

You Snooze, You Lose: Minimizing Channel-Aware Age of Information

Bhishma Dedhia and Sharayu Moharir

Department of Electrical Engineering

Indian Institute of Technology Bombay

Abstract—We propose a variant of the Age of Information (AoI) metric called Channel-Aware Age of Information (CA-AoI). Unlike AoI, CA-AoI takes into account the channel conditions between the source and the intended destination to compute the “age” of the recent most update received by the destination.

We design scheduling policies for multi-sensor systems in which sensors report their measurements to a central monitoring station via a shared unreliable communication channel with the goal of minimizing the time-average of the weighted sum of CA-AoIs. We show that the scheduling problem is indexable and derive low complexity Whittle index based scheduling policies. We also design randomized scheduling algorithms and give optimization procedures to find the optimal parameters of the policy. Via simulations, we show that our proposed policies surpass the greedy policy in several settings. Moreover the Whittle Index based scheduling policies outperform other policies in all the settings considered.

I. INTRODUCTION

With the advent of Internet of Things (IoT), multi-sensor systems have become increasingly ubiquitous. They are used to make control decisions in systems ranging from smart homes to vehicular networks. Sensors in these systems measure time varying quantities and communicate with a central monitoring station. It is crucial that the monitoring station receives timely updates from the sensors to make critical decisions.

In this work, we consider n sensors that measure real time updates and update a monitoring station through a shared unreliable communication channel. Time is divided into slots. We model the channels as ON-OFF channels that have independent realizations across sensors and time slots. In each time slot, at most one source is allowed to communicate with the monitoring station through the communication channel. The goal is to design a scheduling policy that optimizes a freshness objective and hence reduces the staleness in information.

Age of Information (AoI) has emerged as a popular metric to measure freshness of information at a destination and is defined as the time elapsed since the last update from a source. It grows linearly with time and resets to zero when an update is received from a source. AoI minimization has been studied extensively for scheduling applications in [1]-[10]. In a multi-sensor setting with a shared channel, the AoI of a sensor increases due to two reasons. The first is when an attempted update by a sensor fails due to poor channel conditions and the second is when the shared channel is used by some other sensor. The original AoI metric does not distinguish between these two events. It is evident that the first event is fundamental

and is caused by the physics of the problem and the second event is a resource allocation issue and therefore, controllable by the algorithm designer. Given this, we propose a variant of the AoI which distinguishes between these two events by imposing a penalty by increasing the “age” of the information of a sensor at the monitoring station only in the latter case. We refer to this metric as the *Channel-Aware Age of Information* (CA-AoI) and by definition, the CA-AoI of a sensor is a measure of the number of missed opportunities to send an update since its recent most successful update.

The goal of our work is to design scheduling policies to minimize the time-average of the weighted sum of CA-AoIs of the sensors. The following example highlights the difference in schedules designed to minimize the time-average of the sum of the AoIs vs. the CA-AoIs. Consider a two sensor system with equal weights such that the first sensor has near reliable channel conditions and the second sensor has an extremely poor channel connection. On average, the AoI of the second sensor will be higher than the AoI of the first sensor irrespective of the fraction of times it is scheduled for transmission. As a result, a scheduling algorithm designed to optimize for AoI will allocate greater resources to the second sensor in spite of the system conditions limiting its communication capacity. A scheduling algorithm designed to optimize for CA-AoI will thus be fairer across the two sensors.

Optimizing this objective has several challenging aspects. For most sources the channel state information (CSI) is unknown. Hence the scheduler needs to guess when the channel state is ON so that the source can push an update. On the other hand, for some sources CSI is known before every time-slot. But even with CSI, designing a scheduling policy is highly nontrivial because the scheduler is allowed to schedule only a single source in every time slot. Computing the optimal policy using dynamic programming is difficult for such problems as the state-space grows exponentially with the number of sources. Hence low complexity policies are required that can make this constrained scheduling problem computationally efficient.

A. Our Contributions

We consider three cases, namely, systems without CSI, systems with CSI, and systems with partial CSI.

- We propose a new AoI-like freshness metric called Channel Aware-Age of Information (CA-AoI). Unlike vanilla AoI, CA-AoI is not agnostic of channel reliability.

- We view the scheduling problem through the lens of a restless multi-armed bandit framework (RMAB) and prove that all three problem instances are indexable. We derive structural properties for the optimal policy. Therefore we calculate closed form expressions for the Whittle index and propose low complexity scheduling algorithms.
- We propose a suite of randomized scheduling algorithms to optimize the objective for all three problem instances. We find closed form expressions for the cost under these policies and thus optimal parameters for the algorithms.
- We compare the performance of the Whittle index and randomized policies against the greedy policy via simulations. We show the efficacy of our scheduling algorithms and conclude the superiority of the Whittle index policy in diverse settings.

