Vector Resource Allocation Problems in Communication Networks

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***Abstract*—We consider a problem of optimal allocation of a homogeneous resource in spatially distributed systems such as communication networks, where both utilities of users and network expenses must be taken into account. The network is divided into zones which leads to a two-level vector optimization problem and involves non-differentiable functions whose values are computed algorithmically. We propose several approaches to find a solution. Also, new simple subgradient type methods for non-differentiable Pareto optimization problems are suggested. Their performance is illustrated by computational results on test problems.**

***Index Terms*—Resource allocation, spatial systems, communication networks, multi-objective optimization, non-differentiable functions, subgradient methods.**

I. Introduction

The necessity of distribution of limited resources in spatially distributed systems where locations of elements must be taken into account arises now in various applied problems related to contemporary telecommunications, for instance, in such emerging technologies as mobile ad-hoc networks and sensor networks; see e.g. [1], [2]. Mostly, the decision making procedures here are based on solutions of the corresponding scalar optimization problems. As a rule, they are often represented by high-dimensional linear programming problems which admit various decomposition approaches. However, the contemporary models of telecommunication systems lead to complex nonlinear high-dimensional problems. Moreover, experience of dealing with these very complicated systems usually shows that a proper decomposition/clustering approach, which can be based on zonal, time, frequency and other attributes of nodes/units, might be very efficient here; see e.g. [3], [4]. Also, due to the presence of different factors, which must be taken into account, multi-objective optimization problems seem more suitable to their formulation. In this paper, we consider one of such problems of optimal allocation of resources in spatially distributed systems. Its criteria involve non-differentiable functions whose values are computed algorithmically. In the scalar case, such problems can be solved by the usual optimization methods; see [5], [6]. We first describe a right-hand side decomposition approach for the initial problem and then propose several subgradient type methods to define its solution. We present also results of computational experiments which confirm the applicability of the new methods.

II. Notation and the problem statement

Let us consider a network with nodes (attributed to users) which is divided into zones (clusters). The problem of a manager of the network is to find an optimal allocation of a limited homogeneous resource (e.g., the bandwidth in telecommunication networks) among the zones. In what follows we use the following notation:

- n is the number of zones;
- I_k is the index set of nodes (currently) located in zone k ($k = 1, \ldots, n$);
- R is the total resource supply (the total bandwidth) for the system;
- x_k is an unknown quantity of the resource allotted to zone k and $w_k(x_k)$ is the cost of implementation of this quantity of the resource for zone k ($k = 1, \ldots, n$);
- z_j is the unknown resource amount allotted to node j and $U_j(z_j)$ is the utility of node j from the resource consumption value z_j;
- $\alpha_j \geq 0$ and $\beta_j < +\infty$ are, correspondingly, the lower and upper bounds for z_j ($j \in I_k,\ k = 1, \ldots, n$).

For each k we denote by $f_k(x_k)$ the optimal value of the zonal optimization problem:

$$\max \to \sum_{j \in I_k} U_j(z_j) \tag{1}$$

subject to

$$\sum_{j \in I_k} z_j \leq x_k, \tag{2}$$

$$\alpha_j \leq z_j \leq \beta_j,\ j \in I_k. \tag{3}$$

Hence, $f_k(x_k)$ determines the total utility of consumers of zone k if the resource value for this zone equals x_k. However, the implementation expenses $w_k(x_k)$ for providing this quantity of the resource for zone k must be taken into account by the manager.

