Equalization of the Non-Linear Satellite Communication Channel with an Echo State Network

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***Abstract*—Because of the small energy available aboard a satellite, the power amplifier must achieve a very high power efficiency which suggest to work close to the saturation point. This would be power efficient, but unfortunately would add non-linear distortions to the communication channel. Several equalization algorithms have been proposed to compensate for this non-linear behaviour. The Echo State Network (ESN), an algorithm coming from the field of artificial neural networks, has also been proposed for this task but has never been compared to state-of-the-art equalizers for non-linear channel. The aim of this paper is to adapt the ESN to the satellite communication channel and to compare it to the baseband Volterra equalizer. We show that the ESN is able to reach the same performances as the Volterra equalizer, evaluated in terms of bit error rate, and has similar complexity. In addition, we propose a new training strategy for the ESN and the Volterra equalizer to improve their performance.**

***Index Terms*—Satellite Communications, Non-linear communication channel, Equalization, Volterra, Echo State Network**

I. Introduction

The increasing demand of bit rate in wireless communications requires the use of higher modulation orders. Such modulations require a higher signal-to-noise ratio (SNR) on the receiving side to achieve sufficiently low bit error rate (BER). But in the case of the satellite communications, like DVB-S2 [1], the available power is very limited which strongly degrades the link budget. To ensure a high power output, the power amplifier in the satellite must work close to the saturation point which allows a high efficiency. However the power amplifier adds important non-linear distortions in the communication channel which have to be compensated, at the transmitter side with pre-distortion or at the receiver side with equalization, to ensure a low BER. In this paper, we will study the digital compensation of the non-linear channel at the receiver side.

Such a channel can be represented by using a baseband Volterra model that can represent at the same time the memory and the non-linearity of the channel [2]. A baseband Volterra model can also be used as an efficient equalizer for these non-linear communication channels [3][4][5]. In the following, the term Volterra will always refer to the baseband Volterra model.

Other approaches based on artificial recurrent neural network structures have been investigated. One of them is called the Echo State Network (ESN) [6][7], also known as reservoir computer [8]. A recent reviewing of this algorithm is presented in [9]. This algorithm has the advantage to offer an interesting compromise between performances and complexity in comparison with classical recurrent neural networks. Preliminary results on ESN applied to non-linear channel equalization were reported in [10] but with a non-realistic channel model. The performances of this ESN was limited by the fact that it only considered real inputs. The extension to complex symbols has been proposed in [11]. Several papers have evaluated the performances of the ESN for the equalization of a non-linear communication channel but they were mostly compared with a linear filter [11] [12]. In the best of the knowledge of the authors, the ESN has never been compared to state-of-the-art equalizers for non-linear channels, like the Volterra equalizer.

Note that, recently, high performance experimental implementations have been reported on optoelectronic [13][14] and all-optical [15][16] circuits. This suggest that analog equalizers could be efficiently implemented using the ESN approach.

The objective of this paper is to apply the ESN to the satellite communication channel. A comparison will be done with the Volterra equalizer in terms of BER and complexity to evaluate the interest of the ESN for the digital equalization of non-linear communication channels.

Most of the time, the coefficients of the ESN or the Volterra equalizer are evaluated to minimize the mean square error (MSE) between the transmitted signal and the estimated one [4]. So the equalizers have to compensate for the inter-symbol interferences and the compression of the received constellation due to the saturated behaviour of the power amplifier. We will show that we can improve the quality of the equalization if we take this compression into account in the training process.

The outline of this paper is the following. In Section 2, the satellite communication channel is described. In Section 3, the ESN will be introduced. The improvement of the training process will be proposed in Section 4. The Volterra equalizer and the ESN will be compared in terms of BER and complexity in Section 5.