B. Related Work

As mentioned above, there exists a rich body of work that uses AoI minimization in scheduling applications [1]-[10]. Several existing works model the scheduling problem as restless bandits and use Whittle index policies to solve them. The work in [6] derives the Whittle index for AoI minimization in a system having iid/Markovian channels and multiple information settings (without CSI/ with CSI/ delayed CSI). In [10], the authors use Whittle’s methodology for the setting where realization of the stochastic arrival process for a given time-slot is known apriori. In [9], the focus is on designing Whittle index policies for AoI minimization when the system has throughput constraints. In [8], the authors consider Whittle index policies to minimize non-decreasing functions of AoI and prove optimality of the index policy for a system with 2 sources¹ and reliable channels. Very recently in [11], the authors focus on scheduling a web engine under bandwidth constraints to track content changes in sources with different observabilities and propose randomized algorithms to optimize a harmonic freshness objective.

II. SETTING

We consider a system of n sensors, each communicating with a central monitoring station via a shared unreliable communication channel. Each sensor measures a time varying quantity and can push real time updates to the central monitoring station.

A. Communication Model

Time is divided into slots. In a time slot, a maximum of a single sensor can push updates to the monitoring station. Here the sensors are assumed to be active sources and can thus generate updates in every time slot.

B. Channel Model

We model the channels as ON-OFF channels with channel ON probability for channel i denoted as p_i . Here we assume that channel realizations are independent across sensors, and independent and identically distributed (i.i.d.) over time-slots.

¹We use source and sensor interchangeably.

C. Channel State Information (CSI) Models

We consider the following three cases:

- *Systems without CSI*: In this case, the state of the channels for the various sensors (ON/OFF) in a time-slot is not known to the scheduler before making the scheduling decision for that time-slot.
- *Systems with CSI*: In this case, the state of the channels for the various sensors (ON/OFF) in a time-slot is known to the scheduler before making the scheduling decision for that time-slot.
- *Systems with partial CSI*: In this case, the state of the channels for a some (a fixed subset) of sensors is known to the scheduler before making the scheduling decision for that time-slot.

In all three cases, channel statistics are known to the scheduler.

D. Our Metric: Channel-Aware Age of Information (CA-AoI)

Let $X_i(t)$ denote the CA-AoI of sensor i at time t . We denote the channel state at time t with respect to sensor i as $\Lambda_i(t)$, where $\Lambda_i(t) = 1$ means that the channel is ON and $\Lambda_i(t) = 0$ implies that the channel is OFF. We indicate the action taken for sensor i with $a_i(t)$, where $a_i(t) = 1$ means that sensor i is scheduled as time t and $a_i(t) = 0$ otherwise. Given the above definitions, CA-AoI for a sensor evolves as follows:

$$X_i(t+1) = \begin{cases} 0, & \text{if } a_i(t) = 1 \text{ and } \Lambda_i(t) = 1, \\ X_i(t) + 1, & \text{if } a_i(t) = 0 \text{ and } \Lambda_i(t) = 1, \\ X_i(t), & \text{otherwise.} \end{cases}$$

Note that as opposed to the vanilla AoI metric, CA-AoI doesn’t penalize the age of a sensor if the channel is OFF at time t . In other words the age of a sensor is incremented only when the channel is in the ON-state and the sensor isn’t scheduled.

E. Objective

The objective of the scheduling policy π is to minimize the time average of the weighted sum of CA-AoI:

$$\begin{aligned} \text{minimize } J^\pi &= \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}_\pi \left[\sum_{t=1}^T \sum_{i=1}^n w_i X_i(t) \right], \quad (1) \\ \text{subject to } a_i(t) &\in \{0, 1\} \quad \forall i, \sum_{i=1}^n a_i(t) \leq 1. \end{aligned}$$

In (1), w_i denotes the weight assigned to sensor i and \mathbb{E}_π denotes the expected weighted sum of CA-AoIs when a policy π is used.

III. MAIN RESULTS AND DISCUSSION

We discuss our key results here. All proofs are provided in the Appendix of [12]. We first focus on the Whittle index policies followed by randomized scheduling policies.

A. Whittle Index Scheduling Policies

The scheduling problem can be viewed as a Restless Multi Armed Bandit (RMAB) with the sensors as arms whose age evolve even when they are not scheduled for transmission. Solving the RMAB problem is P-SPACE hard. However relaxing the constraints on the number of arms that can be played in every time slot and instead placing a constraint on its expectation, makes this problem tractable. The relaxed problem helps decouple the problem into n -subproblems, one for each sensor and the solution of this problem is called the Whittle index policy. Hence each subproblem consists of a single sensor, a single channel and a playing charge c (Lagrange multiplier) for the arm. Let $c(s(t), a(t))$ denote the cost incurred by action $a(t)$ in state $s(t)$. A policy $\pi = \{a(1), a(2) \dots\}$ decides whether to idle the arm or play the arm at each time step. A policy is cost-optimal if it minimizes the average cost, defined as follows:

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E}_{\pi} [c(s(t), a(t)) | s(0) = 0].$$

Definition 1. (Indexability) Given a playing charge c , let $S(c)$ be the set of states for which the optimal policy is to idle. Then the subproblem is called indexable if $S(c)$ monotonically increases from the empty state to the entire state space as c increase from $-\infty$ to ∞ .

Definition 2. (Whittle index) The Whittle index of a state is the playing charge that makes it equally lucrative to play or idle the arm.

