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Low-complexity 2D coherently distributed sources decoupled DOAs estimation method

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The existing directions-of-arrival (DOAs) estimation methods for two-dimensional (2D) coherently distributed sources need one- or two-dimensional search, and the computational complexities of them are high. In addition, most of them are designed for special angular signal distribution functions. As a result, their performances will degenerate when deal with different sources with different angular signal distribution functions or unknown angular signal distribution functions. In this paper, a low-complexity decoupled DOAs estimation method without searching using two parallel uniform linear arrays (ULAs) is proposed for coherently distributed sources, as well as a novel parameter matching method. It can resolve the problems mentioned above efficiently. Simulation results validate the effectiveness of our approach.

2D coherently distributed source, uniform linear array, direction-of-arrival (DOA), decoupled estimation, quadric rotational invariance property (QRIP)

1 Introduction

In the fields of radar, sonar and wireless communication, etc., most sources appear spatial angular spread, which can be modeled as distributed source. The conventional DOA estimation methods, which are designed for point source, will be deteriorated when deal with the distributed source[1]. Based on the fact that the components from the same sources are correlated or not, the distributed source can be divided into two types[2]: coherently distributed (CD) source and incoherently distributed (ID) source. In this paper, the DOA estimation of the coherently distributed source is considered. Most of the existed parameter estimation algorithms are designed for one-dimensional (1D) coherently distributed source and they all need one- or two-dimensional searches, such as DSPE[2], DISPARE[3], min-minimum eigenvalue method, and max-maximum eigenvalue method[4]. The DOA estimation algorithms based on generalized eigendecomposition[5] and sparse signal representation (SSR)[6] do not need to search, but they are also restricted to the 1D coherently distributed source. The 1D distributed source model assumes that the sources and the local scattering points around them are in the same plane with the receiving array. However, in practice, this assumption does not hold. That is, the sources and the local scattering points are not in the same plane

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with the receiving array, which can be modeled as a 2D distributed source. In this paper, we considered the central DOA estimation problem for 2D coherently distributed source. The 1D coherently distributed source model is the special case of the 2D coherently distributed source, that is, no azimuth angle and its angular spread case.

A 2D coherently distributed source has four parameters: central elevation DOA, central azimuth DOA, elevation angular spread, and azimuth angular spread. Thus, the parameters estimation method will be more complex. Lee et al.[7] proposed a low-complexity parameters estimation method for 2D coherently distributed sources using an L-linear array. It transforms a four-dimensional optimization problem into two two-dimensional optimization problems, so the complexity is decreased. In addition, they proposed a sequential one-dimensional (SOS) method for DOAs estimation of 2D coherently distributed sources using two parallel uniform circular arrays[8]. The main idea is as follows: The central elevation DOA is estimated by TLS-ESPRIT, and the central azimuth DOA can be given by SOS. The method can estimate the two parameters using only one-dimensional search. However, all the methods mentioned above are designed for special angular distribution functions. For the cases where different sources are with different angular signal distribution functions or angular signal distribution functions are unknown, the performance of them will degenerate.

Aiming at the above problems, we presented a low-complexity decoupled DOAs estimation approach for 2D coherently distributed sources using two parallel uniform linear arrays. Firstly, TLS-ESPRIT method is used to estimate the central elevation DOA, and then the central azimuth DOA can be obtained by quadric rotational invariance property (QRIP) of generalized steering vector. The proposed method need not search and can deal with when different distributed sources are with unknown angular signal distribution functions or different distributed sources are with different angular signal distribution functions. In addition, a simple parameter matching approach is addressed. Simulation results validate the effectiveness of our method.

2 The 2D coherently distributed source model

Consider two uniform linear arrays $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ monitoring a wave field of q coherently distributed sources in additive background noise. Each array has L sensors, the distance between the two arrays is d, and the distance between the adjacent elements in each array is Δ. Their geometries are illustrated in Figure 1. The outputs of the arrays $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are expressed as the following continuous integral forms

$$\boldsymbol{x}_1 = \sum_{i=1}^{q} \iint \boldsymbol{a}(\vartheta,\varphi) s_i(\vartheta,\varphi;\mu_i) d\vartheta d\varphi + \boldsymbol{n}_{x_1}, \quad (1)$$

$$\boldsymbol{x}_2 = \sum_{i=1}^{q} \iint \boldsymbol{a}(\vartheta,\varphi) \mathrm{e}^{-\mathrm{j}(2\pi d/\lambda)\cos\vartheta} \cdot s_i(\vartheta,\varphi;\boldsymbol{\mu}_i) d\vartheta d\varphi + \boldsymbol{n}_{x_2}, \quad (2)$$

where $\boldsymbol{a}(\vartheta,\varphi)=[1, \mathrm{e}^{\mathrm{j}\eta\sin\vartheta\cos\varphi}, \cdots, \mathrm{e}^{\mathrm{j}\eta(L-1)\sin\vartheta\cos\varphi}]^{\mathrm{T}}$ is the steering vector of the array in the direction (ϑ,φ), where $\eta = 2\pi\Delta/\lambda$ and λ is the wavelength. $[\cdot]^{\mathrm{T}}$ is transposed operator. $s_i(\vartheta,\varphi;\boldsymbol{\mu}_i)$ is the angular signal density function of the ith source. Notice that the integrating ranges in the above equations are $\vartheta \in (-\pi/2, \pi/2]$ and $\varphi \in (0,\pi]$, but we omit them for the simplicity. The parameters θ_i, σ_{θ_i}, ϕ_i and σ_{ϕ_i} in the vector $\boldsymbol{\mu}_i = [\theta_i, \sigma_{\theta_i}, \phi_i, \sigma_{\phi_i}]$ are the central elevation DOA, elevation angular spread, central azimuth DOA and azimuth angular spread of the ith source, respectively. The central DOAs(θ_i,ϕ_i) are estimated in this study. $\boldsymbol{n}_{x_1}$ and