Therefore, the upper-level manager problem consists in maximizing the total network utility and in minimizing the

total network implementation expenses by means of the optimal resource allocation:

$$\max \rightarrow \sum_{k=1}^{n} f_k(x_k) \tag{4}$$

and

$$\min \rightarrow \sum_{k=1}^{n} w_k(x_k) \tag{5}$$

subject to

$$\sum_{k=1}^{n} x_k \leq R, \tag{6}$$

$$x_k \geq 0,\ k = 1, \ldots, n. \tag{7}$$

Clearly, criteria in (4) and (5) need not be homogeneous in general. Hence, we have obtained a two-objective optimization problem. Note that both the separate scalar optimization problems are separable, however, each function f_k can be non-differentiable and their values are computed algorithmically without any explicit formula. If we set $w_k \equiv 0$, the above problem corresponds to the master one in the so-called right-hand side decomposition approach; see e.g. [7] and the references therein. That is, the main problem is then the following:

$$\max \rightarrow \sum_{k=1}^{n} \sum_{j \in I_k} U_j(z_j)$$

subject to

$$\sum_{k=1}^{n} \sum_{j \in I_k} z_j \leq R,$$
$$\alpha_j \leq z_j \leq \beta_j,\ j \in I_k,\ k = 1, \ldots, n.$$

However, the index sets I_k and user coefficients α_j, β_j may be unknown to the upper level network manager, especially in ad hoc wireless networks, and he/she should be oriented on the total zonal utility depending on the resource share x_k. This justifies usefullness of this right-hand side decomposition even in the scalar case.

Nevertheless, solution of problem (4), (6), (7) may meet serious computational difficulties. In order to create efficient solution methods for the upper-level manager problem we should first define a suitable solution concept and then derive additional properties of cost functions.

III. Basic optimization formulations

We first introduce the following natural assumptions. Let

$$\sum_{k=1}^{n} \sum_{j \in I_k} \alpha_j \leq R,$$

let the function U_j be concave and continuous. Then problem (1)–(3) has a solution, i.e., the value $f_k(x_k)$ is defined if $\min\limits_{j \in I_k} \alpha_j \leq x_k$. It seems also reasonable to suppose that U_j is monotone, i.e., $U_j(\alpha_j) \leq U_j(\beta_j),\ \ j \in I_k$. Moreover, it is easy to see that f_k is concave on $\mathbb{R}_+$, and then (4), (6), (7) becomes a scalar concave maximization problem, however, f_k need not be differentiable. Also, if we suppose also that each cost function w_k is convex on $\mathbb{R}_+$, then (5), (6), (7) becomes a scalar convex minimization problem.

We can apply the following approaches to formulation of the optimality concept for problem (4)–(7).

1) Replace the multi-objective problem (4)–(7) by its scalarization via assigning weights, say, $\gamma_1 > 0$ and $\gamma_2 > 0$ to both criteria. This approach leads to the concave maximization problem:

$$\max \rightarrow \sum_{k=1}^{n} (\gamma_1 f_k(x_k) - \gamma_2 w_k(x_k)) \tag{8}$$

subject to (6), (7). Clearly, the determination of these weights may be rather difficult in the case where the criteria in (4) and (5) are not homogeneous.

2) If there exists the maximal (or desirable) total cost limit C, one can replace the minimization criterion in (5) with the constraint

$$\sum_{k=1}^{n} w_k(x_k) \leq C, \tag{9}$$

thus obtaining the scalar concave maximization problem (4), (6), (9), (7).

3) The general approach consists in considering the Pareto optimization problem:

$$\max_{\succ} \rightarrow F(x) = \sum_{k=1}^{n} F^{(k)}(x_k) \tag{10}$$

subject to (6), (7), where $x = (x_1, \ldots, x_n)^T$, $F^{(k)}(x_k) = (f_k(x_k), -w_k(x_k))^T$, $k = 1, \ldots, n$, $\succ$ denotes the usual Pareto relation:

$$x \succ y \iff x_i \geq y_i\ \forall i \text{ and } x \neq y.$$

That is, the problem is to find a feasible vector x^* such that there does not exist any other feasible vector x' such that $F(x') \succ F(x^*)$; see e.g. [8].

First we observe that both the scalar optimization problems above are separable, but involve non-differentiable functions. Hence, we can apply suitable convex non-differentiable optimization methods to find solutions of both (8), (6), (7) and (4), (6), (9), (7); see e.g. [7], [9]. Also, we can combine these methods with the dual Lagrangian methods. In this case, we have a small-dimensional basic problem and several inscribed nonlinear optimization problems for computation of components of its cost function. Nevertheless, they are then highly parallelizable problems.