We now focus on three cases, namely, systems without CSI, systems with CSI, and systems with partial CSI, separately.

1) *System without CSI:* Here the state of the channel is unknown to the scheduler and can be determined only after sending an information packet. We first formulate the decoupled subproblem. We drop the sensor index i for further analysis as each sub-problem deals with a single sensor.

Decoupled subproblem:

States: The CA-AoI ($X(t)$) of the source defines its state $s(t)$

Actions: The actions $\in \{0, 1\}$ are denoted by $a(t)$, where $a(t) = 1$ indicates that sensor is scheduled to update the central station and $a(t) = 0$ signifies that the sensor is kept idle.

Transition probabilities: The transition probability to state s' after taking action a from state $s = x$ is:

$$\begin{aligned} \mathbb{P}(s' = x | a = 1, s = x) &= 1 - p \\ \mathbb{P}(s' = 0 | a = 1, s = x) &= p \\ \mathbb{P}(s' = x | a = 0, s = x) &= 1 - p \\ \mathbb{P}(s' = x + 1 | a = 0, s = x) &= p. \end{aligned}$$

Cost: Let $\Lambda(t) \in \{0, 1\}$ denote the channel state. Then the cost incurred by choosing action a at state $s = x$ is:

$$c(s, a) = w[x(1 - a\Lambda) + \Lambda(1 - a)] + ca.$$

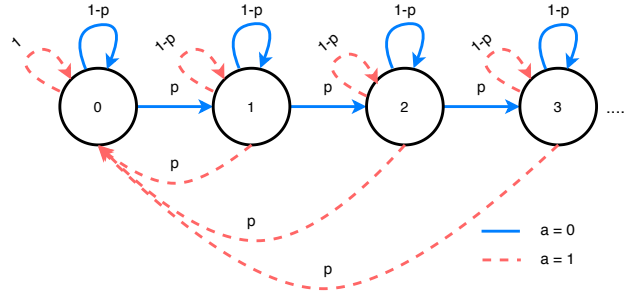


Fig. 1. MDP for subproblems in a system without CSI. (Best viewed in color)

The above cost function can be simply deduced as the sum of the post-action age and the charge incurred when playing an arm.

The Markov Decision Process (MDP) for this subproblem has been shown in Fig. 1. Our first result shows that systems without CSI are indexable and characterizes the Whittle index in closed form.

Theorem 1. The scheduling problem for a system without CSI is indexable and the Whittle index of a sensor in state x is given by:

$$\mathcal{W}(x, p) = \frac{w(x+1)(x+2)}{2(2-p)}.$$

We use this result to design a Whittle index based scheduling policy which is to schedule the sensor with the highest Whittle Index in each time-slot. See Appendix A of [12] for the proof.

2) *System with CSI:* In this case the scheduler knows the states of the channel before scheduling. We begin by describing the subproblem for this case:

Decoupled subproblem:

States: The state $s(t)$ of the arm is defined by the tuple $(X(t), \Lambda(t))$

Actions: $a(t) = 1$ indicates that sensor is scheduled to update the central station and $a(t) = 0$ signifies that the sensor is kept idle.

Transition probabilities: The transition probability to state s' after taking action a from state $s = x$ is:

$$\begin{aligned} \mathbb{P}(s' = (x, 0) | a = 0, s = (x, 0)) &= 1 - p \\ \mathbb{P}(s' = (x, 1) | a = 0, s = (x, 0)) &= p \\ \mathbb{P}(s' = (x, 0) | a = 1, s = (x, 0)) &= 1 - p \\ \mathbb{P}(s' = (x, 1) | a = 1, s = (x, 0)) &= p \\ \mathbb{P}(s' = (x+1, 0) | a = 0, s = (x, 1)) &= 1 - p \\ \mathbb{P}(s' = (x+1, 1) | a = 0, s = (x, 1)) &= p \\ \mathbb{P}(s' = (0, 0) | a = 1, s = (x, 1)) &= 1 - p \\ \mathbb{P}(s' = (0, 1) | a = 1, s = (x, 1)) &= p. \end{aligned}$$

Cost: The cost incurred by choosing action a at state $s = x$ is:

$$c(s, a) = w[x(1 - a\Lambda) + \Lambda(1 - a)] + ca.$$

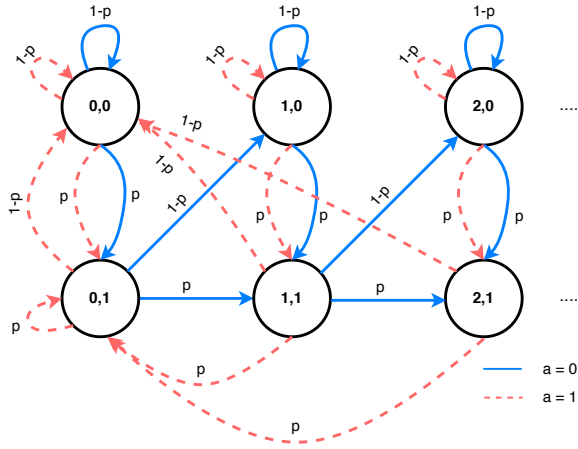


Fig. 2. MDP for subproblems in system with CSI (Best viewed in color)

Our next result shows that systems with CSI are indexable and characterizes the Whittle index in closed form.