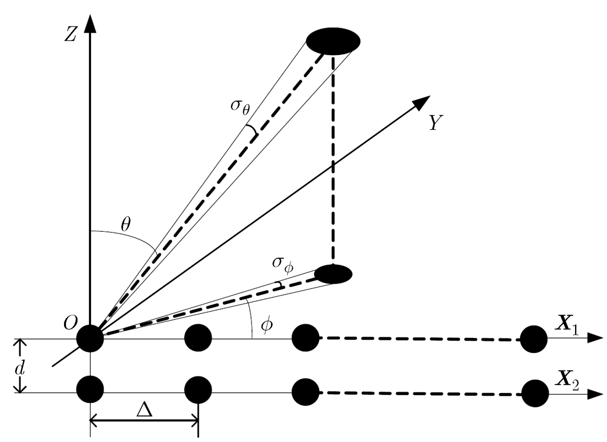

Figure 1 The double parallel uniform linear arrays geometry.

$\boldsymbol{n}_{x_2}$ are the additive white Gaussian noise.

For 2D coherently distributed source, $s_i(\vartheta, \varphi; \boldsymbol{\mu}_i)$ can be expressed as

$$s_i(\vartheta, \varphi; \boldsymbol{\mu}_i) = \gamma_i g_i(\vartheta, \varphi; \boldsymbol{\mu}_i), \tag{3}$$

where γ_i is a random variable and $g_i(\vartheta, \varphi; \boldsymbol{\mu}_i)$ is a deterministic angular distribution function. The index i denotes that different sources may have different deterministic angular signal distribution functions.

Let

$$\boldsymbol{b}(\boldsymbol{\mu}_i) = \iint \boldsymbol{a}(\vartheta, \varphi) g_i(\vartheta, \varphi; \boldsymbol{\mu}_i) d\vartheta d\varphi \tag{4}$$

be the generalized steering vector of the coherently distributed source. If the sources are uncorrelated, then the covariance matrix of the receiving signal vector of array X_1 can be expressed as

$$\begin{aligned}\boldsymbol{R}_{x_1} &= \sum_{i=1}^{q}\sum_{j=1}^{q} \boldsymbol{b}(\boldsymbol{\mu}_i) E\{\gamma_i \gamma_j^*\} \boldsymbol{b}^{\mathrm{H}}(\boldsymbol{\mu}_j) + \sigma_n^2 \boldsymbol{I}_L \\ &= \boldsymbol{B}\boldsymbol{\Gamma}\boldsymbol{B}^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I}_L,\end{aligned} \tag{5}$$

where $\boldsymbol{B} = [\boldsymbol{b}(\boldsymbol{\mu}_1), \boldsymbol{b}(\boldsymbol{\mu}_2), \cdots, \boldsymbol{b}(\boldsymbol{\mu}_q)]$ is the steering matrix, $[\boldsymbol{\Gamma}]_{ij} = E\{\gamma_i \gamma_j^*\}$. Moreover, σ_n^2 is the noise power, and $\boldsymbol{I}_L$ is an $L \times L$ identity matrix. $E\{\cdot\}$, $*$, and H are expectation operator, conjugate operator, and conjugate transposition, respectively. As a result, eq. (1) can be rewritten as

$$\boldsymbol{x}_1 = \boldsymbol{B}\boldsymbol{\gamma} + \boldsymbol{n}_{x_1}, \tag{6}$$

in which $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_q]^{\mathrm{T}}$. Similarly, eq. (2) can also be written as

$$\boldsymbol{x}_2 = \boldsymbol{C}\boldsymbol{\gamma} + \boldsymbol{n}_{x_2}, \tag{7}$$

where $\boldsymbol{C} = [\boldsymbol{c}(\boldsymbol{\mu}_1), \boldsymbol{c}(\boldsymbol{\mu}_2), \cdots, \boldsymbol{c}(\boldsymbol{\mu}_q)]$ with the generalized steering vector $\boldsymbol{c}(\boldsymbol{\mu}_i)$ is defined as

$$\boldsymbol{c}(\boldsymbol{\mu}_i) = \iint \boldsymbol{a}(\vartheta, \varphi) \mathrm{e}^{-\mathrm{j}(2\pi d/\lambda)\cos\vartheta} g_i(\vartheta, \varphi; \boldsymbol{\mu}_i) d\vartheta d\varphi. \tag{8}$$

It can be proved that for $d \ll \lambda$, we have (see Appendix)

$$\boldsymbol{c}(\boldsymbol{\mu}_i) \approx \boldsymbol{b}(\boldsymbol{\mu}_i) \mathrm{e}^{-\mathrm{j}(2\pi d/\lambda)\cos\theta_i}. \tag{9}$$

The matrix form is

$$\boldsymbol{C} \approx \boldsymbol{B}\boldsymbol{\Phi}, \tag{10}$$

where the rotation matrix $\boldsymbol{\Phi}$ is

$$\boldsymbol{\Phi} = \mathrm{diag}(\mathrm{e}^{-\mathrm{j}2\pi(d/\lambda)\cos\theta_1}, \cdots, \mathrm{e}^{-\mathrm{j}2\pi(d/\lambda)\cos\theta_q}). \tag{11}$$

3 Low-complexity 2D coherently distributed sources decoupled DOAs estimation method

3.1 The central elevation DOA estimation using TLS-ESPRIT

The total receiving signal vector of the two arrays can be expressed as

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{B}\boldsymbol{\Phi} \end{bmatrix} \boldsymbol{\gamma} + \begin{bmatrix} \boldsymbol{n}_{x_1} \\ \boldsymbol{n}_{x_2} \end{bmatrix} = \boldsymbol{\Psi}\boldsymbol{\gamma} + \boldsymbol{n}, \tag{12}$$

where $\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{B}\boldsymbol{\Phi} \end{bmatrix}$ and $\boldsymbol{n} = \begin{bmatrix} \boldsymbol{n}_{x_1} \\ \boldsymbol{n}_{x_2} \end{bmatrix}$.