If we follow the basic Pareto approach, we also should take into account the non-differentiability of the functions f_k. The corresponding methods were presented e.g. in [10], [11] (see also the references therein). Within this approach, we intend to utilize the simplest relaxation subgradient method for convex minimization (see [12] and also [13], [9]) as a basis for construction of new methods. The description and substantiation of these methods will be presented in Section V.

IV. Adjustment for the case of moving nodes

In the above statement the numbers of nodes located at each zone are assumed to be known and unchangeable (within the planning time interval). However, the latter trends in telecommunications industry are aimed at data transmission without any heavy and expensive equipment. Instead, mobile devices are used both as receivers and transmitters of signals. In the so-called ad hoc networks, there is no stationary infrastructure. Then nodes locations are ever-changing. However, following the approach suggested by I. Konnov (see e.g. [14]), we can treat each node as a Markovian chain. This allows us to evaluate the probability p_{jk} for each node node j to stay in zone k within some planning time interval.

We now denote by I_k the set of indices of nodes whose probabilities to be in zone k are positive (or greater than a certain positive threshold). Next, for each k we denote by $f_k(x_k)$ the optimal value of the modified optimization problem:

$$\max \rightarrow \sum_{j \in I_k} p_{jk} U_j(z_j) \tag{11}$$

subject to

$$\sum_{j \in I_k} z_j \leq x_k, \tag{12}$$

$$\alpha_j \leq z_j \leq \beta_j, \ j \in I_k. \tag{13}$$

Of course, each function f_k is again concave. So, we can replace (1)–(3) with (11)–(13). Afterwards, we can utilize the same basic problem (4)–(7), and (4), (6), (7) is also a concave maximization problem.

V. Relaxation subgradient methods for Pareto optimization problems

For description of the methods we replace problem (10), (6), (7) by a more general formulation. For vectors $a, b \in \mathbb{R}^m$, together with the standard Pareto preference relation above we will utilize also its strict version:

$$a > b \Longleftrightarrow a_i > b_i \quad \forall i.$$

Let $\varphi_i : \mathbb{R}^n \rightarrow \mathbb{R}$, $i = 1, \ldots, m$, be concave functions and let $h : \mathbb{R}^n \rightarrow \mathbb{R}$ be a concave function. Then we can define the set $D = \{x \in \mathbb{R}^n \mid h(x) \geq 0\}$ which is convex and closed. Set $\varphi(x) = (\varphi_1(x), \ldots, \varphi_m(x))$. In what follows we suppose that

(A) *There exists a point $\bar{x} \in \mathbb{R}^n$ such that $h(\bar{x}) > 0$ (regularity).*

Let us consider the corresponding Pareto maximization problems:

$$\max_{\succ} \rightarrow \varphi(x) \quad \text{subject to} \quad x \in D \tag{14}$$

and

$$\max_{>} \rightarrow \varphi(x) \quad \text{subject to} \quad x \in D. \tag{15}$$

That is, a point x^* is a solution of problem (14) if no point $x \in D$ exists such that $f(x) \succ f(x^*)$. Analogously, a point $\tilde{x}$ is a solution of problem (15) if no point $x \in D$ exists such that $f(x) > f(\tilde{x})$. That is, problem (15) yields weak Pareto solutions. We denote by D_P^* and D_W^* the solution sets of problems (14) and (15), respectively. Clearly, $D_P^* \subseteq D_W^*$. Also, (10), (6), (7) is a particular case of (14) where $\varphi(x) = F(x)$, $h(x) = \min\{R - \sum_{k=1}^{n} x_k, x_1, \ldots, x_n\}$, $m = 2$.