Theorem 2. *The scheduling problem for a system with CSI is indexable and the Whittle index of a sensor in state (x, Λ) is given by:*

$$\mathcal{W}(x, p) = \begin{cases} \frac{w(x+1)(x+2)}{2} & \text{if } \Lambda = 1, \\ 0 & \text{otherwise.} \end{cases}$$

We use this result to design a Whittle index based scheduling policy which is to schedule the sensor with the highest Whittle Index in each time-slot. Note that the Whittle index gives us an online algorithm because it is independent of the channel statistics. See Appendix B of [12] for the proof.

3) *System with partial CSI:* We now consider the case for which CSI is available only for a subset of the sensors. We refer to this setting as systems with partial CSI. As in the previous two cases, the scheduling problem reduces to n subproblems and the previous results hold true for this case as well. Let n^- and n^+ be the set of channels without CSI and with CSI respectively. Our next result shows that systems with partial CSI are indexable and characterizes the Whittle index in closed form. This result follows from Theorems 1 and 2.

Theorem 3. *The scheduling problem for a system having partial CSI is indexable and the Whittle index of a sensor in state x is given by:*

$$\mathcal{W}(x_i, p_i) = \begin{cases} \frac{w_i(x_i+1)(x_i+2)}{2(2-p_i)} & \text{if } i \in n^-, \\ \frac{w_i(x_i+1)(x_i+2)}{2} & \text{if } \Lambda_i = 1, i \in n^+, \\ 0 & \text{otherwise.} \end{cases}$$

B. Randomized Scheduling Policies

We refer to a policy as a randomized scheduling policy if it specifies a probability distribution on the set of scheduling

decisions at every time slot. To characterize the optimal² randomized scheduling policy for each of the three cases, we first find closed form expressions for the objective under a class of randomized policies and give algorithms to find policy parameters that optimize the objective.

1) *System without CSI:* The randomized policy in this case is to schedule source i according to a Bernoulli process with parameter Δ_i . Let n^- denote the set of all sensors without CSI and $\vec{\Delta}$ the vector of parameters Δ_i . Then the policy is:

$$\pi^- = \{\text{Sched}(i) \sim \text{Bernoulli}(\Delta_i) \quad \forall i \in n^- | 1 \geq \Delta_i > 0\}.$$

To optimize the parameters $\vec{\Delta}$ of the policy we relax the constraints on π^- from $\sum_{i=1}^n a_i(t) = 1$ to $\mathbb{E}[\sum_{i=1}^n a_i(t)] = 1$. Our next result characterizes the performance of a randomized policy as a function of the parameters $\vec{\Delta}$.

Theorem 4. *Without loss of generality assume that $\vec{p} > 0$. Then under the policy π^- , the freshness objective J^{π^-} as defined in Eq. 1 is*

$$J^{\pi^-} = \sum_{i \in n^-} \frac{w_i(1 - \Delta_i)}{\Delta_i}. \quad (2)$$

Check Appendix C of [12] for the proof. Next, we use this result to design an optimization problem for finding the optimal parameters $\vec{\Delta}$.

Problem 1: Finding optimal $\pi^ \in \pi^-$ – Finding the best policy reduces to the following convex optimization problem.*

$$\begin{aligned} \text{minimize } J^{\pi^-} &= \sum_{i \in n^-} \frac{w_i(1 - \Delta_i)}{\Delta_i}, \\ \text{subject to } &\sum_{i \in n^-} \Delta_i = 1, 1 \geq \Delta_i > 0 \quad \forall i \in n^-. \end{aligned}$$

Note that the first constraint comes from the relaxation of the scheduling problem constraints. Using the Lagrange multiplier method, finding the optimal policy reduces to solving the following system of equations:

$$\Delta_i = \sqrt{\frac{w_i}{\lambda}} \quad \forall i \in n^-, \quad (3)$$

$$\sum_{i \in n^-} \Delta_i = 1. \quad (4)$$

Note that an exact solution to $\vec{\Delta}$ can be obtained from the above system of equations. The optimization procedure is given in Algorithm 1.

See Appendix D of [12] for more details. Since the policy found through Algorithm 1 is optimal for the relaxed constraint scheduling problem ($\mathbb{E}[\sum_{i=1}^n a_i(t)] = 1$) we need to make some implementational modification for the original problem ($\sum_{i=1}^n a_i(t) = 1$). When more than one sensors are to be scheduled according to π^* we schedule the sensors greedily i.e. the sensor in the scheduling pool having the highest weighted age is scheduled.

²Optimal among policies belonging to the given class of randomized scheduling policies.