Assuming that the covariance matrix of the total receiving signal vector $\boldsymbol{x}$ is $\boldsymbol{R}_x$. The signal subspace of $\boldsymbol{R}_x$ should be a $2L \times q$ matrix, which is denoted as $\boldsymbol{F}_s$. Moreover, it can be divided into the upper and lower $L \times q$ sub-matrices $\boldsymbol{F}_0$ and $\boldsymbol{F}_1$. There is a nonsingular matrix $\boldsymbol{T}$, which satisfies

$$\boldsymbol{F}_s = \begin{bmatrix} \boldsymbol{F}_0 \\ \boldsymbol{F}_1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{B}\boldsymbol{T} \\ \boldsymbol{B}\boldsymbol{\Phi}\boldsymbol{T} \end{bmatrix}. \tag{13}$$

Based on the above equation, total least square—ESPRIT (TLS-ESPRIT) can be used to estimate the central elevation DOA. We can summarize the procedure as follows:

1) Compute the covariance matrix estimation using $\hat{\boldsymbol{R}}_x = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)$ from N snapshots.

2) Eigendecomposition of $\hat{\boldsymbol{R}}_x$ to obtain the signal subspace $\hat{\boldsymbol{F}}_s$ and divide it into the upper and lower $L \times q$ sub-matrices $\hat{\boldsymbol{F}}_0$ and $\hat{\boldsymbol{F}}_1$, which satisfies $\hat{\boldsymbol{F}}_s = \begin{bmatrix} \hat{\boldsymbol{F}}_0 \\ \hat{\boldsymbol{F}}_1 \end{bmatrix}$.

3) Eigendecomposition of the matrix $\boldsymbol{G} = \begin{bmatrix} \hat{\boldsymbol{F}}_0^{\mathrm{H}} \\ \hat{\boldsymbol{F}}_1^{\mathrm{H}} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{F}}_0 & \hat{\boldsymbol{F}}_1 \end{bmatrix}$ as $\boldsymbol{G} = \boldsymbol{F}\boldsymbol{\Lambda}_{\boldsymbol{G}}\boldsymbol{F}^{\mathrm{H}}$, where $\boldsymbol{\Lambda}_{\boldsymbol{G}}$ is a diagonal matrix whose diagonal elements are eigenvalues, and $\boldsymbol{F}$ is the eigenvectors, correspondingly. Partition $\boldsymbol{F}$ into four $q \times q$ sub-matrices, that is, $\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F}_{11} & \boldsymbol{F}_{12} \\ \boldsymbol{F}_{21} & \boldsymbol{F}_{22} \end{bmatrix}$.

4) Eigendecomposition of $\boldsymbol{\Xi} = -\boldsymbol{F}_{11}\boldsymbol{F}_{21}^{-1}$ to obtain the eigenvalue λ_i ($\boldsymbol{F}_{11}, \boldsymbol{F}_{21}$ are the noise subspace matrices), then the central elevation DOA of

the coherently distributed source can be given by $\hat{\theta}_i = \cos^{-1}\left[-\frac{\arg(\lambda_i)}{2\pi d/\lambda}\right]$, $(i = 1, \cdots, q)$.

3.2 The central azimuth DOA estimation based on quadric rotational invariance property (QRIP) of the generalized steering vector

For small angular spread, the generalized steering vector $\boldsymbol{b}(\boldsymbol{\mu}_i)$ can be expressed as (see Appendix)

$$[\boldsymbol{b}(\boldsymbol{\mu}_i)]_k \approx \mathrm{e}^{\mathrm{j}\eta k \sin\theta_i \cos\phi_i}[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k, \qquad (14)$$

where $\boldsymbol{h}(\boldsymbol{\mu}_i)$ is a real function which is determined by $g_i(\vartheta, \varphi; \boldsymbol{\mu}_i)$. Let spatial frequency $\omega_i = \eta \sin\theta_i \cos\phi_i$, and then (14) can be rewritten as

$$[\boldsymbol{b}(\boldsymbol{\mu}_i)]_k = \mathrm{e}^{\mathrm{j}k\omega_i}[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k. \qquad (15)$$

Define the $L-1$ vectors $\bar{\boldsymbol{b}}(\boldsymbol{\mu}_i)$ and $\underline{\boldsymbol{b}}(\boldsymbol{\mu}_i)$ as $\bar{\boldsymbol{b}}(\boldsymbol{\mu}_i) = \boldsymbol{J}_1\boldsymbol{b}(\boldsymbol{\mu}_i)$ and $\underline{\boldsymbol{b}}(\boldsymbol{\mu}_i) = \boldsymbol{J}_2\boldsymbol{b}(\boldsymbol{\mu}_i)$, respectively, where $\boldsymbol{J}_1 = [\boldsymbol{I}_{L-1}, 0]_{(L-1)\times L}$ and $\boldsymbol{J}_2 = [0, \boldsymbol{I}_{L-1}]_{(L-1)\times L}$ are the selection matrices. For point source, the steering vector satisfies the following rotational invariance property

$$\bar{\boldsymbol{b}}(\boldsymbol{\mu}_i) = \mathrm{e}^{-\mathrm{j}\omega_i}\underline{\boldsymbol{b}}(\boldsymbol{\mu}_i).$$

However, for distributed source, it does not hold. Fortunately, it satisfies the QRIP as follows:

$$\bar{\boldsymbol{b}}(\boldsymbol{\mu}_i) \circ \underline{\boldsymbol{b}}^*(\boldsymbol{\mu}_i) = e^{-j2\omega_i}\bar{\boldsymbol{b}}^*(\boldsymbol{\mu}_i) \circ \underline{\boldsymbol{b}}(\boldsymbol{\mu}_i), \qquad (16)$$

where $\circ$ is Hadamard product.