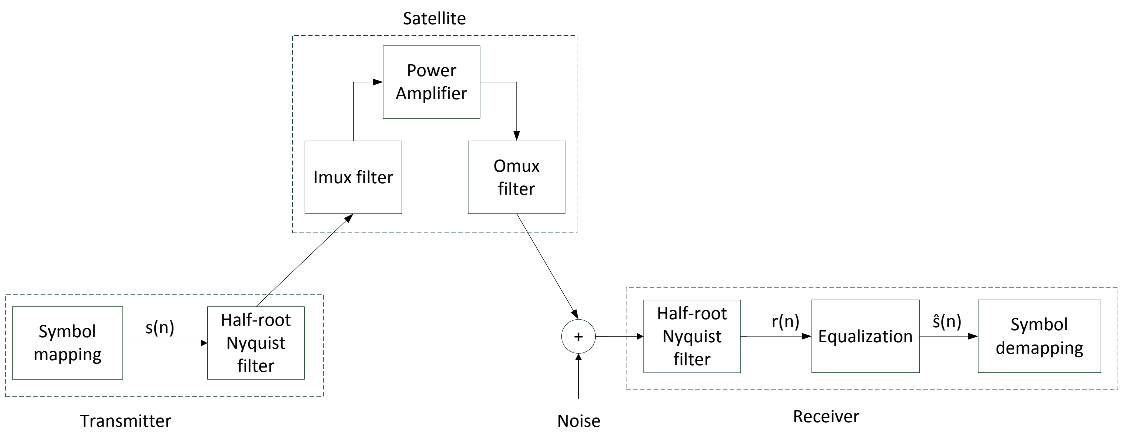

Fig. 1. Block scheme of a satellite communication channel

II. SYSTEM MODEL

The baseband satellite communication channel is described in Fig. 1. The power amplifier aboard the satellite is the source of non-linearity. It is described by its baseband model which gives the amplitude modulation (AM-AM) and phase modulation (AM-PM) characteristics (Fig. 2). If the input of the power amplifier is defined as $z(n) = |z(n)|e^{j\phi(z(n))}$, the output of the amplifier $y(n)$ is

$$y(n) = g(|z(n)|)e^{j(\phi(z(n))+\psi(|z(n)|))} \quad (1)$$

where $g(.)$ represents the AM-AM relation and $\psi(.)$ represents the AM-PM relation.

The operating point is defined by the output back off (OBO) defined as

$$OBO = 20log_{10}\frac{A_{out}}{A_{sat}} \quad (2)$$

where A_{out} is the mean amplitude of the signal at the output of the power amplifier and A_{sat} is the saturation amplitude of the amplifier. A low OBO is required to work in the linear regime but reduces the efficiency of the power amplifier.

The satellite also contains low-pass filters before and after the power amplifier (respectively, imux and omux filters) that can be modelled with a Butterworth response. Half-root Nyquist shaping filters are considered on the ground stations.

Because of the demand for ever increasing bit rates, high symbol rates are used, leading to an increase of the spectral bandwidth. When the bandwidth of the signal becomes comparable or larger than the bandwidth of the imux and omux filters, inter-symbol interferences will occur. In this regime, and with high OBO, the satellite is used more efficiently, but the equalization becomes more difficult.

This communication channel can be described by a Volterra model. This model describing the relation between the transmitted symbols $s(n)$ and the samples at the output of a noiseless channel $r(n)$ is of the form [2]

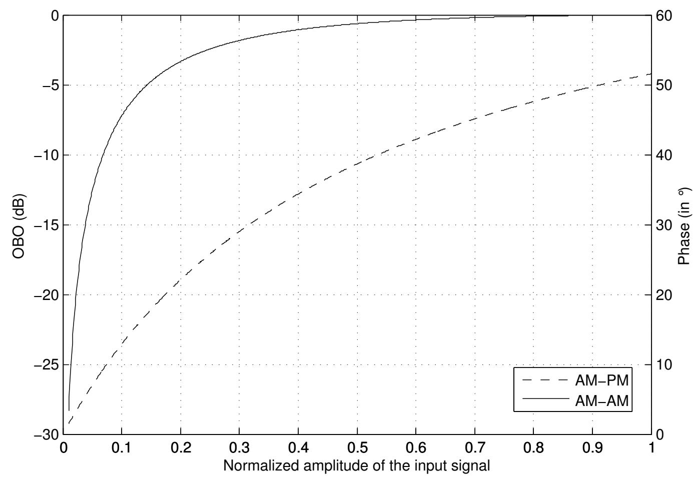

Fig. 2. AM-AM (characterised with the OBO) and AM-PM (characterised with the phase) relations of the power amplifier as a function of the amplitude of the input signal normalised to one for an OBO of 0 dB

$$r(n) = \sum_{p=0} \sum_{n_1=-L'_{2p+1}}^{L_{2p+1}-1} \sum_{n_2=n_1}^{L_{2p+1}-1} \cdots \sum_{n_{p+1}=n_p}^{L_{2p+1}-1} \sum_{n_{p+2}=-L'_{2p+1}}^{L_{2p+1}-1} \cdots \sum_{n_{2p+1}=n_{2p}}^{L_{2p+1}-1} h_{2p+1}(n_1, ..., n_{2p+1}) \prod_{i=1}^{p+1} s(n-n_i) \prod_{j=p+2}^{2p+1} s^*(n-n_j) \quad (3)$$

where $h_{2p+1}(n_1, ..., n_{2p+1})$ are the kernels of the Volterra model and $M_{2p+1} = L'_{2p+1} + L_{2p+1}$ is the size of the memory of order $2p+1$. The restrictions on the summation indices take into account the symmetry of the kernels [17].