We recall that the subdifferential $\partial\mu(x)$ of a convex function $\mu : \mathbb{R}^n \rightarrow \mathbb{R}$ at x is defined as follows:

$$\partial\mu(x) = \{g \mid \mu(y) - \mu(x) \geq \langle g, y - x\rangle \ \forall y \in \mathbb{R}^n\}.$$

For a concave function $\phi : \mathbb{R}^n \rightarrow \mathbb{R}$ we can define its subdifferential at x as follows:

$$\partial^c\phi(x) = \{g \mid \phi(y) - \phi(x) \leq \langle g, x - y\rangle \ \forall y \in \mathbb{R}^n\},$$

i.e., $\partial^c\phi(x) = -\partial(-\phi)(x)$.

In order to solve Pareto optimization problems (14) and (15), we can utilize the following bifunction:

$$\Phi(x, y) = min\{\varphi_1(y) - \varphi_1(x), \ldots, \varphi_m(y) - \varphi_m(x), h(y)\}.$$

Note that Φ is concave in y, $\Phi(x, x) = 0$ for all $x \in D$. So, we can consider the scalar equilibrium problem: Find $x^* \in D$ such that

$$\Phi(x^*, y) \leq 0 \quad \forall y \in \mathbb{R}^n. \tag{16}$$

We denote by D^e the solution set of this problem.

Lemma 1: It holds that

$$D_P^* \subseteq D_W^* = D^e. \tag{17}$$

Proof. Suffice it to prove the equality in (17). Let $x^* \in D_W^*$, but $x^* \notin D^e$. Then there exists $z \in \mathbb{R}^n$, $\Phi(x^*, z) > 0$, hence $h(z) > 0$ and $\varphi_i(z) > \varphi_i(x^*)$, $i = 1, \ldots, m$. It follows that $z \in D$ and $\varphi(z) > \varphi(x^*)$. This is a contradiction, therefore, $D_W^* \subseteq D^e$.

Conversely, let $x^* \in D^e$, but $x^* \notin D_W^*$. Then there exists $z \in D$ such that $\varphi(z) > \varphi(x^*)$. Set $z(\alpha) = \alpha\bar{x} + (1 - \alpha)z$, then due to the concavity of h we have

$$h(z(\alpha)) \geq \alpha h(\bar{x}) + (1 - \alpha)h(z) > 0 \text{ for } \alpha \in (0, 1).$$

Similarly, due to the concavity of φ_i, for $\alpha' \in (0, 1)$ small enough we have

$$\varphi_i(z(\alpha')) \geq \alpha' \varphi_i(\bar{x}) + (1 - \alpha')\varphi_i(z) > \varphi_i(x^*).$$

For $\bar{z} = \alpha'\bar{x} + (1-\alpha')z$ we obtain $h(\bar{z}) > 0$ and $\varphi(\bar{z}) > \varphi(x^*)$. This means that $\Phi(x^*, \bar{z}) > 0$, which is a contradiction. Hence, $D^e \subseteq D_W^*$. □

So we intend to find a solution of problem (16) by a relaxation subgradient method which combines the ideas of methods from [12], [13], [10].

Lemma 2: A point $x^* \in D$ is a solution of problem (16) if and only if

$$\mathbf{0} \in \partial_y^c \Phi(x^*, y)|_{y=x^*}. \tag{18}$$

In fact, (16) means that

$$\Phi(x^*, x^*) = \sup_{y \in \mathbb{R}^n} \Phi(x^*, y)$$

and (18) represents the usual necessary and sufficient optimality condition for this problem.

We shall also use the following condition:

(B) *There exists an index l and a point $\tilde{x} \in D$ such that the set $D_l(\tilde{x}) = \{x \in D \mid \varphi_l(x) \geq \varphi_l(\tilde{x})\}$ is bounded.*

From ***(B)*** it follows that

$$\varphi_l^* = \sup_{x \in D} \varphi_l(x) < +\infty.$$

Moreover, take a fixed point $\bar{z} \in \mathbb{R}^n$, then the function $\Phi(\bar{z}, y)$ is bounded from above in y. In fact, if $y \notin D$, then

$$\Phi(\bar{z}, y) \leq h(y) < 0.$$

Otherwise, if $y \in D$, then

$$\Phi(\bar{z}, y) \leq \varphi_l(y) - \varphi_l(\bar{z}) \leq \varphi_l^* - \varphi_l(\bar{z}) < +\infty.$$

Besides, from ***(B)*** we have that there exists a point $\tilde{x} \in D$ such that $\varphi_l^* = \varphi_l(\tilde{x})$, then $\tilde{x} \in D_W^* = D^e$.

