Q matrix are designed larger than the elements of R matrix because the main control problem is the stabilization and the system states should converge to zero. In other words, the controller is designed such that it is more sensitive to the states of the system than the control input.

Using the system model (14)-(15) and the above weighting matrices, the optimal feedback gain matrix, $K \in R^{2\times 4}$, obtained by YALMIP is:

$$K = \begin{bmatrix} -87.77 & 2.69 & -22.65 & -0.02 \\ -0.64 & -53.56 & -0.083 & -9.59 \end{bmatrix}$$

The controller gains, K_{22} and K_{24} have larger magnitudes than K_{21} and K_{23} because the control input that is applied to the tilt mechanism mostly depends on q_2 and $\dot{q}_2$. Since the pan mechanism does not directly depend on q_2 and $\dot{q}_2$, K_{12} and K_{14} have smaller magnitudes than K_{11} and K_{13}.

The performance of the controller is tested on the nonlinear model and the stabilization is achieved. The states are presented in Figures 9-10. While Figures 9(a) and 10(a) present position and velocity responses of the first joint, Figures 9(b) and 10(b) show the responses for the second joints. Joint angles and velocities converge to zero as expected.

The initial joint positions are approximately 150° and 75°. The joint velocities are assumed as zero. The velocity of the first joint decreases to $-120°/sec$ and becomes zero again to stabilize the joint angle of the pan axis. Similarly, the velocity of the second joint decreases to $-30°/sec$ and becomes zero to make the joint angle of the tilt axis zero.

The control inputs are presented in Figures 11-12. Figures 11(a) and 12(a) depict output responses for 5 seconds and Figures 11(b) and 12(b) present the results at the beginning of the simulation. The control inputs are high at the beginning of the simulation because initial joint angles are multiplied by large controller gains, K_{11} and K_{22}. The control input that is applied to the pan axis converge to zero when the first joint angle is stabilized. However, the control input which is applied to the tilt axis does not converge to zero. Since the center of gravity is located along the tilt axis, the effect of gravity cannot be ignored. Therefore, the control input is needed to stabilize the tilt axis at zero angle. The control input, u_2, converge to 2.45 as depicted in Figure 12(b).