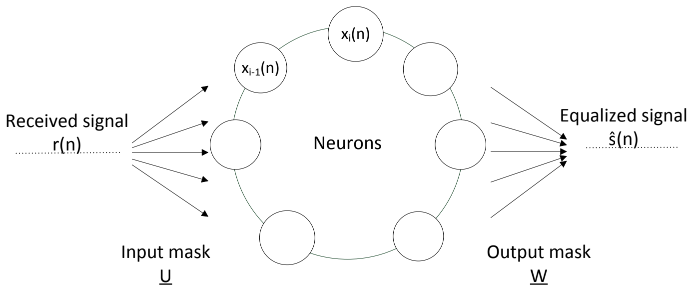

Fig. 3. Structure of the Echo State Network with a ring structure

III. Echo State Networks

Artificial neural networks, such as the recurrent neural network, are often used for linear or non-linear signal processing. Their main source of complexity comes from the adaptation of the different connections which compose the system. The learning task becomes very expensive for large networks [11].

The Echo State Network (ESN) has been proposed to simplify the learning task [6][7][8]. The idea is to keep a recurrent connection between the neurons but the connections between them (inter-connection matrix $\underline{\alpha} = (\alpha_{ij})$) and the connections with the input (input mask $\underline{U} = (u_i)$) are randomly generated. Only the connections with the output (output mask $\underline{W} = (w_i)$) are trained. In this way, the number of connections to adapt is strongly reduced which accelerates the learning task without reducing the performances of the system.

The evolution of an ESN can be defined by:

$$\begin{cases} a_i(n) = \sum_j \alpha_{ij} x_j(n-1) + u_i r(n) \\ x_i(n) = f_{NL}(a_i(n)) \\ \widehat{s}(n) = \sum_i w_i x_i(n) \end{cases} \tag{4}$$

where $a_i(n)$ is the activation signal transmitted to the neuron $x_i(n)$. The non-linear behaviour of the ESN is created with the help of the inter-connection function $f_{NL}(.)$.

The ESN requires the echo state property which specifies that the values of each neuron depend on the past history of the inputs. This property is kept if the spectral radius of the linearised connection matrix between the neurons is lower than 1 [6]. In this way, the network has a fading memory. This condition guarantees the stability of the linearised recurrent network.

It has been shown in [18] that a low complexity structure (which minimizes the number of connections between the neurons) can achieve the same performances as a random inter-connection matrix as initially proposed. In the ESN studied in this paper, we used a circular matrix for the connections between the neurons:

$$\underline{\alpha} = A \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix} \tag{5}$$

where A is the feedback gain. The result is a ring structure as illustrated in Fig. 3. A low feedback gain means that the past history is not relevant in comparison with the actual input. At the opposite, a high feedback gain ensures an important memory of the neurons.

The ESN was first proposed to work with real inputs. In [11], an adaptation of the ESN to work with complex values has been proposed. To make this adaptation, the classical hyperbolic tangent used as activation function has been replaced by a complex function defined by:

$$f_{NL}(a) = th(|a|)e^{j\phi_a} \tag{6}$$

where ϕ_a is the phase of a.

In order to reduce the numerical complexity of the algorithm, we propose a polynomial function of order 3:

$$f_{NL}(a) = a(c_1 + c_3|a|^2) \tag{7}$$

where $c_1 = 0.716$ and $c_3 = -0.0478$. These coefficient have been found empirically to obtain the same results as with the complex hyperbolic tangent. However the amplitude of the input signal must be adapted, through the input mask, in function of the values of the coefficients c_1 and c_3 to ensure sufficient non-linear behaviour and keeping the echo state property of the ESN.

IV. Improvement of the training with the centroids

The non-linear behaviour of the channel creates important inter-symbol interferences but also a compression of the received constellation as we can see in Fig. 4. In a linear communication channel, the cloud of points of the received samples will be centred on the transmitted constellation. But, in a non-linear channel, the compression creates a displacement of the center of the cloud of points, called the centroids [19].