The basic part of the relaxation subgradient method applied to problem (16) can be described as follows. For brevity, set $\phi_k(y) = \Phi(x^k, y)$. We denote by $\mathrm{Nr}X$ the element of X nearest to origin.

Algorithm 1. Input: a point $x^0 \in D$, numbers $\alpha > 0$, $\eta > 0$. **Parameter:** a number $\theta \in (0, 1)$. **Output:** a point $\tilde{z}$.
Step 0. Set $k = 0$.
Step 1. Choose $g^0 \in \partial^c \phi_k(x^k)$, set $i = 0$, $p^i = g^i$.
Step 2. If $\|p^i\| \leq \eta$, set $\tilde{z} = x^k$ and stop. Otherwise set $x^{k,i+1} = x^k + \alpha\, p^i / \|p^i\|$.
Step 3. If

$$\phi_k(x^{k,i+1}) \geq \theta \alpha \|p^i\|, \tag{19}$$

set $x^{k+1} = x^{k,i+1}$, $k = k + 1$ and go to **Step 1**.
Step 4. Choose $g^{i+1} \in \partial^c \phi_k(x^{k,i+1})$, set $p^{i+1} = \mathrm{Nr}\,\mathrm{conv}\{p^i, g^{i+1}\}$, $i = i + 1$, and go to **Step 2**.

Note that $x^k \in D$, hence $\tilde{z} \in D$ as well. We will call one change of index i in Algorithm 1 as an inner step. By construction, its expenses per step are similar to those in the usual (non relaxation) subgradient method (see e.g. [7], [9]), i.e. it requires no linear searches and no a priori constants to be evaluated.

Lemma 3: If **(A)** and **(B)** hold, then the number of inner steps in Algorithm 1 is finite.

PROOF. Suppose that k is fixed, but $i \to +\infty$. Then (19) does not hold and

$$\phi_k(x^{k,i+1}) < \phi_k(x^k) + \theta \alpha \|p^i\|,$$

but

$$\begin{aligned} &\phi_k(x^k) - \phi_k(x^{k,i+1}) \leq \langle g^{i+1}, x^k - x^{k,i+1} \rangle \\ &= -\alpha \langle g^{i+1}, p^i \rangle / \|p^i\|. \end{aligned}$$

It follows that

$$\alpha \langle g^{i+1}, p^i \rangle / \|p^i\| \leq \theta \alpha \|p^i\|,$$

i.e.,

$$\langle g^{i+1}, p^i \rangle \ \leq \theta \|p^i\|^2.$$

Hence, the sequence $\{p^i\}$ satisfies the known decreasing property (see, e.g., [12], [13])

$$\|p^i\| \leq \frac{C}{(1-\theta)\sqrt{i+1}},$$

where $C \geq \|g^i\|$, $i = 0, 1, \ldots$ It follows that

$$i \leq \left(\frac{C}{(1-\theta)\eta} \right)^2 + 1.$$

Next, if $k \to +\infty$, then by (19),

$$\Phi(x^k, x^{k+1}) = \phi_k(x^{k+1}) \geq \theta \alpha \|p^i\| \geq \theta \alpha \eta > 0.$$

Therefore, $x^{k+1} \in D$ and

$$\begin{aligned} \varphi_l(x^{k+1}) - \varphi_l(x^k) &\geq \Phi(x^k, x^{k+1}) \\ &\geq \Phi(x^k, x^k) + \theta \alpha \eta = \theta \alpha \eta. \end{aligned} \tag{20}$$