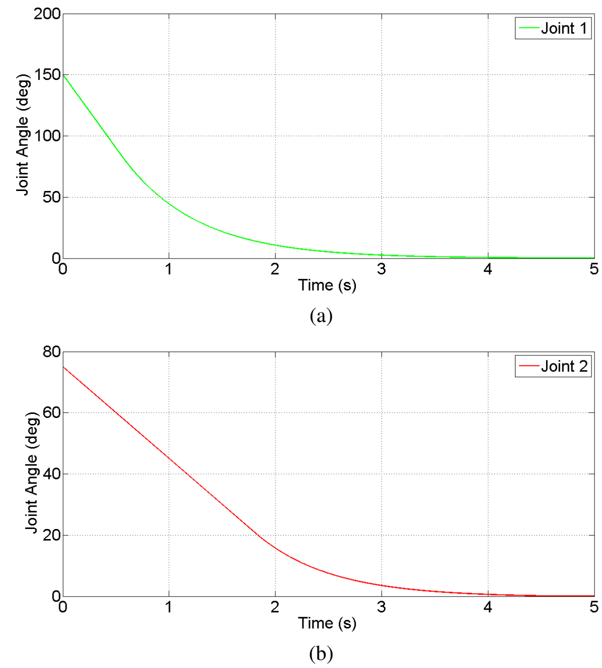


Fig. 9. Stabilized joint angles (a) q_1 (b) q_2

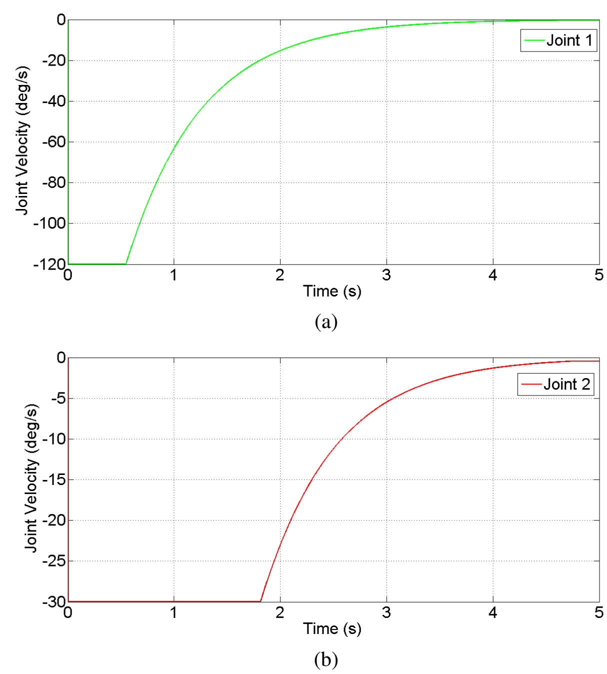
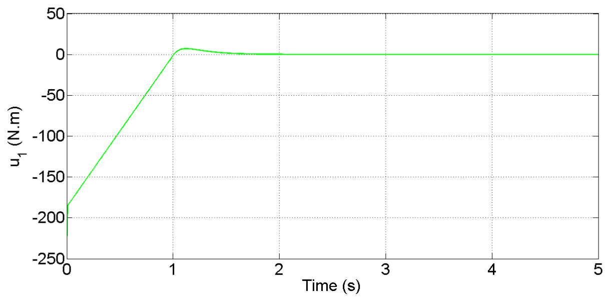
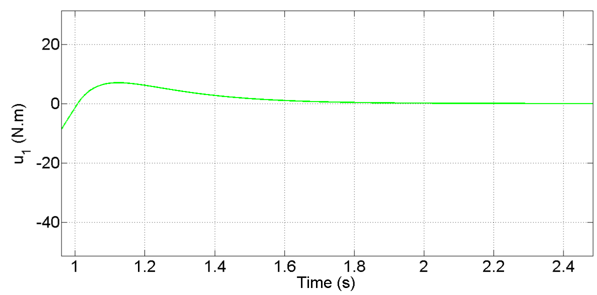


Fig. 10. Stabilized joint velocities (a) $\dot{q}_1$ (b) $\dot{q}_2$

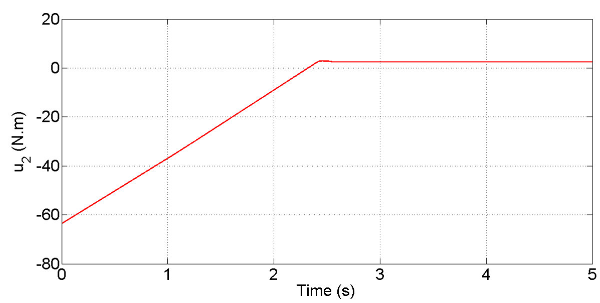


(a)

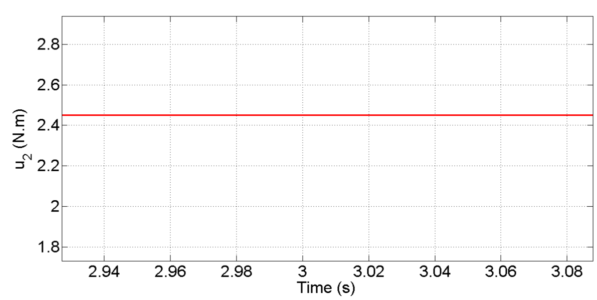


(b)

Fig. 11. Control input applied to the pan axis (a) 0-5 sec (b) 1-2.5 sec



(a)



(b)

Fig. 12. Control input applied to the tilt axis (a) 0-5 sec (b) 2.9-3.1 sec

VI. Conclusion and Future Work

We have now presented a polytopic quasi-LPV model of the nonlinear pan-tilt system. LQR controller is synthesized based on the developed LPV model using YALMIP toolbox. Since the dimension of the parameter vector is 4, the total number of vertices is 16. The feedback gain matrix is designed by interpolating LMIs at each vertex. The performance of the feedback gain matrix is tested on the nonlinear system for stabilization purposes. The LQR controller decreases all states to zero with less control effort by selecting the elements of Q matrix is higher than the ones in R matrix. Thus, the selection of the weighting matrices is critical to solve the stabilization problem efficiently.

As a future work, different control algorithms that utilize acceleration feedback will be developed based on the polytopic quasi-LPV models and compared with the performance of the controller used in this work. Experimental verification of the control algorithm on a physical system will be also realized.

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