This compression is a memoryless effect. So the position of the centroids can be evaluated from the memoryless part of the Volterra model of the channel. In the case of a 16-QAM constellation, we have 16 centroids in the received constellation. Their positions $(\overline{X}_i)_{i=1}^{16}$ in the complex plan are defined by:

$$\overline{X}_i = H_0(X_i) = \sum_{p=0} h_{2p+1}(0, ..., 0) X_i |X_i|^{2p} \tag{8}$$

where $(X_i)_{i=1}^{16}$ are the positions of the centroids of the transmitted constellation. In practice, it is difficult to evaluate exactly the Volterra model of the channel. So we can evaluate the position of the centroids $\overline{X}_i$ by averaging the N_i received

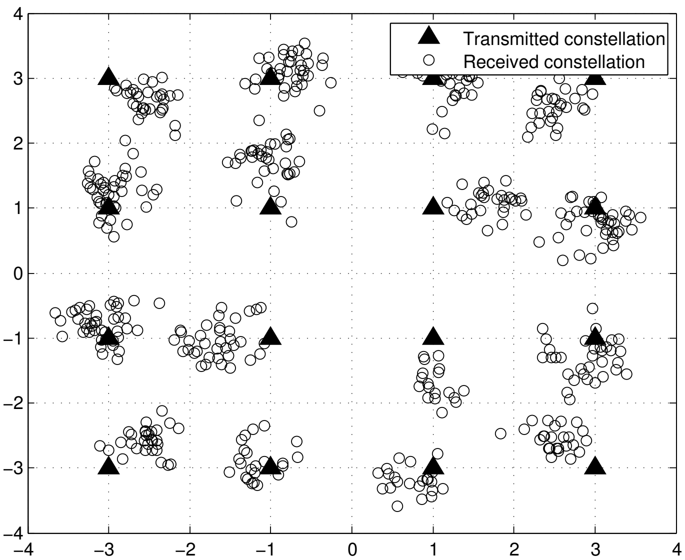

Fig. 4. 16-QAM modulation before and after a noiseless satellite communication channel with an OBO of -1 dB

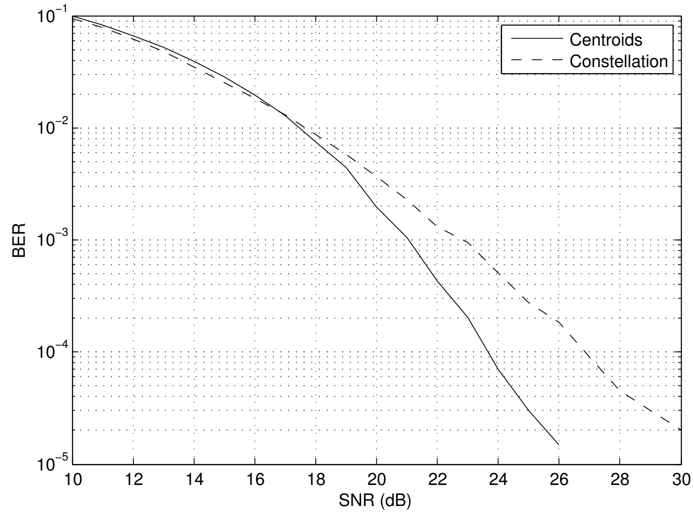

Fig. 5. Evolution of the BER in function of the SNR for two different training strategy for the ESN with 300 neurons and a feedback gain of 0.7

symbols corresponding to X_i over a finite learning sequence $s_T(n)$ of length N [19]:

$$\overline{X}_i = \frac{1}{N_i} \sum_{j=0}^{N-1} r(j|s_T(j) = X_i) \tag{9}$$

where

$$r(j|s_T(j) = X_i) = \begin{cases} 0, & s_T(j) \neq X_i \\ r(j), & s_T(j) = X_i \end{cases} \tag{10}$$

In previous works [4][5][11], the Volterra equalizer and the ESN were trained to minimize the MSE between the estimated sequence $\widehat{s}(n)$ and the transmitted sequence $s(n)$. This means that a part of the complexity of the equalizers is devoted to compensating for the displacement of the centroids. But, as the new positions of the centroids, defined by $\overline{X}_i$, are taken into account for the demapping operation, recovering the initial position of the centroids during the equalization process gives no information gain. Furthermore, this correction requires important displacements of the received symbols which depend on their amplitude. As this last one is affected by the noise, a noise amplification can occur if the equalizer try to compensate for the displacement of the centroids.