We now obtain

$$\varphi_l^* \geq \varphi_l(x^k) \geq \varphi_l(x^0) + k \theta \alpha \eta,$$

or

$$k \leq \frac{\varphi_l^* - \varphi_l(x^0)}{\theta \alpha \eta}.$$

Thus, the total number of inner steps must be finite. □

Set

$$G(x, \alpha) = \mathrm{conv} \bigcup_{y \in B(x, \alpha)} \partial_y^c \Phi(x, y),$$

where $B(x, \alpha) = \{z \mid \|z - x\| \leq \alpha\}$. Since the subdifferential mapping of a concave function is upper semicontinuous (u.s.c.), using the properties of compositions of u.s.c. mappings, we conclude that the mapping $(x, \alpha) \mapsto G(x, \alpha)$ is also u.s.c. on $\mathbb{R}^n \times [0, +\infty)$.

Now we describe the construction of a converging iteration sequence.

Basic Scheme. Choose a point $z^0 \in D$, sequences $\{\alpha_s\} \searrow 0$, $\{\eta_s\} \searrow 0$, and a number $\theta \in (0, 1)$.
For each $s = 1, 2, \ldots$, we have a point $z^{s-1} \in D$, numbers α_s and η_s, apply Algorithm 1 with $x^0 = z^{s-1}$, $\alpha = \alpha_s$, $\eta = \eta_s$, and set $z^s = \tilde{z}$.

Theorem 1: It **(A)** and **(B)** hold, then the sequence $\{z^s\}$ has limit points and all these limit points are solutions of problem (16) or, equivalently, (15).

PROOF. First we note that (20) gives

$$z^s \in D \quad \text{and} \quad \varphi_l(z^{s+1}) \geq \varphi_l(z^s),$$

but the set $D_l(z^0)$ must be bounded, hence $\{z^s\}$ is a bounded sequence and has limit points. Next, by construction, for each s we have

$$\|d^s\| \leq \eta_s,$$

where $d^s \in G(z^s, \alpha_s)$. If $\bar{z}$ is a limit point of $\{z^s\}$, then $\bar{z} \in D$ and

$$\mathbf{0} \in G(\bar{z}, 0) = \partial_y^c \Phi(\bar{z}, y) \mid_{y = \bar{z}}.$$

On account of Lemma 2 we obtain that $\bar{z} \in D^e$ and the result follows. □

We can also utilize a modification of Algorithm 1 with explicit usage of constraints. Set

$$\Psi(x,y) = min\{\varphi_1(y) - \varphi_1(x), \ldots, \varphi_m(y) - \varphi_m(x)\}$$

and $\psi_k(y) = \Psi(x^k, y)$.

Algorithm 2. Input: a point $x^0 \in D$, numbers $\alpha > 0$, $\eta > 0$. **Parameter:** a number $\theta \in (0,1)$. **Output:** a point $\tilde{z}$.
Step 0. Set $k = 0$.
Step 1. Choose $g^0 \in \partial^c \psi_k(x^k)$, set $i = 0$, $p^i = g^i$.
Step 2. If $\|p^i\| \leq \eta$, set $\tilde{z} = x^k$ and stop. Otherwise set $x^{k,i+1} = x^k + \alpha\, p^i/\|p^i\|$.
Step 3. If

$$\psi_k(x^{k,i+1}) \geq \theta\alpha\|p^i\|,\ h(x^{k,i+1}) \geq 0,$$

set $x^{k+1} = x^{k,i+1}$, $k = k+1$ and go to **Step 1**.
Step 4. Choose

$$g^{i+1} \in \begin{cases} \partial^c \psi_k(x^{k,i+1}) & \text{if } h(x) \geq 0, \\ \partial^c h(x^{k,i+1}) & \text{if } h(x) < 0, \end{cases}$$

set $p^{i+1} = \mathrm{Nr}\,\mathrm{conv}\{p^i, g^{i+1}\}$, $i = i+1$, and go to **Step 2**.

The creation of the main iteration sequence and substantiation of Algorithm 2 are similar to those of Algorithm 1 and omitted.