Algorithm 1: Finding $\pi^* \in \pi^-$ (Problem 1)

input : p_i - Channel ON probability $\forall i \in n^-$
 w_i - Weight of sensor $i \forall i \in n^-$
output: $\vec{\Delta}^*$
 $\sqrt{\lambda^*} \leftarrow \sum_{i \in n^-} \sqrt{w_i}$;
for $i \in n^-$ **do**
 $\Delta_i^* \leftarrow \frac{\sqrt{w_i}}{\sqrt{\lambda^*}}$
return $\vec{\Delta}^*$

2) *System with CSI:* For systems with CSI, the scheduling problem is again non-trivial since only one sensor can be scheduled in every time slot. Hence we schedule each sensor for only a fraction of the times for which the channel is in the ON-state. Let n^+ denote the set of sensors with CSI and $\vec{\alpha}$ denote the vector of parameters α_i . The policy is:

$$\pi^+ = \{\text{Schedule sensor } i \text{ with probability } \alpha_i \\ \text{if channel state} = \text{ON } \forall i \in n^+ | 1 \geq \alpha_i > 0.\}$$

Like the previous case we find an optimal policy for relaxed constraints. Our next result characterizes the performance of a randomized policy as a function of the parameters $\vec{\alpha}$.

Theorem 5. *Without loss of generality assume that $\vec{p} > 0$. Then under the policy π^+ , the freshness objective J^{π^+} as defined in Eq. 1 is*

$$J^{\pi^+} = \sum_{i \in n^+} \frac{w_i(1 - \alpha_i)}{\alpha_i}. \quad (5)$$

Check Appendix E of [12] for the proof. Note that sensor i has channel ON probability p_i and is scheduled with probability α_i when channel state is ON. Thus the average rate at which it is scheduled is $p_i \alpha_i$. This problem can also be setup as an optimization problem.

Problem 2: Finding optimal $\pi^ \in \pi^+$* – Finding the optimal policy can be reduced to the following convex optimization problem.

$$\text{minimize } J^{\pi^+} = \sum_{i \in n^+} \frac{w_i(1 - \alpha_i)}{\alpha_i}$$

$$\text{Subject to } \sum_{i \in n^+} p_i \alpha_i = 1, 1 \geq \alpha_i > 0 \quad \forall i \in n^+.$$

These constraints are derived from the relaxation of the constraints of the original scheduling problem. We ignore the $\alpha_i \leq 1$ constraint and optimize this using the Lagrange multiplier method. We describe how we account for this omitted constraint ahead. The optimal solution is obtained by solving the following system of equations:

$$\alpha_i = \sqrt{\frac{w_i}{p_i \lambda}} \quad \forall i \in n^+, \quad (6)$$

$$\sum_{i \in n^+} p_i \alpha_i = 1. \quad (7)$$

An exact solution of the above set of equations can be found. Algorithm 2 describes the optimization procedure.

In Algorithm 2, $\sqrt{\lambda^*}$ is obtained by solving the following set

Algorithm 2: Finding $\pi^* \in \pi^+$ (Problem 2)

input : p_i - Channel ON probability $\forall i \in n^+$
 w_i - Weight of sensor $i \forall i \in n^+$
output: $\vec{\alpha}^*$
 $R \leftarrow 1$;
 $\text{ActiveSet} \leftarrow n^+$;
while $\text{ActiveSet} \neq \phi$ **do**
 violation \leftarrow False ;
 $\sqrt{\lambda^*} \leftarrow \sum_{i \in \text{ActiveSet}} \frac{\sqrt{w_i p_i}}{R}$;
 for $i \in \text{ActiveSet}$ **do**
 $\alpha_i^* = \sqrt{\frac{w_i}{p_i \lambda^*}}$;
 if $\alpha_i^* \geq 1$ **then**
 $\alpha_i^* = 1$;
 violation = True ;
 $R = R - p_i$;
 $\text{ActiveSet} = \text{ActiveSet} \setminus \{i\}$;
 if violation == False **then**
 Break
return $\vec{\alpha}^*$

of equations:

$$\alpha_i = \sqrt{\frac{w_i}{p_i \lambda^*}} \quad \forall i \in \text{ActiveSet} \quad (8)$$

$$\sum_{i \in \text{ActiveSet}} \alpha_i p_i = R. \quad (9)$$

In Algorithm 2, we initialize the active set with n^+ . If any policy parameter exceeds the inequality constraints i.e. $\alpha_i^* \geq 1$ then we set $\alpha_i^* = 1$, remove it from the active set and modify the constraints. This loop is run until the active set is empty or all policy parameters satisfy the inequality constraints. Like the previous case, when more than one sensors are to be scheduled according to the policy, we schedule them greedily. See Appendix F of [12] for more details.

3) *System with partial CSI:* We combine the policies derived for systems with and without CSI dealt with in the previous sections to design a scheduling policy for the system having partial CSI. Let n^- and n^+ denote the set of channels without CSI and with CSI respectively. Like the previous cases we relax the constraints and make implementational modifications to account for them. The policy in this case is:

$$\pi = \begin{cases} \{\text{Schedule}(i) \sim \text{Bernoulli}(\Delta_i) \quad \forall i \in n^- | 1 \geq \Delta_i > 0\}, \\ \{\text{Schedule}(i) \text{ with probability } \alpha_i \\ \text{if channel state} = \text{ON } \forall i \in n^+ | 1 \geq \alpha_i > 0\}. \end{cases}$$

Our next result characterizes the performance of a randomized policy as a function of the parameters $\vec{\Delta}$ and $\vec{\alpha}$.