Here we propose that the equalizer only compensate for the interferences and ignore the displacement of the centroids. The training sequence to recover is therefore

$$s_T^c(n) = H_0(s_T(n)) \tag{11}$$

A simple look-up table can also be used to replace the symbols of $s_T(n)$ defined on X by the new sequence of symbols $s_T^c(n)$ defined on $\overline{X}$. With such a training sequence, the training algorithm will consider that the positions of the centroids are correct and will only try to compensate for the interferences.

V. Numerical results

In these simulations, we will consider a 16-QAM modulation. The imux and omux filters have a bandwidth of 36 MHz. The roll-off factor of the half-root Nyquist shaping filters on the ground stations is fixed at 0.2. The symbol rate is 33 MHz. The operating point of the power amplifier will be defined by a -1 dB OBO.

The coefficients of the Volterra equalizer and the ESN have been evaluated to minimize the MSE between the estimated sequence and the training sequence. In both case, we consider that the training sequence was long enough to converge to the optimal weight to minimize the MSE. The convergence speed is not studied in this paper. We can see in Fig. 5 that, if the training sequence is defined with the centroids of the received constellation, we can reduce the BER for a SNR higher than 18 dB.

The performances of the Volterra equalizer and the ESN, both trained on the centroids, are compared in Fig.6. We can see that an ESN with 300 neurons is able to reach the same performances as a Volterra equalizer for all SNR. A lower number of neurons implies an important reduction of the performance. The input mask $\underline{u}$ is composed by random complex numbers with uniform phase distribution. Their amplitude is uniformly distributed between $[0; U]$ where U depends on the amplitude of the input signal. The feedback gain of the ESN is equal to 0.8.

The performance of the Volterra equalizer depends on the memory M_{2p+1} allowed for each order of non-linearity $2p+1$. In these simulations, a Volterra equalizer of order 3 has been used with a linear memory of $M_1 = 21$ and an order 3 memory of $M_3 = 7$. A higher non-linear memory gives no improvement.

For the Volterra equalizer, the complexity depends on the number of kernels. For the ESN, we can see that this complexity is proportional to the number of neurons. The main source of

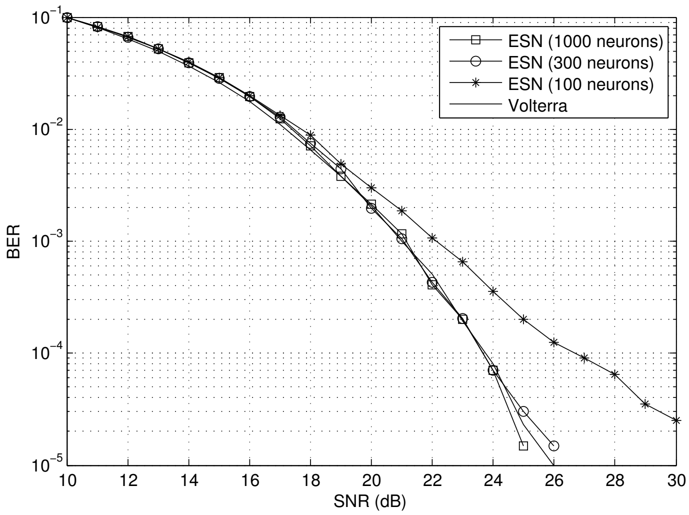

Fig. 6. Comparison of the Volterra equalizer with an ESN for different numbers of neurons, both trained on the centroids

complexity is the activation function. This is why we replaced the hyperbolic tangent by an order 3 polynomial function. If we evaluate the complexity in function of the number of real summations and products, we can observe that both solutions have the same order of complexity. For the parameters of the present simulations, we have complexity ratio of 4 in favour of the Volterra equalizer.

VI. Conclusion

The problem of the equalization of a satellite non-linear channel has been investigated for both the Volterra equalizer and ESN equalizer. An improvement of the training sequence, using the centroids of the received sequence, has been proposed. We showed that, without adding any complexity in the equalization algorithms, a significant performance gain can be achieved. We showed that the ESN can reach the same performances as a Volterra filter but with a slightly higher complexity. The interest of the ESN lies in particular in the possibility of efficient analog implementation.

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