VI. Computational experiments

We first tested the efficiency of the above methods on the following two-criteria problem:

$$\max_{\succ} \to \{\varphi_1(x),\ \varphi_2(x)\}$$

subject to

$$x_1 + x_2 \leq 12,\ x_1 \geq 0,\ x_2 \geq 0.$$

Here $\varphi_1(x) = -(x_1-15)^2 - x_2^2 + 225$, $\varphi_2(x) = -(x_1+15)^2 - x_2^2 + 225$. Evidently, the set of Pareto-optimal solutions for this problem is the segment $[(0,0);(12,0)]$. We performed 100 experiments with the basic scheme utilizing Algorithm 1, randomly choosing initial points from the square $[0;20] \times [0;20]$ (and deleting the points that do not belong to the admissible set). We chose the initial values of parameters as follows: $\alpha_0 = 1.5, \eta_0 = 0.01$ and put $\alpha_s = \alpha_0/s$, $\eta_s = \eta_0/s$ for each $s \geq 1$; we fixed $\theta = 0.9$. For the stopping criterion we used the following rule: the time consumed exceeds 0.5 seconds and/or the neighboring points of the constructed sequence are sufficiently close (the Euclidean norm of their difference does not exceed 0.1). See Table I for results of the first 10 tests (the rest results are analogous).

In the next series of 100 tests we used the same initial values of parameters $\alpha_0 = 1.5$ and $\eta_0 = 0.01$ tending them to zero in a geometric rate with the common ratio $q = 1/1.3$. See Table II for results of the first 10 tests (the rest results are analogous).

We have also performed 100 numerical tests for solving the same problem by the basic scheme utilizing Algorithm 2. We used the same stopping criterion and the same initial values of parameters. We also varied the parameters as harmonic or

TABLE I
Results for Algorithm 1 with $\alpha_s = \alpha_0/s, \eta_s = \eta_0/s$

N	Initial point	Appr. solution
1	(3.16, 7.78)	(0.43, 0.30)
2	(0.06, 6.83)	(0.00, 0.08)
3	(1.04, 8.26)	(0.12, 0.07)
4	(4.52, 4.34)	(2.11, 0.03)
5	(1.13, 9.95)	(0.08, 0.01)
6	(6.06, 2.91)	(4.51, 0.02)
7	(5.33, 5.98)	(1.16, 0.01)
8	(3.77, 6.78)	(0.67, 0.04)
9	(1.28, 8.44)	(0.12, 0.02)
10	(6.83, 2.86)	(5.27, 0.04)

TABLE II
Results for Algorithm 1 with $\alpha_s = q^s\alpha_0, \eta_s = q^s\eta_0$

N	Initial point	Appr. solution
1	(9.55, 1.21)	(9.49, 0.05)
2	(3.80, 6.45)	(0.71, 0.34)
3	(1.02, 10.43)	(0.04, 0.09)
4	(2.21, 6.70)	(0.32, 0.07)
5	(2.59, 4.22)	(0.69, 0.12)
6	(1.73, 6.82)	(0.23, 0.08)
7	(4.24, 7.13)	(0.63, 0.03)
8	(3.43, 1.15)	(3.36, 0.00)
9	(8.03, 1.28)	(7.95, 0.13)
10	(9.68, 1.16)	(9.62, 0.01)

TABLE III
Results for Algorithm 2 with $\alpha_s = \alpha_0/s, \eta_s = \eta_0/s$

N	Initial point	Appr. solution
1	(4.76, 6.45)	(0.96, 0.14)
2	(0.94, 3.75)	(0.23, 0.09)
3	(0.08, 4.49)	(0.00, 0.00)
4	(2.40, 7.13)	(0.34, -0.04)
5	(2.90, 6.49)	(0.42, 0.02)
6	(8.57, 2.86)	(6.97, 0.15)
7	(4.18, 3.10)	(2.76, -0.02)
8	(2.49, 4.44)	(0.78, 0.35)
9	(1.01, 6.69)	(0.13, 0.01)
10	(2.92, 2.57)	(1.58, 0.10)