Theorem 6. Without loss of generality assume that $\vec{p} > 0$. Then under the policy π , the freshness objective J^π as defined in Eq. 1 is

$$J^\pi = \sum_{i \in n^-} \frac{w_i(1 - \Delta_i)}{\Delta_i} + \sum_{i \in n^+} \frac{w_i(1 - \alpha_i)}{\alpha_i}. \quad (10)$$

This result is a straightforward extension of Theorems 4 and 5. Hence the scheduling problem can be setup as an optimization problem as follows:

Problem 3: Finding optimal $\pi^ \in \pi$* – Finding the optimal policy can be reduced to the following convex optimization problem.

$$\begin{aligned} \text{minimize } J^\pi &= \sum_{i \in n^-} \frac{w_i(1 - \Delta_i)}{\Delta_i} + \sum_{i \in n^+} \frac{w_i(1 - \alpha_i)}{\alpha_i}, \\ \text{subject to } &\sum_{i \in n^-} \Delta_i + \sum_{i \in n^+} p_i \alpha_i = 1, \\ &1 \geq \alpha_i > 0 \quad \forall i \in n^+, 1 \geq \Delta_i > 0 \quad \forall i \in n^-. \end{aligned}$$

We ignore the inequality constraints for α_i and use the Lagrange multiplier method to reduce the optimization problem to solving the following set of equations:

$$\Delta_i = \sqrt{\frac{w_i}{\lambda}} \quad \forall i \in n^-, \quad (11)$$

$$\alpha_i = \sqrt{\frac{w_i}{p_i \lambda}} \quad \forall i \in n^+, \quad (12)$$

$$\sum_{i \in n^-} \Delta_i + \sum_{i \in n^+} p_i \alpha_i = 1. \quad (13)$$

An exact solution to the above system of equations can be found and the optimization procedure is described in Algorithm 3.

In Algorithm 3 $\sqrt{\lambda^*}$ is obtained by solving the following set of equations:

$$\Delta_i = \sqrt{\frac{w_i}{\lambda^*}} \quad \forall i \in n^-, \quad (14)$$

$$\alpha_i = \sqrt{\frac{w_i}{p_i \lambda^*}} \quad \forall i \in \text{ActiveSet}, \quad (15)$$

$$\sum_{i \in n^-} \Delta_i + \sum_{i \in \text{ActiveSet}} p_i \alpha_i = R. \quad (16)$$

It is straightforward to check that Algorithm 3 utilizes the optimization procedure formulated in Algorithms 1 and 2. When more than one sensors are to be scheduled according to the policy, we schedule the sensors in a greedy manner.

IV. SIMULATION RESULTS

In this section we compare the performance of our policies against the greedy policy via simulations. We normalize the weights in all experiments so that they sum to unity. The greedy policy for the system without CSI is to schedule the sensor having the highest weighted age times the channel ON probability in that time slot, i.e., $\text{argmax}_i (w_i X_i(t) p_i)$. In Fig. 3 we simulate a system without CSI consisting of three sensors. We set $p_1 = 0.1$, $p_2 = 0.9$ and vary the

Algorithm 3: Finding $\pi^* \in \pi$ (Problem 3)

input : p_i - Channel ON probability $\forall i \in n$
 w_i - Weight of sensor $i \forall i \in n$
output: $\vec{\Delta}^*, \vec{\alpha}^*$
 $R \leftarrow 1$;
ActiveSet $\leftarrow n^+$;
while **ActiveSet** $\neq \phi$ **do**
 violation \leftarrow **False**;
 $\sqrt{\lambda^*} \leftarrow \sum_{i \in n^-} \frac{\sqrt{w_i}}{R} + \sum_{i \in \text{ActiveSet}} \frac{\sqrt{w_i p_i}}{R}$;
 for $i \in \text{ActiveSet}$ **do**
 $\alpha_i^* = \sqrt{\frac{w_i}{p_i \lambda^*}}$;
 if $\alpha_i^* \geq 1$ **then**
 $\alpha_i^* = 1$;
 violation = **True**;
 $R = R - p_i$;
 ActiveSet = **ActiveSet** $\setminus \{i\}$
 if **violation** == **False** **then**
 | **Break**
 for $i \in n^-$ **do**
 | $\Delta_i^* \leftarrow \sqrt{\frac{w_i}{\lambda^*}}$
return $\vec{\alpha}^*, \vec{\Delta}^*$