Assuming that the signal subspace of covariance matrix $\boldsymbol{R}_{x_1}$ is $\boldsymbol{U}_s = [\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_L]^{\mathrm{H}}$, where $\boldsymbol{u}_1^{\mathrm{H}}, \boldsymbol{u}_2^{\mathrm{H}}, \cdots, \boldsymbol{u}_L^{\mathrm{H}}$ are the row vectors of $\boldsymbol{U}_s$. From subspace theory, the signal subspace $\boldsymbol{U}_s$ and the generalized vector $\boldsymbol{b}(\boldsymbol{\mu}_i)$ have the following relationship[4]

$$\boldsymbol{b}(\boldsymbol{\mu}_i) = \boldsymbol{U}_s\boldsymbol{g}, \qquad (17)$$

where $\boldsymbol{g}$ is a $q \times 1$ vector. Based on eqs. (15)–(17), it follows that

$$\boldsymbol{u}_k^{\mathrm{H}}\boldsymbol{g}\boldsymbol{g}^{\mathrm{H}}\boldsymbol{u}_{k-1} = \mathrm{e}^{\mathrm{j}2\omega_i}\boldsymbol{u}_{k-1}^{\mathrm{H}}\boldsymbol{g}\boldsymbol{g}^{\mathrm{H}}\boldsymbol{u}_k, (k = 1, \cdots, L). \qquad (18)$$

Using $\mathrm{vec}(\boldsymbol{ABC}) = (\boldsymbol{C}^{\mathrm{T}} \otimes \boldsymbol{A})\mathrm{vec}(\boldsymbol{B})$, where $\mathrm{vec}(\cdot)$ is vector operator and $\otimes$ is Kronecker product, eq. (18) can be rewritten as

$$\boldsymbol{Q}_1\boldsymbol{\xi} = \mathrm{e}^{\mathrm{j}2\omega_i}\boldsymbol{Q}_2\boldsymbol{\xi}, \qquad (19)$$

where the $q^2 \times 1$ vector $\boldsymbol{\xi} = \mathrm{vec}(\boldsymbol{g}\boldsymbol{g}^{\mathrm{H}})$ and the $(L-1) \times q^2$ matrices $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ are

$$\boldsymbol{Q}_1 = \begin{bmatrix} \boldsymbol{u}_1^{\mathrm{T}} \otimes \boldsymbol{u}_2^{\mathrm{H}} \\ \boldsymbol{u}_2^{\mathrm{T}} \otimes \boldsymbol{u}_3^{\mathrm{H}} \\ \vdots \\ \boldsymbol{u}_{L-1}^{\mathrm{T}} \otimes \boldsymbol{u}_L^{\mathrm{H}} \end{bmatrix}, \quad \boldsymbol{Q}_2 = \begin{bmatrix} \boldsymbol{u}_2^{\mathrm{T}} \otimes \boldsymbol{u}_1^{\mathrm{H}} \\ \boldsymbol{u}_3^{\mathrm{T}} \otimes \boldsymbol{u}_2^{\mathrm{H}} \\ \vdots \\ \boldsymbol{u}_L^{\mathrm{T}} \otimes \boldsymbol{u}_{L-1}^{\mathrm{H}} \end{bmatrix}.$$

Eq. (19) shows that the spatial frequency of coherently distributed source can be obtained from the generalized eigendecomposition of the pencil of matrix $(\boldsymbol{Q}_1^{\mathrm{H}}\boldsymbol{Q}_1, \boldsymbol{Q}_1^{\mathrm{H}}\boldsymbol{Q}_2)$ and $e^{j2\omega_i}$ is the generalized eigenvalue. Therefore, the central azimuth DOA procedure can be summarized as follows:

1) Compute the covariance matrix estimation using $\hat{\boldsymbol{R}}_{x_1} = \frac{1}{N}\sum_{t=1}^{N}\boldsymbol{x}_1(t)\boldsymbol{x}_1^{\mathrm{H}}(t)$ from N snapshots.

2) Eigendecomposition of $\hat{\boldsymbol{R}}_x$ to obtain the signal subspace $\hat{\boldsymbol{U}}_s$, then $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$ can be computed correspondingly.

3) Compute the generalized eigendecomposition of pencil of matrix $(\boldsymbol{Q}_1^{\mathrm{H}}\boldsymbol{Q}_1, \boldsymbol{Q}_1^{\mathrm{H}}\boldsymbol{Q}_2)$, using the phases of the q generalized eigenvalues whose modulus are near to one to estimate the spatial frequency $\hat{\omega}_i$.

4) The central azimuth DOA of coherently distributed source can be given by $\hat{\phi}_i = \tilde{f}(\hat{\theta}_i, \hat{\omega}_i) = \cos^{-1}\left[\frac{\hat{\omega}_i}{\eta \sin\hat{\theta}_i}\right]$.

3.3 Min-minimum eigenvalue spectrum parameter matching method

The proposed method can give a decoupled DOAs estimation for 2D coherently distributed. For the single source, there is no parameter matching problem. However, if there are multiple sources, parameter matching must be given for obtaining the correct DOAs estimation. A simple parameter matching method based on the orthogonality of subspace was addressed in ref. [9], but it is designed for point source only. For distributed source, it does not hold.