TABLE IV
Results for Algorithm 2 with $\alpha_s = q^s\alpha_0, \eta_s = q^s\eta_0$

N	Initial point	Appr. solution
1	(1.94, 2.74)	(0.90, 0.03)
2	(5.77, 5.73)	(1.33, 0.01)
3	(3.41, 5.34)	(0.80, 0.06)
4	(9.52, 1.55)	(9.41, 0.05)
5	(4.03, 3.50)	(1.58, 0.05)
6	(0.30, 4.51)	(0.00, 0.02)
7	(6.03, 5.71)	(1.47, 0.09)
8	(2.18, 2.75)	(1.05, 0.09)
9	(6.43, 5.53)	(1.70, 0.10)
10	(2.42, 8.70)	(0.25, -0.01)

geometric series (see Tables III and IV, respectively, for the results of the first 10 tests).

According to the performed numerical experiments, both the algorithms give points close to Pareto optimal ones within 0.5 seconds independently of the initial point and their results are rather similar.

Next, due to the similarity of results, we applied Algorithm 1 to several vector two-level problems of form (10), (6), (7), where all the utility functions were chosen quadratic and strongly concave and all the implementation expense functions were chosen linear, i.e. $w_k(x_k) = w_k x_k$, where $w_k > 0$ for all k. We chosen value $10 \times n$ (sec) of the processor time as stopping criterion. The results for four examples are given in Tables V– VIII. The obtained points also approximated solutions of the scalarized problem (8), (6), (7) with $\gamma_1 = \gamma_2 = 1$ computed by scalar optimization methods.

TABLE V
TWO-LEVEL PROBLEM 1: 2 ZONES, 10 NODES, $R = 210$

zone	nodes	w_k	x_k	utility	expense
1	7	1	99.1227	4915.37	99.1227
2	3	5	23.1227	1087.58	115.964
total	10		122.245	6002.95	214.736

TABLE VI
TWO-LEVEL PROBLEM 2: 3 ZONES, 12 NODES, $R = 295$

zone	nodes	w_k	x_k	utility	expense
1	3	8	63.2889	2718.37	506.311
2	5	1	87.96	3793.11	87.96
3	4	3	37.6608	3127.71	112.982
total	10		188.91	9639.19	707.254

TABLE VII
TWO-LEVEL PROBLEM 3: 6 ZONES, 20 NODES, $R = 474$

zone	nodes	w_k	x_k	utility	expense
1	3	8	34.1179	1795.36	272.943
2	4	1	76.1179	3091.89	76.1179
3	8	3	64.1179	6581.25	192.354
4	1	9	11.1179	204.356	100.061
5	1	12	10.1179	497.233	121.415
6	3	6	46.1179	2649.37	276.707
total	20		241.707	14819.5	1039.6

VII. CONCLUSIONS

In this work, we considered a vector problem of managing limited resources in a network. We proposed a new decomposition approach, which extends the so-called right-hand side decomposition one. In order to solve the problem obtained we suggested some subgradient type methods whose efficiency is confirmed by the results of numerical experiments.

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TABLE VIII
TWO-LEVEL PROBLEM 4: 12 ZONES, 24 NODES, $R = 559$

zone	nodes	w_k	x_k	utility	expense
1	2	8	32.1877	1546.13	257.502
2	3	1	46.1877	1257.26	46.1877
3	2	3	42.1877	2022.44	126.563
4	3	9	30.1877	1090.31	818.617
5	1	12	24.1877	1817.43	290.253
6	1	6	36.1877	1202.45	217.126
7	2	8	32.1877	1521.13	257.502
8	3	12	18.1877	240.783	218.252
9	2	8	32.1877	1051.16	257.502
10	3	2	44.1877	1990.72	88.3755
11	1	7	4.1877	29.3633	29.3139
12	1	9	30.1877	2886.03	271.69
total	24		372.253	16655.2	2331.96

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