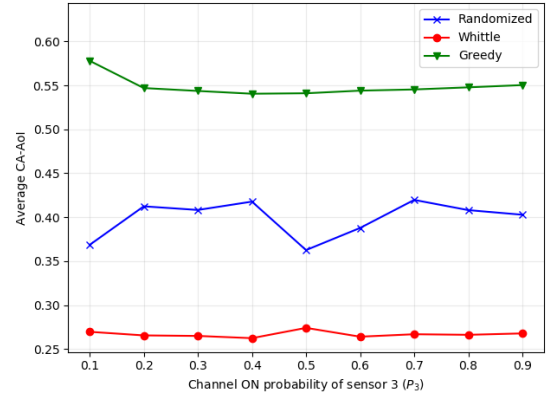


Fig. 3. Average sum of weighted age on changing channel ON probability of sensor 3 (P_3) in a system without CSI

channel ON probability of sensor 3 (p_3) from 0 to 1. The corresponding weights are set as $w_1 = 1$, $w_2 = 1$, $w_3 = 100$. We give sensor 3 a considerably larger weight to clearly observe the effect of changing its channel ON probability. The Whittle's index policy outperforms the other two policies in this system. In Fig. 4 we test our policies on a system without CSI consisting of two sensors where an important sensor has a poor connection. We fix $p_1 = 0.1$ and vary p_2 between 0 to 1. The weights are set as $w_1 = 1000$, $w_2 = 1$. The Whittle's index policy and randomized policy outperform the greedy policy by a considerable margin in this system. In Fig. 5 we investigate the effect of the size of the system on the performance of the policies. We vary the number of sensors from 10 to 40. In each system the channel statistics

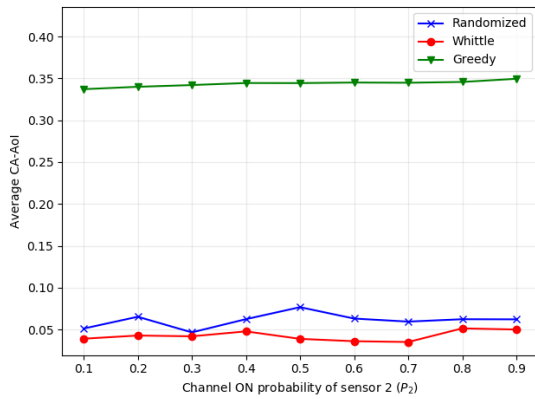


Fig. 4. Average sum of weighted age on changing channel ON probability of sensor 2 (P_2) in a system without CSI

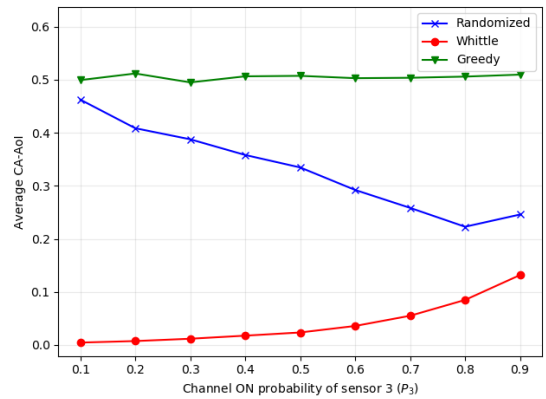


Fig. 6. Average sum of weighted age on changing channel ON probability of sensor 3 (P_3) in a system with CSI

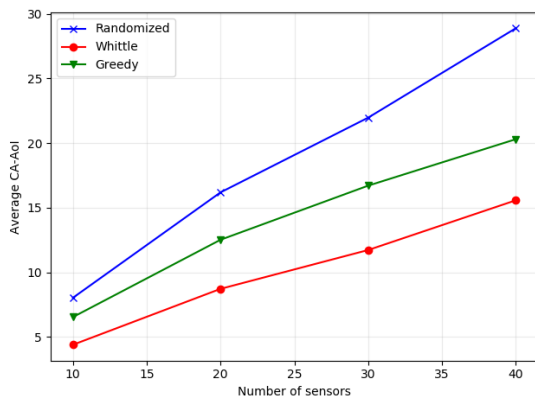


Fig. 5. Average sum of weighted age on varying number of sensors in a system without CSI

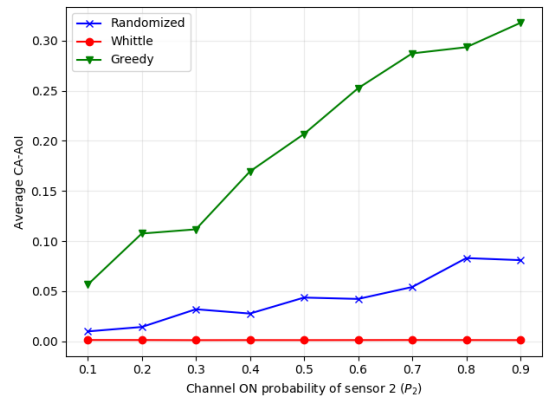


Fig. 7. Average sum of weighted age on changing channel ON probability of sensor 2 (P_2) in a system with CSI

are chosen uniformly from 0 to 1. Similarly the weights are chosen uniformly at random from 1 to 100. The Whittle index policy outperforms both the policies.

We run similar experiments for a that system with CSI. The greedy policy for a system having CSI is to schedule the sensor having the highest weighted age i.e. $\text{argmax}_i (w_i X_i(t))$. Fig. 6 shows the simulation of a system with CSI having 3 sensors. The parameters of the system remain the same. Once again the Whittle index policy performs the best and the randomized policy attains similar performance at higher channel ON probabilities. In Fig. 6 we simulate our policies in a system with CSI having important sensors transmitting through poor channels. The parameters remain the same as before. The Whittle index policy and the randomized policy perform substantially better than the greedy policy. We simulate the effect of varying system size on the performance of the policies in Fig. 8. The greedy policy and the Whittle index policy yield good performance.