In order to solve this problem, 1D min-minimum eigenvalue spectrum[3] is generalized to 2D case for parameter matching. For q distributed sources, one needs to compute the q^2 2D min-minimum eigenvalue spectrum values and compares them with each other to finish the parameter matching.

Let $\hat{\boldsymbol{G}}_n$ be the noise subspace of the covariance matrix $\boldsymbol{R}_{x_1}$, it follows $\hat{\boldsymbol{G}}_n^{\mathrm{H}}\boldsymbol{b}(\boldsymbol{\mu}_i) = 0$. Therefore, the parameter of coherently distributed source is the solution of the following minimization problem:

$$\boldsymbol{\mu}_i = \arg\min_{\mu} \boldsymbol{b}^{\mathrm{H}}(\boldsymbol{\mu})\hat{\boldsymbol{G}}_n\hat{\boldsymbol{G}}_n^{\mathrm{H}}\boldsymbol{b}(\boldsymbol{\mu}). \quad (20)$$

Substitute A3 into eq. (20), and one has

$$(\theta_i, \phi_i; \boldsymbol{h}(\boldsymbol{\mu}_i)) = \arg\min_{(\theta,\phi;\boldsymbol{h})} \boldsymbol{h}^{\mathrm{H}}\boldsymbol{D}^{\mathrm{H}}(\theta,\phi) \cdot \hat{\boldsymbol{G}}_n\hat{\boldsymbol{G}}_n^{\mathrm{H}}\boldsymbol{D}(\theta,\phi)\boldsymbol{h}. \quad (21)$$

Here, $\boldsymbol{D}(\theta,\phi) = \mathrm{diag}(\boldsymbol{a}(\theta,\phi))$ and the expression of $\boldsymbol{h}(\boldsymbol{\mu}_i)$ can be seen from Appendix. As a result, the eigenvalue of $\boldsymbol{D}^{\mathrm{H}}(\theta,\phi)\hat{\boldsymbol{G}}_n\hat{\boldsymbol{G}}_n^{\mathrm{H}}\boldsymbol{D}(\theta,\phi)$ will be minimum when (θ,ϕ) equals to (θ_i,ϕ_i) and $\boldsymbol{h}(\boldsymbol{\mu}_i)$ is the corresponding eigenvector. Notice that $\boldsymbol{D}^{\mathrm{H}}(\theta,\phi)\hat{\boldsymbol{G}}_n\hat{\boldsymbol{G}}_n^{\mathrm{H}}\boldsymbol{D}(\theta,\phi)$ is a nonnegative definite matrix with nonnegative eigenvalues and real eigenvector $\boldsymbol{h}(\boldsymbol{\mu}_i)$, so eq. (21) can be rewritten as

$$(\theta_i, \phi_i; \boldsymbol{h}(\boldsymbol{\mu}_i)) = \arg\min_{(\theta,\phi;\boldsymbol{h})} \boldsymbol{h}^{\mathrm{H}}\bar{\boldsymbol{Q}}(\theta,\phi)\boldsymbol{h},$$

where $\bar{\boldsymbol{Q}}(\theta,\phi) = \mathrm{Re}[\boldsymbol{D}^{\mathrm{H}}(\theta,\phi)\hat{\boldsymbol{G}}_n\hat{\boldsymbol{G}}_n^{\mathrm{H}}\boldsymbol{D}(\theta,\phi)]$. For uniform linear array, the constrain condition $\boldsymbol{h}(1)$=1 holds, so the central DOA of coherently distributed source can be obtained from

$$(\theta_i, \phi_i; \boldsymbol{h}(\boldsymbol{\mu}_i)) = \arg\min_{(\theta,\phi;\boldsymbol{h}(1)=1)} \boldsymbol{h}^{\mathrm{H}}\bar{\boldsymbol{Q}}(\theta,\phi)\boldsymbol{h}.$$

In what follows, we use $\boldsymbol{h}^{\mathrm{T}}\boldsymbol{w} = 1$ instead of $\boldsymbol{h}(1) = 1$, where $\boldsymbol{w} = [1, 0, \cdots, 0]^{\mathrm{T}}$ is an $L \times 1$ vector. The Lagrange cost function is

$$L(\theta,\phi;\boldsymbol{h}) = \boldsymbol{h}^{\mathrm{T}}\bar{\boldsymbol{Q}}(\theta,\phi)\boldsymbol{h} + 2\beta(1 - \boldsymbol{h}^{\mathrm{T}}\boldsymbol{w}),$$

where β is Lagrange coefficient. With Lagrange multiplier method, the 2D min-minimum eigenvalue spectrum can be expressed as

$$f(\theta_i,\phi_i) = \arg\max_{\theta,\phi} \boldsymbol{w}^{\mathrm{T}}\bar{\boldsymbol{Q}}^{\dagger}\boldsymbol{w}, \quad (22)$$

where $\bar{\boldsymbol{Q}}^{\dagger}$ is the Moor-Penrose inverse of $\bar{\boldsymbol{Q}}(\theta,\phi)$. Obviously, the 2D min-minimum eigenvalue spectrum function value will be the largest one when (θ,ϕ) equals to (θ_i,ϕ_i). This property can help us to realize the parameter matching. The matching procedure is summarized as follows:

1) Compute $\hat{\phi}_{ij} = \cos^{-1}\{\hat{\omega}_j/(\eta\sin\hat{\theta}_i)\}$ from the central elevation DOAs estimation $\{\hat{\theta}_1,\cdots,\hat{\theta}_q\}$ and the spatial frequency estimation $\{\hat{\omega}_1,\cdots,\hat{\omega}_q\}$.

2) Substitute the q^2 combinations $(\hat{\theta}_i,\hat{\phi}_{ij})$ into (22) and calculate the function value $f(\hat{\theta}_i,\hat{\phi}_j)$, respectively.

