

# Coalgebras and Higher-Order Computation: a GoI Approach\*

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## Abstract

Girard's *geometry of interaction (GoI)* [3] can be seen – in one practical aspect of it – as a compositional compilation method from functional programs to sequential machines (see e.g. [8, 2]). There tokens move around and express interactions between (parts of) programs. Intrigued by the combination of abstract structures and concrete dynamics in GoI, our line of work [4, 5, 10, 11, 6, 9] has aimed at exploiting, in GoI, results from the theory of *coalgebra* – a categorical abstraction of state-based transition systems that has found its use principally in concurrency theory. Such reinforced connection between higher-order computation and state-based dynamics is made possible thanks to an elegant categorical axiomatization of GoI by Abramsky, Haghverdi and Scott [1], where *traced monoidal categories* [7] are identified to be the essential structure behind. In the talk I shall lay out these basic ideas, together with some of our results on: GoI semantics for a *quantum programming language* [4, 5]; and our “memoryful” extension of GoI [10, 11, 6, 9] with *algebraic effects* [12].

The talk is based on my joint work with my colleague Naohiko Hoshino (RIMS, Kyoto University) and my (former) students Koko Muroya (University of Birmingham) and Toshiki Kataoka (University of Tokyo), to whom I owe special thanks.

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