In Fig. 9 we simulate a system having partial CSI. The system has 2 sensors with CSI and 2 sensors without CSI. The channel statistics of the sensors without CSI are $p_1^- = 0.1$, $p_2^- = 0.9$. The channel ON probability of one of the

sensors with CSI is set to $p_1^+ = 0.1$ and the probability of the other sensor is varied from 0 to 1. The weights are setup as $w_1^- = w_2^- = w_1^+ = 1$, $w_2^+ = 100$. The Whittle index policy performs considerably better than the other policies. In Fig. 10 we simulate a similar system albeit with parameters between sensors with and without CSI exchanged. The channel ON probabilities are $p_1^- = p_1^+ = 0.1$, $p_2^+ = 0.9$ and p_2^- is varied from 0 to 1. The weights are $w_1^+ = w_2^+ = w_1^- = 1$, $w_2^- = 100$. Once again the Whittle index policy outperforms the others.

From these results, and others which we omit due to lack of space, we conclude that the randomized scheduling policies work well when important sensors have poor channel communication. However the Whittle index policy is effective across all types of system settings and achieves minimum cost. It outperforms the randomized policy by a substantial margin in larger systems and systems that have better connections.

V. CONCLUSIONS

We propose a variant of the AoI metric called Channel-Aware Age of Information (CA-AoI). The CA-AoI of a source at the intended destination is defined as the number of time-slots elapsed since the recent most update from the source was received in which the channel conditions were good enough

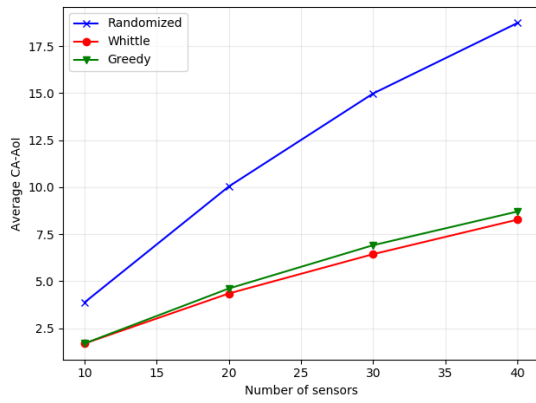


Fig. 8. Average sum of weighted age on varying number of sensors in a system with CSI

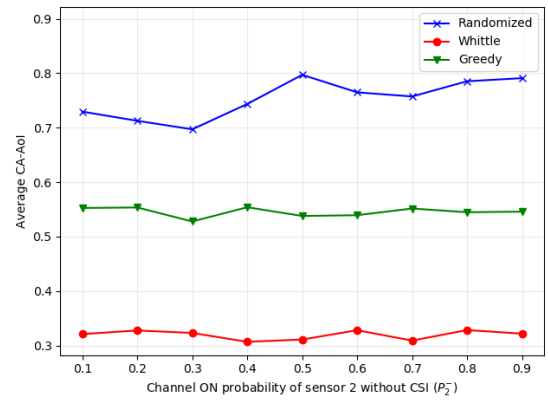


Fig. 10. Average sum of weighted age on varying channel ON probability (p_2^-) of sensor 2 without CSI in a partial CSI system

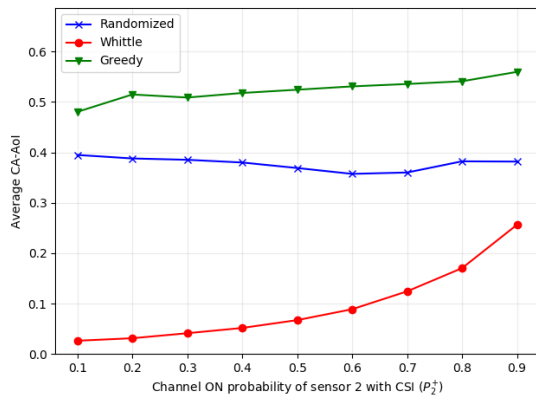


Fig. 9. Average sum of weighted age on varying channel ON probability (p_2^+) of sensor 2 with CSI in a partial CSI system

for the source to send an update. The CA-AoI of a sensor is therefore, a measure of the number of missed opportunities to send an update since its recent most successful update. We focus on a multi-source system that updates a monitoring station through a shared unreliable channel under different CSI models. We model our scheduling problem as a Restless Multi Armed Bandit (RMAB) problem and prove indexability under all the CSI models. We show the threshold type structure of the optimal policy and derive the Whittle index for the problems. Besides this we also design randomized scheduling policies and show that finding the optimal parameters reduces to simple convex optimization procedures. Via extensive simulations we show that the proposed policies outperform the greedy policies in several settings. It turns out that the Whittle index policy is the most effective across all system settings. An interesting future extension to our work can be designing online scheduling algorithms when the channel statistics of the system are unknown.

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