3) If $f(\hat{\theta}_m,\hat{\phi}_n)$ is the largest, then (θ_m,ϕ_n) is the correct matching. Also, calculate $\omega_m = \eta\sin\theta_m\cos\phi_n$.

4) Get rid of the $\{\hat{\theta}_m,\hat{\omega}_m\}$ from $\{\hat{\theta}_1,\cdots,\hat{\theta}_q\}$ and $\{\hat{\omega}_1,\cdots,\hat{\omega}_q\}$, then let $q = q-1$ and repeat steps 1)–4).

4 Simulation results

The sensor elements number is $L = 8$, the distance of adjacent sensors in each array is $\Delta = \lambda/2$, and the distance between the two arrays is $d = \lambda/10$.

Case 1. Two sources are with the same deterministic angular signal distribution functions.

Consider two coherently distributed sources with Gaussian deterministic angular signal distribution $\boldsymbol{\mu}_1 = [10°, 3°, 25°, 4°]$ and $\boldsymbol{\mu}_2 = [30°, 5°, 40°, 6°]$. The signal-noise ratio (SNR) is 15 dB and the snapshot number is $N = 500$. Figure 2 shows the two spatial frequencies estimation with 50 independent trials. Correspondingly, the spatial frequencies estimation are $\hat{\omega}_1 = 0.4944$ and $\hat{\omega}_2 = 1.2033$. The central elevation DOAs estimation using TLS-ESPRIT are $\hat{\theta}_1 = 9.9923°$ and $\hat{\theta}_2 = 30.0542°$, respectively. Therefore, one can build the corresponding parameter matching Table 1. At first, we substitute $(\hat{\theta}_1,\hat{\phi}_{11})$ and $(\hat{\theta}_1,\hat{\phi}_{12})$ into eq. (22) when $\hat{\theta}_1 = 0.3458°$. Compare the function values of (22) and select the largest one as the correct parameter matching result. Based on the same method, we can give the true parameter matching result when $\hat{\theta}_2 = 30.0542°$. By this mean, the true central azimuth DOAs $\hat{\phi}_1 = 23.8144°$ and $\hat{\phi}_2 = 41.5118°$ can be obtained. With 20 independent trials, the central DOAs estimation results of coherently distributed sources are plotted in Figure 3.

Table 1 The spatial frequency and central elevation DOA matching

Azimuth DOA matching		$\hat{\omega} = \pi\sin\hat{\theta}\cos\hat{\phi}$	
		$\hat{\omega}_1$	$\hat{\omega}_2$
$\hat{\theta}$	$\hat{\theta}_1$	$\hat{\phi}_{11} = \tilde{f}(\hat{\theta}_1,\hat{\omega}_1)$	$\hat{\phi}_{12} = \tilde{f}(\hat{\theta}_1,\hat{\omega}_2)$
	$\hat{\theta}_2$	$\hat{\phi}_{21} = \tilde{f}(\hat{\theta}_2,\hat{\omega}_1)$	$\hat{\phi}_{22} = \tilde{f}(\hat{\theta}_2,\hat{\omega}_2)$

With 500 Monte Carlo simulations, the root-mean-square-error (RMSE) of the central azimuth DOA estimation versus SNR of our method and SOS[8] are depicted in Figure 4. As can be seen

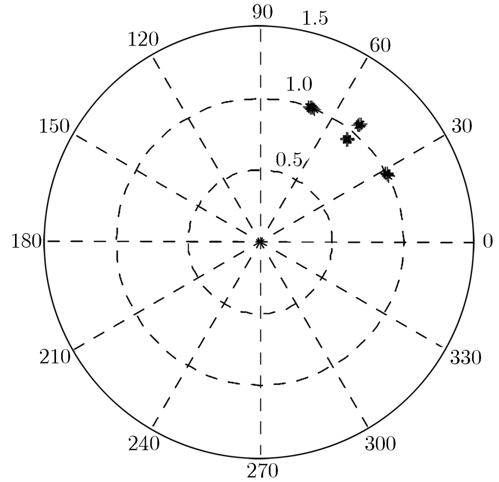

Figure 2 The spatial frequency estimation using QRIP (50 tirals).

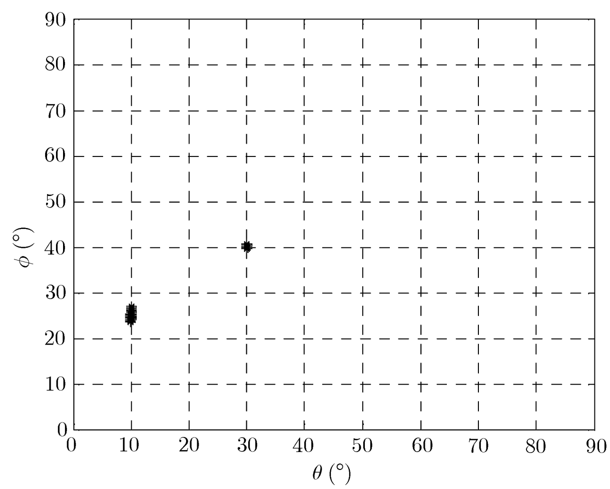

Figure 3 The 2D DOA estimation results (20 trials).

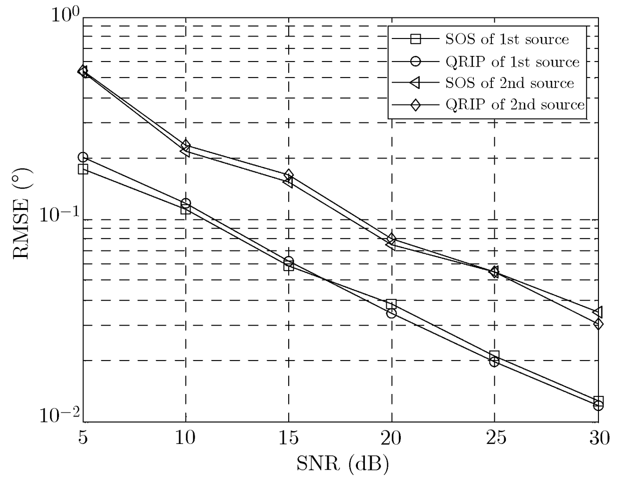

Figure 4 RMSE of the central elevation DOA estimation versus SNR.

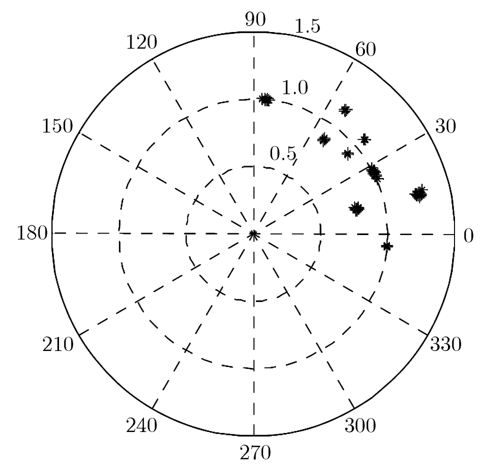

Figure 5 The spatial frequency estimation using QRIP (50 tirals) (three sources are with different deterministic angular signal distribution functions).

from the figure, our method has the same performance as SOS whenever the SNR is low or high. However, our method has lower computational complexity than that of the SOS method. In addition, SOS cannot give the correct DOA estimation results for the case that different sources are with different deterministic angular signal distributions.

Case 2. Three sources are with the different deterministic angular signal distribution functions.

Three coherently distributed sources with $\boldsymbol{\mu}_1 = [20°, 3°, 95°, 2°]^{\mathrm{T}}$, $\boldsymbol{\mu}_2 = [10°, 3°, 30°, 4°]^{\mathrm{T}}$, and $\boldsymbol{\mu}_3 = [70°, 5°, 60°, 6°]^{\mathrm{T}}$ are impinging on the receiving arrays. Their deterministic angular signal distribution functions are Gaussian and uniform. The SNR and snapshot number are the same as case 1. At first, the central elevation DOAs estimation using TLS-ESPRIT are $\hat{\theta}_1 = 9.8977°$, $\hat{\theta}_2 = 69.9958°$, and $\hat{\theta}_3 = 20.0488°$, and the spatial frequency estimation based on QRIP of the generalized steering vector are $\hat{\omega}_1 = 84.5723°$, $\hat{\omega}_2 = 27.0691°$ and $\hat{\omega}_3 = -5.3656°$, which can be seen in Figure 5. After parameter matching, the correct central azimuth estimation are $\hat{\phi}_1 = 30.0567°$, $\hat{\phi}_2 = 60.0554°$, and $\hat{\phi}_3 = 94.9918°$, respectively.

From the two different cases, it can be seen that the proposed method can give more exact DOAs estimation results. It need not search and can deal with that different sources are with different angular signal distribution functions or unknown angular signal distribution functions. Thus, the robustness of our method is good.

5 Conclusion

Herein, a low-complexity decoupled DOAs estimation method without searching using two parallel uniform linear arrays (ULAs) is proposed for coherently distributed sources, as well as a simple parameter matching method. Compared with the existed methods, our approach need not search and can deal with that different sources are with different angular signal distribution functions or unknown angular signal distribution functions. In addition, a simple parameter matching method, which is generalized from 1D min-minimum eigenvalue spectrum, is derived to solve parameter matching problem. Out method is suited to fast parameter estimation and tracking.

Appendix The approximated steering vector and rotational invariance property for small angular spread

For arbitrary DOA ϑ, φ, we have $\vartheta = \theta_i + \tilde{\theta}$ and $\varphi = \phi_i + \tilde{\varphi}$, where $\tilde{\vartheta}, \tilde{\varphi}$ are the small deviation between ϑ, φ and the central DOA θ, ϕ. For small angular spread, one has

$$\begin{aligned}[\boldsymbol{a}(\vartheta,\varphi)]_k &= \mathrm{e}^{\mathrm{j}\eta k \sin\vartheta\cos\varphi} = \mathrm{e}^{\mathrm{j}\eta k \sin(\theta_i+\tilde{\vartheta})\cos(\phi_i+\tilde{\varphi})} \\ &\approx \mathrm{e}^{\mathrm{j}\eta k (\sin\theta_i + \tilde{\vartheta}\cos\theta_i)(\cos\phi_i - \tilde{\varphi}\sin\phi_i)} \\ &= \mathrm{e}^{\mathrm{j}\eta k \sin\theta_i\cos\phi_i} \\ &\quad \cdot \mathrm{e}^{\mathrm{j}\eta k(\tilde{\vartheta}\cos\theta_i\cos\phi_i - \tilde{\varphi}\sin\theta_i\sin\phi_i)}. \end{aligned} \tag{A1}$$

Here, we use the facts that $\sin\tilde{\vartheta} \approx \tilde{\vartheta}$, $\sin\tilde{\varphi} \approx \tilde{\varphi}$, $\cos\tilde{\vartheta} \approx 1$, $\cos\tilde{\varphi} \approx 1$, and $\tilde{\vartheta}\tilde{\varphi} \approx 0$ for small $\tilde{\vartheta}$ and $\tilde{\varphi}$. Substitute (A1) into (4), and one obtains

$$[\boldsymbol{b}(\boldsymbol{\mu}_i)]_k \approx \mathrm{e}^{\mathrm{j}\eta k \sin\theta_i\cos\phi_i}[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k. \tag{A2}$$

With the vector form as follows:

$$\boldsymbol{b}(\boldsymbol{\mu}_i) \approx \boldsymbol{D}(\theta_i,\phi_i)\boldsymbol{h}(\boldsymbol{\mu}_i), \tag{A3}$$

where

$$\boldsymbol{D}(\theta_i,\phi_i) = \mathrm{diag}(\boldsymbol{a}(\theta_i,\phi_i)), \tag{A4}$$

$$\begin{aligned}[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k = \iint \mathrm{e}^{\mathrm{j}\eta k(\tilde{\vartheta}\cos\theta_i\cos\phi_i - \tilde{\varphi}\sin\theta_i\sin\phi_i)} \\ \cdot g_i(\theta_i+\tilde{\vartheta}, \phi_i+\tilde{\varphi}; \boldsymbol{\mu}_i) d\tilde{\vartheta} d\tilde{\varphi}. \end{aligned} \tag{A5}$$

For the common deterministic angular signal distribution functions, such as Gaussian and uniform, we can give the closed forms of $[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k$.

For example, if $g_i(\vartheta,\varphi;\boldsymbol{\mu}_i)$ has Gaussian shape, that is,

$$g_i(\vartheta,\varphi;\boldsymbol{\mu}_i) = \frac{1}{2\pi\sigma_{\theta_i}\sigma_{\phi_i}} \mathrm{e}^{-1/2((\vartheta-\theta_i)^2/\sigma_{\theta_i}^2 + (\varphi-\phi_i)^2/\sigma_{\phi_i}^2)}.$$

Then, the expression of $\boldsymbol{h}(\mu_i)$ are

$$[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k^G = \mathrm{e}^{-\frac{\eta^2 k^2(\sigma_{\phi_i}^2 \sin^2\theta_i \sin^2\phi_i + \sigma_{\theta_i}^2\cos^2\theta_i\cos^2\phi_i)}{2}}.$$

If $g_i(\vartheta,\varphi;\boldsymbol{\mu}_i)$ has uniform shape, that is,

$$g_i(\vartheta,\varphi;\boldsymbol{\mu}_i) = \begin{cases} 1/(2\sqrt{3}\sigma_{\theta_i}) - \sqrt{3}\sigma_{\theta_i} < \tilde{\vartheta} < \sqrt{3}\sigma_{\theta_i}, \\ 1/(2\sqrt{3}\sigma_{\phi_i}) - \sqrt{3}\sigma_{\phi_i} < \tilde{\varphi} < \sqrt{3}\sigma_{\phi_i}, \\ 0 \qquad \text{otherwise}. \end{cases}$$

Then, the expression of $\boldsymbol{h}(\boldsymbol{\mu}_i)$ are

$$\begin{aligned}[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k^U &= \frac{\sin(\sqrt{3}\eta k\sigma_{\theta_i}\cos\theta_i\cos\phi_i)}{\sqrt{3}\eta k\sigma_{\theta_i}\cos\theta_i\cos\phi_i} \\ &\quad \cdot \frac{\sin(\sqrt{3}\eta k\sigma_{\phi_i}\sin\theta_i\sin\phi_i)}{\sqrt{3}\eta k\sigma_{\phi_i}\sin\theta_i\sin\phi_i}. \end{aligned}$$

Substitute (A1) into (8), and we have

$$\begin{aligned}[\boldsymbol{c}(\boldsymbol{\mu}_i)]_k &= \mathrm{e}^{\mathrm{j}\eta k\sin\theta_i\cos\phi_i} \\ &\quad \cdot \iint \mathrm{e}^{\mathrm{j}\eta k(\tilde{\vartheta}\cos\theta_i\cos\phi_i - \tilde{\varphi}\sin\theta_i\sin\phi_i)} \\ &\quad \cdot \mathrm{e}^{-\mathrm{j}(2\pi d/\lambda)(\cos\theta_i - \tilde{\vartheta}\sin\theta_i)} \\ &\quad \cdot g_i(\theta_i+\tilde{\vartheta}, \phi_i+\tilde{\varphi}; \mu_i) d\tilde{\vartheta} d\tilde{\varphi}. \end{aligned}$$

If $d \ll \lambda$, it follows that $e^{j(2\pi d/\lambda)(\tilde{\vartheta}\sin\theta_i)} \approx 1$, so we can rewrite the above formula as follows:

$$[\boldsymbol{c}(\boldsymbol{\mu}_i)]_k = \mathrm{e}^{-\mathrm{j}(2\pi d/\lambda)\cos\theta_i}\mathrm{e}^{\mathrm{j}\eta k\sin\theta_i\cos\phi_i}[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k, \tag{A6}$$

where the expression of $[\boldsymbol{h}(\boldsymbol{\mu}_i)]_k$ can be seen from (A5). From (A2) and (A6), the following relationship holds:

$$\boldsymbol{c}(\boldsymbol{\mu}_i) \approx \mathrm{e}^{-\mathrm{j}(2\pi d/\lambda)\cos\theta_i}\boldsymbol{b}(\boldsymbol{\mu}_i).$$

With the matrix form as

$$\boldsymbol{C} \approx \boldsymbol{B}\boldsymbol{\Phi}, \tag{A7}$$

where the rotational matrix $\boldsymbol{\Phi}$ is

$$\boldsymbol{\Phi} = \mathrm{diag}(\mathrm{e}^{-\mathrm{j}2\pi(d/\lambda)\cos\theta_1}, \cdots, \mathrm{e}^{-\mathrm{j}2\pi(d/\lambda)\cos\theta_q}).$$

Obviously, the steering matrices $\boldsymbol{B}$ and $\boldsymbol{C}$ have a rotational invariance property.

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